Do investors gain from forecasting the asymmetric return comovements of financial and real assets?

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Do Investors Gain from Forecasting the Asymmetric Return Comovements of Financial and Real Assets?

Abstract

Recent research on asset allocation emphasizes the importance of considering non-traditional asset classes such as commodities and real estate—the former for their diversification properties, and the latter due to its importance in the average investor's portfolio. However, modelling and forecasting asset return comovements is challenging because the dependence structure is dynamic, regime-specific, and non-elliptical. Moreover, little is known about the economic source of this time-varying dependence or how to use this information to improve investor portfolios. We use a flexible framework to assess the economic value to investors of incorporating better forecasting information about return comovements between equities, bonds, commodities, and real estate. The dependence structure is allowed to be dynamic and non-elliptical, while the state variables follow Markov-switching stochastic volatility processes. We find that the predictability of return comovements is significantly improved by incorporating macro and non-macroeconomic variables, in particular inflation uncertainty and bond illiquidity. The economic value added to investors is significant across levels of risk aversion, and the model outperforms traditional multivariate GARCH frameworks.

Keywords: Forecast, Comovement, Asset allocation, Portfolio optimisation, Economic value added, Volatility, Copula, Markov models, Macroeconomic variables, Regimes.

1. Introduction

Classical portfolio theory assumes that asset returns follow a multivariate normal distribution, and further that the dependence structure between assets can be fully characterised by the variance-covariance matrix of returns, i.e. dependence is symmetric. However, industry practitioners and researchers have reported that asset return dependence tends to increase significantly during bear markets and, in particular, during financial crises such as in 2008-09 (e.g., Aloui et al., 2011). If asset return comovements are asymmetric, models based on mean, variance and covariance may be inadequate for estimating portfolio risk and determining optimal asset allocations. Yet measuring, forecasting and explaining asymmetric return comovements remains a challenge in the asset allocation literature.

This paper's contribution is to link multivariate return comovements to macroeconomic and non-macroeconomic factors, across four asset classes (equities, bonds, commodities, and real estate). We use a flexible framework to show how these factors drive the dynamic dependence structure of return comovements. We further show how this information helps forecast return comovements, and we assess the economic significance of the improvements for typical investors. Specifically, we aim to answer the following questions: 1) How does the economic market environment affect the joint dependence structure of return comovements between financial and real assets? 2) What factors explain a potentially non-elliptical joint return distribution in a multi-asset portfolio? 3) What is the economic value to investors that is added by improving forecasts of comovements using (2)? These questions have significant implications for asset allocation decisions.

Our findings are as follows. First, the expansion regime of the dependence structure lasts longer than the contraction regime, and return comovements are less volatile during expansions than contractions. Second, inflation and the risk-free rate are significant drivers of return comovements during expansion regimes. However, during contraction regimes inflation and risk aversion are significant. Indeed, during economic contractions the intensity of return comovements increases with risk aversion. Furthermore, we find evidence that non-macroeconomic variables matter. In particular, inflation uncertainty and bond illiquidity are significant, but their effect differs according to the economic regime. Third, we find that a Markov-switching stochastic volatility (MSSV) model outperforms non-switching models to forecast return comovements across four asset classes. Fourth, our results suggest that investors benefit economically from incorporating the information from macro and non-macro risk factors in their asset allocation decisions. This result is established by measuring Expected Utility certainty-equivalence (Willingness-to-Pay) for a range of reasonable risk aversion levels. Fifth, we provide robustness checks using quantile regressions which support our main findings.

Thus, our paper fills a specific gap in the literature on return comovements across asset classes and their economic implications for asset allocation. Indeed, to our knowledge this is the first study to show in a unified setting: (i) the dynamic structure of return comovements across asset classes, using a flexible non-elliptical framework, (ii) the macro and non-macroeconomic determinants of return comovements across economic regimes, and (iii) how investors with different risk appetites could benefit from such information.

The remainder of the paper is organized as follows: We begin by reviewing some related research in Section 2. Section 3 presents the theoretical framework and describes the empirical methodology. Section 4 describes the sources of data, and presents and discusses the empirical findings. Section 5 reports the robustness checks and specification tests, and lastly Section 6 concludes the paper. Appendices A-C present additional information regarding the data and estimation methods.

2. Literature Review

The existing literature on forecasting and explaining asset return comovements has mainly focused on modelling equities and bonds (e.g. Baele et al., 2010; Wu and Lin, 2014). However, expanding the set of asset classes to include real assets such as commodities and real estate is highly relevant. First, this is because commodities and, more generally, alternative investments may provide useful diversification (Bodie and Rosansky, 1980; Erb and Harvey, 2006; Gorton and Rouwenhorst, 2006; Hillier, Draper and Faff, 2006).[1] Indeed, over the past two decades there has been substantial growth in the importance of real assets and other alternative investments in portfolio allocation. For example, investment in commodity-based traded products has reached $125.8 billion (Carpenter, 2010). Second, real estate represents, for the middle-class investor, the largest share of his or her assets (Campbell, 2006; Guiso and Sodini, 2013). Indeed, the average US household holds 40-45% of their wealth in real estate (Tracy, Schneider, and Chan, 1999; Tracy and Schneider, 2001). Thus, a better understanding of the portfolio implications of asset comovements is valuable for

[1] However, Daskalaki and Skiadopoulos (2011) argue, on the basis of spanning tests, that the diversification benefits of commodities may have been overstated.

middle-class investors, who may over-invest in real estate, as well as wealthy investors, who seek to diversify beyond stocks and bonds (Campbell, 2006).

The theory for asymmetric return comovements has evolved across different literatures and has been studied for different asset classes (equities, bonds, etc.). The basic theory concerns return comovements and the potential for diversification within and across asset classes (e.g. Karolyi and Stulz, 1996). If comovements increase, diversification potential decreases. Return comovements are said to be asymmetric when they are stronger for downside moves than upside moves. The asset pricing interpretation is that a greater proportion of risk in each asset class is systematic or factor-based (with common factors that affect all asset classes) and a smaller proportion of risk is idiosyncratic and diversifiable. Indeed, Karolyi and Stulz (1996) argue that a significant gap exists between international finance theory and financial risk management practice (e.g., RiskMetrics). Accounting for comovements is important because they affect risk premia and thus the pricing of financial or real assets. This reasoning can be extended from covariances to higher-order moments if we allow a departure from multivariate normality. Then, asymmetric return dependence can be interpreted as the risk of financial contagion across asset classes or countries or both (e.g. Piplack and Straetmans, 2010). Lower tail dependence is of particular interest because investors are most concerned about diversification failing, and losses compounding, during crisis periods.

For equities, the motivation starts with developments in two literatures, namely on conditional volatility and covariance, and on asset allocation. Kroner and Ng (1998) argue that the literature on multivariate GARCH models fails to account for asymmetry in the dependence structure. Thus, stylized facts such as Black’s leverage effect cannot be captured. Their goal is to show how to model time-varying covariances that are more consistent with financial data. Moreover, they show that various available multivariate GARCH models impose different restrictions, and that such restrictions may be difficult to justify empirically. Ang and Chen (2002) further show that individual stock return conditional correlations with the market return are sharply asymmetrical. Correlations are higher conditional on downside moves, and clearly non-Gaussian. Ang and Bekaert (2002) extend this empirical analysis to international stocks, and show the importance of regime-specific (bull or bear market) asset allocation strategies. Indeed, a sound understanding of the behavior of asset return comovements is essential for efficient asset allocation decisions. Guidolin and Timmermann (2007) show that since asset return comovements are time-varying, investors require information about the conditional distribution of asset returns to implement dynamic asset allocation strategies. Furthermore, asset return comovements change due to variation in economic conditions and/or changes in non-macroeconomic factors.

Regarding equity-bond comovements, most research confirms that stock and bond return correlations vary inversely with stock market volatility. Connolly et al. (2005 and 2007) relate this result to the 'flight-to-safety' phenomenon. Moreover, real interest rate and inflation volatility have a significant influence on stock-bond return correlations (e.g. d'Addona and Kind, 2006; Boyd et al., 2005 and Andersen et al., 2007). Piplack and Straetmans (2010) provide pairwise analysis of lower tail dependence for equities, bonds, T-bills, and gold. They find evidence of greater co-crash risk for stocks and bonds or T-bills, and for bonds and T-bills. Gold, however, appears to perform as a 'safe haven'.

Models used in the literature often impose time-stability restrictions when examining the dynamics of the dependence structure. These assumptions may not hold in practice, however, as it is well established that the joint distribution of daily asset returns within and across asset classes is not multivariate normal (e.g. Longin and Solnik, 2001; Patton, 2009). Thus, linear correlation may not be an adequate measure of the dependence structure of returns.[2] Moreover, there is evidence showing that the dependence structure is time-varying, in particular during financial crisis periods. For example, Piplack and Straetmans (2010) show that asset return comovements change during periods of market stress. Thus, in constructing an optimal portfolio, one should consider how asset return comovements might change during different economic conditions.

Two further issues arise from the existing empirical evidence. First, research shows that it would be desirable to obtain a more reliable, alternative density specification for a higher-dimensional case (i.e. many asset classes) is desirable (e.g. Creal and Tsay, 2015). Second, a richer framework for the dependence structure would better account for information contained in the distributional tails. In this study, we address both of these issues by employing: i) a dynamic conditional copula model to measure the joint distribution of return comovements, and ii) a Markov-Switching Stochastic Volatility (MSSV) model to examine their determinants.

Much of the research in this general area has focused on international links between equity markets, rather than across asset classes. Rodriguez (2007) uses copulas to characterize and document financial contagion for countries affected by the East Asia and Mexican crises. The results show the dependence structure can break down (through tail risk) even if the evidence suggests no

[2] For problems linked to linear correlation, see e.g. Embrechts et al. (2002).

change in correlation patterns. Li (2013) develops a new, nonparametric approach to test the symmetry of stock market return comovements. This test is applied pairwise to the US market and five other developed countries. Symmetry is rejected in all cases except for US-Japan. Garcia and Tsafack (2011) consider both stocks and bonds for two pairs of country, namely Canada-USA and France-Germany. Their approach allows for two regimes, one where comovements is symmetric and the other where it is not. They find a strong and nonlinear (asymmetric) dependence between the two countries in each pair within an asset class, but not across asset classes. Chollete, De La Pena and Lu (2011) also look at international equity market index returns for a larger set of countries. They find, first, that dependence has increased over time, linked to closer financial integration between countries. They further find asymmetric dependence to be particularly strong in Latin America but less so in the G5 countries or in East Asia. They conclude that the evidence of asymmetric tail dependence in returns data suggests limits to international diversification. Weiß and Scheffer (2015) show how to improve portfolio risk management and reduce Value-at-risk using mixture copulas. Chui and Yang (2012) find that in the US and UK, stock and bond return correlations are exceptionally high when markets are either very bullish or bearish. In Germany, however, bonds offer diversification potential when stock returns fall.

The literature for asset classes other than stocks and bonds is smaller. Liu, Ji and Fan (2017) use half-rotated copulas to capture time-varying nonlinear dependence between equity and commodity returns. Their results confirm asymmetric tail dependence and also that the dependence structure is most affected by crisis events. Ji et al. (2018) show that time-varying copulas help capture the link between BRICS equity market returns and crude oil returns and that risk spills over from oil to equities. Paltalidis and Patsika (2019) examine currency markets and find evidence of tail dependence and stronger links during high-volatility periods, suggesting market contagion risks. Chan et al. (2011) look at different asset classes in the US and characterize, using a Markov-switching model, two regimes – tranquil and crisis. They find evidence of flight-from-quality and contagion between asset classes during the crisis period. Case et al. (2012) show that multivariate GARCH models can help improve portfolio performance by adding and dynamically rebalancing real estate investments.

Concerning asset allocation and forecast performance, Wu and Liang (2011) show that using dynamic copulas allows them to improve stock-bond comovements forecasting, which translates to economically sizable gains to investors who implement such strategies. Wu and Lin (2014) further show that investors have significant gains models when their stock-bond investment strategies are based on a combination of Student-t copulas and GJR-GARCH marginal, compared with passive or multivariate GARCH approaches.

Despite a large literature on this subject in general, a gap remains. Specifically, there is to our knowledge no study which addresses in a single setting the questions raised here. For example, Chan et al. (2011), Case et al. (2012), and Delatte and Lopez (2013) examine the dynamics of correlation between different asset classes. However, they do not explore how different economic regimes affect return comovements, nor do they look at why comovements might be asymmetric. Some recent studies use a copula approach to capture tail dependencies (see e.g., Aloui et al., 2011; Chollete et al., 2011; Weiß and Scheffer, 2015; Wu and Lin, 2014), but these studies examine bivariate return comovements, rather than a wider set of asset classes in a single joint setting. Yang et al. (2010) use a bivariate regime-switching model to characterize coskewness, but only consider stock-bond returns.

3. Model Development and Empirical Methodology

In this section, we present the model and describe the methodology. First, we describe a time-varying dependence structure to capture asset return comovements. Second, we present a conditional volatility framework for the asset returns. Third, we adopt a New Keynesian dynamic framework to link the economic source of return comovements to macro and non-macroeconomic state variables. Lastly, we present the Markov-Switching Stochastic Volatility framework for the state variables.

3.1 Estimating the Dependence Structure of Returns

Recent work in finance has shown the usefulness of multivariate copula approaches to estimate the joint return comovements of assets, e.g. to better describe international diversification (Christoffersen et al., 2012). Using this general approach, we estimate our models using the inversion method by substituting the marginal densities of the asset returns. We characterize a multivariate p-dimensional copula as $X \sim t(0, R)$, where t denotes the CDF of a standard t-distribution, R is the correlation matrix and u is the realization of probability integrals of the asset return marginal distributions, X. The choice of the t-copula is motivated by the existing literature. Although no single copula is ideal in every setting, the t-copula has been found to perform very well. Indeed in this literature on return dependence in different asset classes, the t-copula often performs either best or nearly best among a wide class of copula functions. This is documented for example in Chollete, De La Pena and Lu (2011) and in Wu and Lin (2014). In addition, in their study of a wide range of mixture pair copulas, Weiß and Scheffer (2015) find that "For all fitted Mixture-PCCs, the key component is the Student's t copula." Lastly, in his survey of copula models for economic and financial time series, Patton (2012) writes that on the basis of statistical tests, "the Student's t copula is preferred" although no single copula model is always preferred. Then, the corresponding log likelihood function is

$$L_t(R,d;u_t) = -T\ln\frac{\Gamma\left(\frac{d+p}{2}\right)}{\Gamma\left(\frac{d}{2}\right)} - p\,T\ln\frac{\Gamma\left(\frac{d+1}{2}\right)}{\Gamma\left(\frac{d}{2}\right)} - \left(\frac{d+p}{2}\right)\sum_{t=1}^{T}\left(\ln\left(1+\frac{H_t^T\times R^{-1}\times H_t}{d}\right)\right) - \sum_{t=1}^{T}\ln|R| + \left(\frac{d+1}{2}\right)\sum_{t=1}^{T}\sum_{i=1}^{p}\ln\left(1+\frac{H_{it}^2}{d}\right) \quad (1)$$

For the t-copula we have $H_t = \left(t_d^{-1}(u_{1,t},\dots,u_{p,t})\right)$, which is the vector of transformed standardized residuals, where t^{-1} represents the inverse of the Student's t-distribution, d are the degrees of freedom, and R is the correlation matrix. To accommodate for a potentially time-varying dependence structure corresponding to conditional copulas, we allow the dependence parameter ρ_t to vary according to an ARMA (1, q) process (see Hafner and Manner, 2012), which is defined as

$$\rho_t = \Theta\left(\beta_0 + \beta_1\rho_{t-1} + \beta_2\frac{1}{q}\sum_{i=1}^{q}\varphi^{-1}(u_{t-i}) + \beta_3 D\right) \quad (2)$$

where β_1 is the autoregressive term, and β_2, the average of the sum-product of the transformed variables u and v. The term $\beta_1\rho_{t-1}$ accounts for the persistence effect while the term $\beta_2\frac{1}{q}\sum_{i=1}^{q}\varphi^{-1}(u_{t-i})$ captures the variation effect of the dependence parameter. We add a dummy variable term $\beta_3 D$, which enables us to examine the impact of the 2007-08 financial crisis. The dummy variable takes the value '0' prior to the subprime crisis, July 2007, and thereafter takes the value '1'. Based on the AIC value, we select the lags of the moving average term. However, for forecasting purposes, we do not use the dummy variable in the equation.

Let asset returns be denoted $X_{i,t}$ for asset *i* and period *t*. Then we specify the marginal distribution of the asset returns in eq. (3) using an Autoregressive Moving Average ARMA (*p*, *q*)-EGARCH (1, 1)-*t* process (Nelson, 1991). The model is characterised as

$$X_{i,t} = \theta_i + \sum_{j=1}^{p} \beta_j X_{i,t-j} + \sum_{k=1}^{q} \alpha_k \varepsilon_{i,t-k} + \varepsilon_{i,t} \tag{3}$$

$$\ln(\sigma_t^2) = a_0 + \sum_{j=1}^{p} a_{1j} \ln(\sigma_{t-j}^2) + \sum_{i=1}^{q} a_{2i} \left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| + \sum_{j=1}^{q} a_{3j} \left(\frac{\varepsilon_{t-j}}{\sigma_{t-j}}\right) \tag{4}$$

$$\left(\frac{d}{\sigma_{i,t}^2 (d-2)}\right)^{1/2} . \varepsilon_{i.t} | I_{t-1} \sim \mathrm{i.i.d.}\, t_{di} \tag{5}$$

where θ_i and $\varepsilon_{i,t-1}$ are the conditional mean and error terms, reflecting news relating to lagged volatility, β_j is the autoregressive component, and α_k is the moving average parameter. Further, in eq. (4) σ_{t-j}^2 is the GARCH component, ε_{t-1} is the ARCH component capturing the information content of volatility, and a_3 captures the leverage effect. In eq. (5), the noise process ε_t follows a skewed Student-*t* distribution with (d) degrees of freedom and conditional variance σ_t^2. The information set is the conditioning vector 'k'. The ARMA term order 'p' is determined using the Akaike Information Criteria (AIC).

3.2 Estimating the Macro and Non-Macroeconomic State Variables

As we are interested in the *economic* source of return comovements, we consider a variety of standard macro and non-macroeconomic variables for forecasting purposes. We include three macro-economic factors: the risk-free rate (rf_t), output gap (o_t), and inflation (i_t). These variables are expected to affect asset values through their impact on cash flows and discount rates (d'Addona and Kind, 2006). To capture the impact of long-term interest rates, we include the nominal risk-free rate, expected inflation, and the term premium.

To investigate the influence of the term premium and inflation premium we use two 'economic' risk proxies, i.e. output uncertainty (ou_t) and inflation uncertainty (iu_t). Furthermore, we include a proxy for liquidity, which can affect asset prices in two significant ways. First, in illiquid markets, beta (or multiple beta-factors) may fail to reflect quickly the impact of economic shocks. Second, economic shocks that increase liquidity may have a positive impact on asset returns by lowering liquidity risk or the cost of liquidity. Moreover, monetary policy can affect liquidity by increasing borrowing constraints and/or triggering trading activity, thereby influencing asset return comovements. Prior research is inconclusive about such liquidity effects (Chordia et al., 2005; Goyenko et al., 2009). To address this issue in our model, we consider unconstrained proxies of liquidity shocks. We capture stock market illiquidity (lr_t) using the capitalization-based proportion of zero daily returns across all listed firms in the U.S. market (see Lesmond et al., 1999). For bond market illiquidity (ds_t), we use bid-ask spreads across all securities, i.e. one month, three months, and one, two, three, five, seven, ten, twenty and thirty years of maturity.

Plosser and Rouwenhorst (1994) and Estrella and Hardouvelis (2012) use the term spread (ts_t) as a leading indicator of economic activity. However, recent evidence shows that the spread has become less informative over time. Dotsey (1998) and Henry et al. (2004) show that the relationship between business cycles and economic output behaves asymmetrically. Öcal (2006) provides evidence of an asymmetric relationship between economic output and growth. Therefore, it is reasonable to assume the existence of a non-linear relationship between these variables. We use an alternative measure to capture the different regimes of the business cycle. Specifically, our measure of the modified depth of recession (dr_t) is based on Lee and Wang's (2012) estimate of a business cycle proxy. This measure enables us to study its impact for both recession and expansion regimes.

To identify structural shocks, we split the state variables into two sets: i) "macro variables (mv)", $K_{t,mv} = [rf_t, o_t, i_t, ra_t]'$ and ii) "other variables (ov)", i.e. $K_{t,ov} = [ou_t, iu_t, gilr_t, ds_t, lr_t, vp_t, ts_t, dr_t]'$. The 'other variables' (ov) include the non-macroeconomic variables. To model $K_{t,mv}$ we employ a New Keynesian model, which we discuss below. To identify the $K_{t,ov}$ shocks we characterize a simple empirical model where the 'other variables' *ov* depend on the macro variables mv.

To model $K_{t,mv}$, we formulate the structural model for $X_{t,mv}$ employing New Keynesian dynamics (see Bekaert et al., 2010). The model contains three equations: i) the forward-feeding monetary policy *(MP)* rule (eq. (6)), (ii) the demand *(IS)* equation (7), and iii) the aggregate supply *(AS)* equation (8). Further, eq. (9) describes the risk aversion proxy (*ra*). This approach allows us to capture the potentially time-varying risk aversion dynamics in the structural model.

$$rf_t = \alpha_{MF} + \tau rf_{t-1} + (1-\tau)\left[a\left(S_t^{MP}\right)E_t(i_{t+1}) + b\left(S_t^{MP}\right)o_t\right] + X_t^{rf} \qquad (6)$$

$$o_t = \alpha_{IS} + \omega E_t(o_{t+1}) + (1-\omega)o_{t-1} + \theta ra_t - \varphi\left(rf_t - E_t(i_{t+1})\right) + X_t^o \qquad (7)$$

$$i_t = \alpha_{AS} + \lambda E_t(i_{t+1}) + (1-\lambda)i_{t-1} + \varphi o_t + X_t^i \qquad (8)$$

$$ra_t = \beta_{ra} + \gamma ra_{t-1} + X_t^{ra} \qquad (9)$$

(τ) in eq. (6) represents the forward-looking monetary policy smoothing estimate. Cho and Moreno (2006) show that changes in monetary policy significantly influence macro dynamics and structural shocks. We therefore introduce a standard Markov-chain process that allows the monetary policy to vary across two regimes $\left(S_t^{MP}\right)$, expansion and recession, with constant transition probabilities.

Parameters (ω) and (λ) in eq. (7) and (8) represent the degree of IS and AS forward-looking behaviour, respectively. (φ) estimates the impact of the real interest rate on the output gap, and (ϕ) the effect of the output gap on inflation. A high positive value of φ and ϕ will indicate that monetary transmission mechanism has a significant influence on the economy's output and inflation. The state variable (ra_t) accommodates stochastic risk aversion in the demand equation of the New-Keynesian model, nesting Campbell and Cochrane's (1995) external habit model. In particular, (ra_t) represents the local curvature of the utility function. (θ) measures the counteracting effect of consumption smoothing and precautionary savings of risk aversion on the real economy. For the non-macroeconomic variables, we characterize the structural model as

$$K_{t,ov} = \alpha_{ov}(S_t) + \beta_{ov}K_{t-1,ov} + \sum\nolimits_{ov}^{mv} K_{t,mv} + X_{t,ov} \qquad (10)$$

where S_t represents the set of regime variables that drive the coefficient matrices. $K_{t,ov}$ is modelled based on Hamilton's (1989) specifications. β_{ov} is a diagonal matrix, Σ_{ov}^{mv} is a 7×4 matrix, which appropriates contemporaneous covariance with the macro variables $K_{t,mv}$, and $X_{t,mv}$ is the vector of uncorrelated structural shocks of the "other variables". In eq. (10), the non-macroeconomic factors may partially exhibit autoregressive dynamics of the macroeconomic state variables. Moreover, $X_{t,ov}$ represents shocks related to non-macroeconomic variables. Finally, allowing the drifts to depend on the regime variable S_t enables us to model the structural changes in the liquidity parameters (Hasbrouck, 2009).

3.3 The Dynamic Dependence Structure Model

To investigate the dynamics of the dependence structures, we employ a Markov-Switching framework, according to which each state variable follows an evolutionary process. Although autoregressive conditional heteroskedasticity (ARCH) models can be used for this purpose (Bollerslev et al., 1988; Engle, 1982), the Normally and Independently Distributed (NID) assumption of the error term is often violated in practice. We therefore specify a model for the state variables that allows each of the vectors to follow an Independent Stochastic Volatility (ISV) process. The combination of the Markov-switching and stochastic volatility frameworks has been particularly successful in the interest rate modelling literature (e.g. Smith, 2002; Maheu and Yang, 2016). The Stochastic Volatility (SV) specification builds in a time-varying variance process for each of the elements of the structural factors by allowing the variance to be a latent process. We specify the MS model as follows, first defining the dependence structure $\left(y_t\right)$ as

$$y_t = \sum_{l=1}^{L} \varphi_l S_t x_{l,t}^S + \varepsilon_t \tag{11}$$

where L denotes the number of switching coefficients, $x_{l,t}$ represents the explanatory state variables, S_t represents the regime of the variable at time *t*, and $\varepsilon_t \sim P\left(\phi_{S_t}\right)$ with $P(\phi)$ as the probability density function of the innovations defined by the vector (ϕ). Each of the independent state variables x_t follows a MSSV process, which we discuss next.

We let the log volatility of the state variables evolve stochastically over time, which stands in contrast to (G)ARCH-type models. The model has the advantage of being parsimonious yet flexible. Following convention (Ball and Torous, 1999; Shephard, 1996), we characterize the SV model as a discrete-time extension of the time-diffusion process

$$\Delta x_t = a + bx_{t-1} + \sigma_t x_{t-1}^{\gamma} \varepsilon_t \quad (12)$$

where γ represents the diffusion term, $\Delta x_t = x_t - x_{t-1}$ and ε_t is a standard normal random variable. The residual of the above equation is $e_t = \sigma_t x_{t-1}^{\gamma} \varepsilon_t$. The volatility parameter (σ) evolves stochastically, following a first-order autoregressive process

$$\ln \sigma_t^2 = \omega + \varphi \ln \sigma_{t-1}^2 + \eta_t \quad (13)$$

where $\eta_t \sim \text{i.i.d.} N(0, \sigma_\eta^2)$ is the disturbance term. As a result, the variance is subject to random shocks. We transform the residuals in eq. (12) to $e_t = \Delta x_t - a - bx_{t-1}$, which allows us to formulate a quasi-likelihood function using Kalman filtering. The log of the squared residuals is

$$\ln e_t^2 = \ln \sigma_t^2 + 2\,\gamma \ln x_{t-1} + \ln \varepsilon_t^2 \quad (14)$$

Further defining $z_t = \ln e_t^2$ and $g_t = \ln \sigma_t^2$, it follows that eq. (14) reduces to

$$z_t = g_t + 2\gamma \ln x_{t-1} + \ln \varepsilon_t^2 \quad (15)$$

where $g_t = \omega_m + \varphi g_{t-1} + \eta_t$. Next, we discuss the complete MSSV model, which is used to examine the dynamics of the dependence structure.

The above model is a generalization of the SV and the MS model. This framework allows volatility parameters to vary across different regimes, better fitting the data. Thus, a key motivation to use MSSV is that it allows for different estimates of the elasticity of variance (γ). By combining (12)-(15), the MSSV model is characterised as

$$z_t = g_t + 2\gamma \ln x_{t-1} + \ln \varepsilon_t^2 \quad (16)$$

$$g_t = \omega_m + \varphi g_{t-1} + \eta_t$$

with all variables and parameters defined above. In contrast to the standard SV model, in (16) we define $\omega_m = \ln \sigma_m^2$, allowing us to capture the different regimes at a particular point in time. With the regimes governing the dynamic behaviour of the estimated state variables, we condition a particular regime and calibrate the density of the variable of interest. In this parameterization of the MS model, the transition probabilities from state *m* to state *n* at time *t* are defined as $p_{mn} =$

$\Pr[S_t = m|S_{t-1} = n]$. Note that for $m = 1, \dots, M$, only $M(M-1)$ needs to be specified, as $p_{mn} = \Pr[S_t = M|S_{t-1} = n] = 1 - \sum_{m=1}^{M-1} \Pr[S_t = m|S_{t-1} = n]$. The unconditional volatility can change between different states by allowing (σ_i) to take values of $m \in \{1, \dots, M\}$ at time t. The corresponding equation becomes

$$\Delta x_t = a + bx_{t-1} + \sigma_m x_{t-1}^{\gamma} \varepsilon_t \tag{17}$$

An important component of the Markov-switching model is that the switching of the regimes follows a stochastic process. Thus, identifying regimes based on distributional characteristics of the regime-switching variable, such as $(\mu \pm \sigma)$, would lead to a restricted form of the switching model that may not capture the true dynamics of the dependence structure. However, using a weak regime classification will imply that the model is unable to successfully distinguish between the regimes only based on the behaviour of the data, leading to misspecification. To address this issue, we identify the regimes based on a regime-switching classification. Ideally, the switching model should classify the regimes such that they are mutually exclusive. Following Ang and Bekaert (2002), we construct the regime classification statistic (RCS) for *M* states as

$$RCS(M) = 100M^2 \frac{1}{T} \sum_{t=1}^{T} \left(\prod_{m=1}^{M} p_{mt} \right) \tag{18}$$

where $p_{mt} = \Pr(S_t = m|I_T)$ indicates the regime transition probabilities and $100M^2$ serves as a normalizing constant to keep the statistic between 0 and 100, where a value of 0 indicates a perfect regime classification, while a value of 100 implies that the regimes cannot distinguish the behaviour of the dependence structure. We use the Kalman filter to estimate the MSSV model. To remove any path dependence, we compute the conditional expectation of the log-volatility forecast by taking the weighted average output of the previous iteration. The estimation process is described in Appendix A.

To assess the forecasting of asset return comovements robustly, it is standard practice to consider an adequate hold-off sample period. Therefore, we choose 15 years (1987 to 2002) as the observation period to estimate the model parameters, and then we forecast return comovements for a period of 9 years (2003 to 2012). Moreover, since we make no assumption regarding whether our switching model outperforms a single-regime model, the forecasting exercise is repeated for different subsamples. Specifically, we fit the model for four years, estimate the one-step-ahead forecast, delete the first observation, and add the next one and then again re-estimate the forecast.

To assess forecast quality, the median of squared errors is calibrated for the regime-switching MSSV model and the non-regime switching stochastic volatility model. Furthermore, following Pagan and Schwert (1990) we run a forecast efficiency regression to examine whether the regime-switching model outperforms the non-regime model (NRM). We model the forecast efficiency regression as $v_{rc,t} = \alpha + \beta \hat{v}_{rc,t} + \epsilon_t$. In this framework, if the mean and the variance forecast of the asset return comovements ($v_{rc,t}$) are unbiased, then the regression implies that $\alpha = 0$ and $\beta = 1$. To test forecasting efficiency, the regression model is estimated using OLS with Newey and West (1987) heteroskedasticity and autocorrelation-corrected standard errors. We also correct the standard errors for possible factor estimation uncertainty. Since we use a rolling sample, we follow West and McCracken (1998) and multiply the Newey-West standard errors by $\sqrt{(1-\pi^2/3)}$, where $\pi = 9/15$, i.e. the forecasting period by parameter estimation period.

4. Data, Empirical Analysis and Results

4.1 Data

We require data on four asset classes, namely equities, bonds, commodities, and real estate. We extract US data for the Standard and Poor's (S&P) 500 index (E), U.S. 10 year Government bond return index (B), two commodity benchmarks, i.e. the S&P GSCI Gold index (G) and West Texas Intermediate – WTI (Cushing, Ok.) crude oil prices (O), and the S&P Case-Shiller Composite-10 home price index (RE). The data cover the period from the fourth quarter of 1987 to the fourth quarter of 2012. Lastly, we compute daily returns for each index. More detail about the data sources is presented in Appendix B.

<<Insert Table 1>>

Table 1 presents the summary statistics of the asset returns. Panel (A) of Table 1 shows that the annualized mean return of crude oil is highest (3.39 percent) over the sample period, followed by equity (6.27 percent) and bond returns (5.52 percent). Similarly, the standard deviation is also highest for crude oil returns (33 percent) followed by equity returns (16.42 percent). All asset returns are negatively skewed, except for gold. Further, asset returns display excess kurtosis. The

Jarque-Bera test statistics in Panel (B) of Table 1 reject the hypothesis that the unconditional distributions of the asset returns are normally distributed.

4.2 Dynamics of Multi-Asset Return Comovements

We begin by looking at whether multi-asset return comovements show any evidence of regime-switching behaviour. We refer to the trajectory of the multi-asset return comovements as the Joint Dependence Structure (JDS). Panel A of Table 2 reports the transition probabilities and the expected durations[3] of the two regimes, which are identified using the Regime Classification Statistic (RCS) as described in eq. (19).

Our findings indicate significant transition probabilities for both regimes. The identified regimes represent: i) the Dependence Structure High State or contraction regime (DSHS or High) (Regime 1) and the Dependence Structure Low State or expansion regime (DSLS or Low) (Regime 2). Based on the transition probability and the expected duration values presented in Panel A of Table 2, the Low regime tends to be considerably longer than the High regime. The practical implication of this result is that investing in different asset classes leads to considerable diversification because the multi-asset return comovements tend to stay in a lower state. Panel A further indicates that return comovements are more volatile during the economic contraction regime (DSHS), compared to the economic expansion regime (DSLS).

Panel A of Figure 1 presents the time path of the multi-asset return comovements. The graph shows that the regimes of the return comovements closely align with the economic expansionary and contractionary phases of the dating cycle proposed by the NBER and which are conventionally accepted (the dating cycles are reported in Appendix C). The shaded region in the figure represents the economic contractionary periods. Panel B of Figure 1 shows the average value of return comovements from August 1987 to September 2012. Comovements during the economic contractionary period are higher than during the economic expansionary period, consistent with known stylized facts. The findings provide evidence of an increase in return comovement after August 2007 and leading into the financial crisis. However, our findings show no evidence of extreme

[3] Following Hamilton's (1989) formula we estimate the expected duration of the regimes as $\sum_{i=0}^{\infty} i p_{11(22)}^{i-1}\left(1-p_{11(22)}\right)$, where $p_{11}(p_{22})$ are the transition probabilities in Regime 1 (Regime 2).

events in either of the tails. This suggests that investment in real assets can lead to significant portfolio diversification during economic contraction regimes.

<<Insert Table 2>>

<<Insert Figure 1>>

4.3 Macroeconomic and Non-Macroeconomic Factor Exposure

The impact of macroeconomic and non-macroeconomic variables during DSHS (economic contraction phase, i.e. Regime 1) and DSLS (economic expansion phase, i.e. Regime 2) are reported in Table 2, Panel B. In the economic contraction phase, inflation and risk aversion are statistically significant, while in the economic expansion phase, inflation and the risk-free rate are significant. The findings suggest that an increase in risk aversion during the contraction phase increases the intensity of return comovements. Inflation has a positive influence on return comovements during both regimes. This result suggests that (expected) inflation may reflect information about the real interest rate and may induce positive correlation in asset returns.

In the economic contraction regime, non-macroeconomic variables, i.e., output uncertainty, inflation uncertainty, bond market illiquidity, and depth of recession are all significant, while in the economic expansion regime, inflation uncertainty and bond illiquidity are significant. Overall, the factor coefficients presented in the table indicate that non-macroeconomic variables, especially inflation uncertainty (IU) and bond illiquidity, play a more important role in explaining multi-asset return comovements. While IU negatively influences return comovements during the economic contraction phase, it has a positive influence during the economic expansion regime. The positive impact of IU suggests that during the economic expansion phase, risk premiums and asset valuations are affected by increasing economic uncertainty. This finding is also consistent with the learning models of Veronesi (1999), in which uncertainty decreases the equity risk premium.

More interesting yet is the evidence of a negative impact during an economic contraction phase. Indeed, this result suggests that during periods of economic recession, an increase in economic uncertainty lowers interest rates through the channel of a "precautionary savings effect".[4] In contrast, the effect of bond illiquidity is positive during an economic contraction phase and

[4] The "precautionary saving effect" refers to the delay in an individual's consumption and saving in the current period due to uncertainty relating to his or her future earnings.

negative during an economic expansion phase. The former result provides evidence as to how liquidity shocks co-move across markets, while the latter result is consistent with the research suggesting that economic recovery may drive investors and traders from less liquid, safer Treasury bonds toward highly liquid but riskier assets like stocks. As a result, the resulting price pressure effects may induce negative return correlations among the more and the less risky financial assets. Thus, the liquidity effect is consistent with the "flight-to-safety" phenomenon.

4.4 Factor Contributions

In this sub-section, we show the extent to which macro and non-macroeconomic variables contribute to explain multi-asset return comovements. To answer this question, we re-estimate the MSSV model by including the macro and the non-macroeconomic variables separately. Panel C of Table 2 reports the results of this estimation. The implications of the AIC and BIC criteria are clear. Indeed, the model fit worsens considerably when we drop the non-macro factors. Overall, amongst the macroeconomic variables, inflation plays a significant role in both regimes, whereas among the non-macroeconomic variables, inflation uncertainty and bond illiquidity variables are dominant in both phases of the economy. The findings suggest that non-macro variables do indeed play a significant role in explaining the dynamics of multi-asset return comovements. In particular, non-macro variables influence return comovements differently in different regimes. For instance, the bond illiquidity measure loads positively during the economic contraction phase, suggesting that liquidity variation induces positive correlation among asset returns across classes.

4.5 Forecasting Performance of Asset Return Comovements

Panel A of Table 3 presents the median of squared errors and the coefficients of the forecast efficiency regression for the MSSV and the non-regime switching models. It is clear that the median of squared errors is significantly lower for the MSSV model, and the null hypotheses of $\alpha = 0$ and $\beta = 1$ cannot be rejected. This indicates that the MSSV models adequately capture the dynamics of the asset return comovements.

In contrast, the null hypotheses of $\alpha = 0$ and $\beta = 1$ are rejected for the non-regime switching models. This result suggests that the non-regime switching models are not efficient in capturing extreme return comovements. The findings of $\alpha \neq 0$ and $\beta \neq 1$ indicate that the non-regime switching model either underestimates or overestimates the true volatility of asset return comovements. To distinguish between the two cases, i.e. high and low volatility of return comovements, we re-estimate the forecast efficiency regression by allowing a break in the regression line at the median forecast. That is, there are two pairs of (α, β) estimates, one pair (α^{+}, β^{+}) used for forecasts above the median and (α^{-}, β^{-}) below the median. Table 3, Panel B, presents these results. We find that (α^{+}, β^{+}) are significantly different from $(0,1)$. The estimated coefficients of β^{+} indicate that non-regime models overestimate the true variance.

Taken together, these findings imply that the MSSV framework enhances the flexibility of the model in accommodating the persistence of volatility shocks. For instance, if shocks are more persistent in periods of economic contraction than in periods of economic recovery, this feature should be adequately captured by the regime parameters. Moreover, our Markov switching model can capture the 'pressure smoothening' effects of those shocks that are not persistent.

4.6 Economic Value of Asset Return Comovements

A better understanding of the variables that drive asset return comovements should be useful for investors in improving their asset allocation decisions. In this subsection, we examine whether this is empirically true for a short-horizon dynamic strategy. In this setting, investors maximize their one-period expected utility (EU) and do not hedge against future changes in the investment opportunity set (Fleming et al., 2001). Since a short-horizon dynamic strategy ignores the hedging component, it is expected to underperform the optimal strategy under Merton's (1973) framework. Therefore, compared to an optimal strategy, a short-horizon strategy sets a higher bar in terms of what represents significant economic value added.

The framework in Fleming et al. (2001) does not allow for an analytical solution to the optimal portfolio. Therefore, they evaluate their short-horizon dynamic strategy by examining two sub-optimal portfolios relating to maximum-mean and minimum-variance. To overcome this issue, we assume a power utility function over terminal wealth, i.e., $U(W_T) = W_T^{1-\gamma}/(1-\gamma)$, where γ is the investor's constant relative risk aversion coefficient. Based on Campbell and Viceira (2002), the one-period optimal asset allocation is

$$A_t^w = \frac{1}{\gamma}\Sigma_t^{-1}\left(E_t r_{t+1} - Rf_t . I - \sigma_t^2/2\right) \tag{19}$$

where A_t^w is the vector of asset weights, Σ_t is the conditional asset return covariance matrix, $E_t r_{t+1}$ is the expected asset return vector, Rf_t is the risk-free rate, $I = [1, 1]'$ and σ_t^2 is the vector of asset variances.

Below, we present a comparison of two approaches: a multivariate conditional covariance (MCC) approach, and a dynamic approach. In the MCC approach, investors forecast one-period ahead return comovements using a DCC-GARCH model,[5] while in the dynamic approach, investors base their forecast on macroeconomic and non-macroeconomic variables. Investors form their portfolio based on eq. (19) and rebalance it at the end of each quarter. The portfolio formation period is 1987 to 2002 while the investment period is from 2003 to 2012.

To assess economic significance, we use Willingness-to-Pay (WTP) as a measure of certainty-equivalence. WTP is defined as the maximum fee (f) an investor is willing to pay for holding a dynamic strategy over the alternative strategy. WTP is defined as

$$WTP = sup\left\{f \middle| E\left(U\left(W^{EMA/MCC}\right)\right) \leq E\left(U\left(W^{dynamic} - f\right)\right)\right\} \tag{20}$$

Note that terminal wealth is thus $W_T = W_i \prod_{t=1}^{T}(1 + r_t)$, where W_i is the investor's initial wealth, and where expected log-utility is defined using

$$\begin{aligned} W \ln\left(U(W_T)\right) &= (1-\gamma)\sum_{t=1}^{T} \ln(1 + r_t) + (1-\gamma)\ln W_i - \ln(1-\gamma) \\ &= (1+\gamma)T.\overline{\ln(1 + r_t)} + (1-\gamma)\ln W_i - \ln(1-\gamma) \end{aligned} \tag{21}$$

Since eq. (16) suggests that $U(W_T)$ is log-normally distributed, EU is computed as

$$\begin{aligned} \overline{U(W_T)} = exp\Bigg(&(1+\gamma)T.\overline{\ln(1 + r_t)} \\ &+ \frac{1}{2}(1-\gamma)^2 T^2 \widehat{Var}\left(\overline{\ln(1 + r_t)}\right)\Bigg).\frac{W_i^{1-\gamma}}{1-\gamma} \end{aligned} \tag{22}$$

Table 4 compares the performance of the two approaches for various levels of risk aversion and of the risk-free rate. The last column reports the bootstrapped *p*-values of the hypothesis: $H_{null}: WTP \leq 0$.

[5] A direct extension of univariate GARCH models to multivariate GARCH models (such as VEC-GARCH model of Bollerslev et al. (1988) and BEKK-GARCH model defined in Engle and Kroner (1995)) suffers from dimensionality issues and thus cannot be used to estimate covariance matrices of many financial assets. In this study, we use the DCC-GARCH model of Engle (2002) which is more appropriate to estimate large covariance matrices.

The main findings are as follows. First, for constant relative risk aversion investors, the dynamic approach using macro and non-macro variables outperforms the MCC approach. That is, in all instances, the hypothesis $WTP \leq 0$ is rejected. Second, the findings show that the dynamic approach is riskier, as volatility is higher. However, the computed Sharpe ratios suggest that investors are better rewarded under the dynamic approach. Third, WTP decreases as risk aversion (γ) increases. Indeed, this result is consistent with theory as it suggests that a higher level of risk aversion discourages investors from holding riskier assets. Fourth, WTP increases as the risk-free rate increases. Indeed, there is an informational advantage, under the dynamic approach, of accounting for the effect of the risk-free rate on return comovements, thereby improving diversification opportunities.

Overall, the findings reported in Table 4 indicate that the dynamic approach using macro and non-macro variables outperforms the conventional MCC method. The findings also imply that the investor's optimal portfolio choices are improved by a better understanding of the dynamics and influence of these variables on asset return comovements.

<<Insert Table 3>>

<<Insert Table 4>>

Finally, to support our main findings, we present several robustness checks and specification tests in the next section. These include univariate tests, covariance tests, and quantile regressions.

5. Robustness and Specification Tests

To ensure that the state variable and MSSV models are adequate in recovering the dynamics of the state variables and factor exposures, the estimated model must satisfy some basic requirements. To this end, we perform univariate tests and covariance tests on the residuals of the model.

5.1 Univariate Test

The following equation defines the reduced-form model that identifies the structural factors and the factor exposure of the variables.

$$y_t = \mu(R_s) + \beta K_{t-1} + \sigma(R_s)\varepsilon_t \tag{23}$$

Note that R_s assumes two values representing regimes 1 and 2. Let the conditional probability for $R_s = 1$ be p_{t-1} and the corresponding conditional probability for $R_s = 2$ be $(1 - p_{t-1})$. Considering these conditional probability estimates, the residual of the above model is defined as

$$resid_t = y_t - \beta K_{t-1} - \left(p_{t-1}\mu_1 + \left(1 - p_{t-1}\right)\mu_2\right) \quad (24)$$

where $\mu_1$and μ_2 are the means of regime 1 and regime 2, respectively. The conditional variance $(CV_{resid,t-1})$ of $resid_t$ is

$$CV_{resid,t-1} = p_{t-1}\sigma_1^2 + \left(1 - p_{t-1}\right)\sigma_2^2 + p_{t-1}\left(1 - p_{t-1}\right)\left(\mu_1 - \mu_2\right)^2 \quad (25)$$

where σ_1^2 and σ_2^2 are the variances of regime 1 and regime 2, respectively. Based on Timmermann (2000), the conditional skewness $(CSk_{resid,t-1})$ and the conditional kurtosis $(CKr_{resid,t-1})$ are given by

(26)

$$CSk_{resid,t-1} = \frac{p_{t-1}\left(1 - p_{t-1}\right)\left(\mu_1 - \mu_2\right)(3(\sigma_1^2 - \sigma_2^2) + \left(1 - 2p_{t-1}\right)\left(\mu_1 - \mu_2\right)^2}{\left[p_{t-1}\sigma_1^2 + \left(1 - p_{t-1}\right)\sigma_2^2 + p_{t-1}\left(1 - p_{t-1}\right)\left(\mu_1 - \mu_2\right)^2\right]^{\frac{3}{2}}}$$

$$CKr_{resid,t-1} = \frac{p_{t-1}\left[3\sigma_1^2 + \left(\mu_1 - \mu\right)^4 + 6\sigma_1^2\left(\mu_1 - \mu\right)^2\right] + \left(1 - p_{t-1}\right)\left[3\sigma_1^2 + \left(\mu_1 - \mu\right)^4 + 6\sigma_1^2\left(\mu_1 - \mu\right)^2\right]}{\left[p_{t-1}\sigma_1^2 + \left(1 - p_{t-1}\right)\sigma_2^2 + p_{t-1}\left(1 - p_{t-1}\right)\left(\mu_1 - \mu_2\right)^2\right]^{3/2}}$$

In the univariate specification tests, we test for zero mean and no higher order correlation for five lags, i.e. whether or not μ_0, l_1, l_2, l_3, l_4 and l_5 are zero in the following equations:

$E[resid_t] - \mu_0 = 0$ and $E\left[\left(resid_t - \mu_0\right)\left(\sum_{i=1}^{5} resid_{t-i}\, resid_{t-1} - \mu_0\right)\right] - \sum_{i=1}^{5} l_i = 0$

To test for excess skewness and kurtosis, we examine whether e_{sk} and e_{kr} are equal to zero in the following equations, respectively

$$\frac{E\left[\left(resid_t - \mu_0\right)^3\right]}{E\left[\left(resid_t - \mu_0\right)^2\right]^{\frac{3}{2}}} - CSk_{resid} - e_{sk} = 0$$

$$\frac{E\left[\left(resid_t - \mu_0\right)^4\right]}{E\left[\left(resid_t - \mu_0\right)^2\right]^2} - CSk_{resid} - e_{kr} = 0$$

The estimates of μ_0, l_1, l_2, l_3, l_4, l_5, e_{sk} and e_{kr} are obtained using a General Methods of Moments (GMM) framework using a Newey-West (1987) weighting matrix accommodating for 5 lags. The univariate test of zero means, unit variance, and the presence of zero excess skewness and kurtosis follows a chi-squared distribution with one degree of freedom. The test of no autocorrelation up to 5 lags follows a chi-squared distribution with degrees of freedom equal to the number of lags.

5.2 Covariance Test

We carry out the covariance test to ensure that the state variables adequately capture the covariance between the factor shocks. The following condition is tested:

$$E\left[resid_{i,t} resid_{j,t}\right] = 0, \text{ for } i = 1, \dots, N\ ;\ j = 1, \dots, N\ ;\ i \neq j$$

where N denotes the number of state variables. The joint test follows a chi-squared distribution with degrees of freedom equal to $N(N-1)/2$. Furthermore, we test whether the shocks of each of the state variables have zero covariance with the factor shocks. This test follows a chi-square distribution with 10 degrees of freedom. Table 5 presents the specification test results. The hypothesis of zero covariance cannot be rejected at the 5 percent level for all the state variables. This result confirms that the MSSV model is correctly specified.

<<Insert Table 5>>

5.3 Quantile Regressions

We estimate quantile regression models to further investigate the factors that drive the dependence structure. Although this approach allows us to estimate various quantiles (Koenker and Bassett, 1978), we rely on least absolute deviation (LAD) regression to overcome the low-power problem of the OLS regressions (Connolly, 1989). Furthermore, the results from the different quantile regressions help us to provide a more robust description of the variables that drive multi-asset return comovements. We estimate the coefficients of the quantile regression at θ (denoting the quartiles for which the relation between the dependence structures and the explanatory variables is estimated) from 0.05 to 0.95, with 0.05 increments.[6] We include two additional extreme percentiles at the 0.99 and 0.01 levels to document changes in the dependence structure when large deviations are present. The statistical inferences from these regression models are drawn by the bootstrapping method (for details, see Andrews and Buchinsky 2000; Angelis et al. 1993). Note that, consistent with the earlier notation, lower θ values indicate an economic expansion phase (DSLS), while higher θ values indicate an economic contraction phase (DSHS).

We present the results of these regressions in Table 6. The findings are consistent with our earlier MSSV model estimations. Among the macroeconomic variables, inflation and the risk-free rate are found to have a significant effect, while among the non-macroeconomic variables, inflation uncertainty and bond illiquidity significantly affect return comovements. The coefficient signs are generally consistent with our previous findings. Interest rate shocks have a negative effect during an economic expansion, reflecting the effect of the discount rate on asset returns. Surprisingly in light of the recent literature, the variance premium is not significant in either of the regimes. This variable allows us to capture nonlinearities in consumption growth. Since the variance premium correlates positively with the implied volatility of returns for risky assets (such as stocks), but negatively with observed volatility, we can establish whether the "flight-to-safety" effect is due to the risk premium component. Therefore, the non-significance of the variance premium suggests that during recession periods, the exposure of asset returns to cash flow shocks (such as the output gap uncertainty) increased in absolute terms.

<<Insert Table 6>>

5.4 Alternative Copula Specifications

The measurement of asymmetric tail dependence may be sensitive to the choice of copula function.[7] To address this issue, we investigate this measurement using alternative copulas, in particular the Joe-Clayton copula. The results (omitted for brevity—see the appendix) reveal no substantive impact on measuring tail dependence or on the subsequent analysis.

[6] Since the inferences based on the results of all the quintiles are similar, and given space constraints, we report only a subset of the results (the full results can be provided upon request).

[7] We thank a reviewer for suggesting this point.

6. Conclusion

A growing literature documents the diversification properties of non-traditional asset classes including commodities and real estate. There is, however, a significant gap in the literature on optimal asset allocation. Indeed, little is known about how investors should adjust their allocations across asset classes to reflect two stylized facts, namely that the joint dependence of asset returns is non-elliptical, and that the dependence significantly changes between economic regimes, e.g. from economic expansions to recessions/crisis periods. The paper aims to fill this gap in the literature by presenting evidence of the characteristics, determinants, and predictability of asymmetric return comovements across asset classes. We contribute to the existing literature--which is largely restricted to modelling stock-bond correlations--by including real assets and relaxing the normality assumption. We characterise return comovements and show that their behaviour as well as their determinants vary across different economic regimes. We also propose a new approach to forecast multi-asset return comovements and show that it improves certainty-equivalent wealth for investors across a range of plausible risk aversion levels, by helping them make more informed optimal portfolio choices.

Our findings suggest that amongst macroeconomic variables, inflation plays a particularly important role in explaining the joint dependence structure in both expansionary and contractionary economic regimes. Thus, inflation reflects expectations about real interest rates, inducing positive asset return comovements. Among non-macroeconomic variables, inflation uncertainty and bond illiquidity significantly influence return comovements. The positive impact of inflation uncertainty suggests that during an economic expansion phase, increasing economic uncertainties affect risk premiums and asset valuations. In contrast, the negative effect during periods of economic recession highlights a precautionary savings effect. The impact of bond illiquidity suggests that liquidity shocks co-move across markets, leading to a "flight to safety" phenomenon. In terms of factor contributions, we find that the model fit worsens considerably when we drop non-macroeconomic factors. Our results are robust to alternative specification tests.

We then test the economic significance of the variables which we find to be key determinants by assessing their economic value-added. We conduct this assessment by examining forecasts of the joint distribution of asset return comovements. We find that the dynamic model, using a regime-switching framework and using information from macro and non-macro explanatory variables, outperforms the multivariate conditional covariance model in terms of accurately forecasting multi-asset return comovements. Finally, we show that investors—across a range of plausible risk aversion levels--benefit from improved comovement forecasting by enhancing their portfolio optimisation choices using our model.

Data Availability Statement

The data that support the findings of this study are available from Datastream (published by Thomson Reuters). Restrictions apply to the availability of these data, which were used under license for this study. Data are available from the authors with the permission of Thomson Reuters.

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Table 1: Summary Statistics

Panel A: Descriptive Statistics of Asset Returns (1987 – 2012)

	Equity (E)	Bond (B)	Real Estate (RE)	Gold (G)	Oil (O)
Mean (%)	6.274	5.524	3.394	5.438	6.331
Standard Deviation (%)	16.428	1.293	2.730	15.449	33.000
Kurtosis	3.854	0.138	0.611	1.986	1.687
Skewness	-1.114	-0.165	-0.726	0.064	-0.357

Panel B: Diagnostics (1987-2012)

	Equity (E)	Bond (B)	Real Estate (RE)	Gold (G)	Oil (O)
Jarque-Bera statistics	208.3** (0.000)	7.7** (0.020)	31.5** (0.000)	45.7** (0.000)	48.4** (0.000)
ARCH LM statistic (1)	31.586** (0.000)	17.737** (0.000)	1741.764** (0.000)	4.586** (0.033)	13.676** (0.000)
ARCH LM statistic (5)	17.489** (0.000)	8.571** (0.000)	371.920** (0.000)	3.003** (0.016)	4.563** (0.000)
ARCH LM statistic (10)	12.804** (0.000)	4.903** (0.000)	190.231** (0.000)	1.927** (0.041)	2.913** (0.001)
Ljung-Box statistic (1)	8.293** (0.045)	9649.404** (0.000)	4232.160** (0.000)	4.433** (0.036)	5.757** (0.017)
Ljung-Box statistic (5)	1.254 (0.282)	1932.252** (0.000)	914.690** (0.000)	3.005** (0.011)	3.223** (0.007)
Ljung-Box statistic (10)	0.869 (0.562)	971.691** (0.000)	452.606** (0.000)	1.619 (0.100)	2.156** (0.022)

Note: Panel A represents the descriptive statistics of the asset returns for the sample period: fourth quarter of 1987 to the fourth quarter of 2012, yielding 302 observations. The return and the standard deviations are annualized from the monthly observations. Panel B provides the diagnostic test results. Under the normality null hypothesis, Jarque-Bera test statistic follows a Chi-square distribution with fixed (2) degrees of freedom. The null hypothesis of the ARCH-LM test is: there is no evidence of ARCH effect. We conduct the test at lags 1, 5 and 10 with corresponding 1, 5, 10 degrees of freedom. Tests using other lags yield the same results. We conduct the Ljung-Box test for serial correlation, corrected for heteroskedasticity at lags 1, 5 and 10. The p-values are reported in the parentheses. ** signifies rejection of the null hypothesis at 5 percent level.

Table 2: MSSV model estimates and factor exposure

Panel A: Model Characteristics

	Tr. Prob.	Std. Dev.	Exp. Dur.	AIC
Regime 1 (DSHS)	0.850	0.050	6.657	
Regime 2 (DSLS)	0.904	0.011	10.385	-311.587

Panel B: Coefficient Estimates

		Macroeconomic Variables				Non-Macroeconomic Variables						
	Constant	RF	O	I	RA	OU	IU	LR	DS	TS	VP	DR
Regime 1 (EC)	-0.173	0.100	-1.180	0.546**	0.066***	0.532***	-4.522***	-0.001	5.238**	0.097	0.013	-0.012**
	(0.164)	(0.779)	(0.120)	(0.012)	(0.006)	(0.005)	(0.003)	(0.258)	(0.038)	(0.421)	(0.914)	(0.032)
Regime 2 (EE)	-0.111	-1.552***	-0.456	0.722**	0.026	-0.135*	3.010***	0.000	-3.544***	0.048	0.037	0.002
	(0.479)	(0.000)	(0.167)	(0.044)	(0.448)	(0.052)	(0.000)	(0.721)	(0.000)	(0.676)	(0.672)	(0.356)

Panel C: Model Performance

	Full Model	(-) non-Macro	(-) Macro	(-) non-Macro & I	(-) Macro, IU & DS
AIC	-311.587	-240.045	-259.311	-232.11	-233.075
BIC	-238.363	-203.433	-207.008	-200.091	-203.011

Table 2 notes are presented on the following page.

Table 2 (continued)

Note: The table reports the MSSV Model characteristics, the model estimates, and the factor contributions to the model performance. Regime 1 corresponds to the expansion regime of the dependence measure (DSHS), and Regime 2 corresponds to the contraction regime of the dependence measure (DSLS). The expansion regime of the dependence structure (DSHS, i.e. Regime 1) relates to the economic contraction (EC) phase and the contraction regime of the dependence structure (DSLS, i.e. Regime 2) relates to economic expansion (EE) phase. Panel A: Tr. Prob. (TP) corresponds to the transition probabilities of the two states. TP for state 1 refers to the probability that the dependence measure will stay in the expansion regime, and TP for State 2 corresponds to the probability that the dependence measure will stay in the contraction regime. Std. Dev. reports the standard deviation of the regime states. The Standard errors are reported in parentheses. Expected (Exp.) Duration (Dur.) corresponds to the expected duration of the dependence measure in the expansion regime (Regime 1) and in the contraction regime (Regime 2). The sample period is from the fourth quarter 1987 to the fourth quarter 2012. Panel B: In the set of macroeconomic state variables, RF is risk-free rate, O is the output gap, I is inflation and RA is (constant relative) risk aversion. In the set of non-macro factors, OU is output uncertainty, IU is inflation uncertainty, LR is a measure of equity illiquidity, DS is a bond illiquidity measure, TS is the term spread, VP is the variance premium, and DR is the depth of recession. Panel C: This reports the factor contributions of five different model characteristics. The corresponding AIC and BIC values are reported. It is clear that non-macroeconomic variables play a central role in enhancing the model fit. Furthermore, among the non-macro variables, the illiquidity and uncertainty factors are significantly important. Among the macroeconomic variables, inflation plays an important role in defining the JDS.

* corresponds to 10 percent level, ** corresponds to 5 percent significance level and *** corresponds to one percent significance level.

Table 3: Out-of-sample forecasting of asset return comovements

Panel A: Forecasting performance of MSSV and non-regime switching stochastic volatility model

	MSE (MSSV-NRSM)	α		β	
		MSSV	NRSM	MSSV	NRSM
Joint Dependence Structure	0.024	0.000	0.000	1.015	1.709
	(0.019)	(0.630)	(0.037)	(0.743)	(0.048)

Panel B: Below and above median forecasting performance of MSSV and non-regime switching stochastic volatility model

	Below Median Forecast				Above Median Forecast			
	α^-		β^-		α^+		β^+	
	MSSV	NRSM	MSSV	NRSM	MSSV	NRSM	MSSV	NRSM
Joint Dependence Structure	0.000	0.000	0.931	2.376	0.000	0.000	1.019	0.107
	(0.273)	(0.028)	(0.119)	(0.000)	(0.119)	(0.042)	(0.529)	(0.039)

Note: Panel A of this table reports the difference between the median of square errors of MSSV models and the non-regime switching models (NRSM), as well as the forecast efficiency regression estimates of the MSSV model and the non-regime switching model (NRSM). The parameters are estimated for the period 1987 to 2002 and the forecasting is done for the period 2003 to 2012. The forecasting estimates are calibrated for the joint dependence structure. It is clear that the MSSV model's median square errors are significantly lower than the non-regime switching models. The panel also shows the forecast efficient regression estimates of (α, β). In the forecast efficiency regression framework, if the mean and the variance forecast of the asset return comovements are unbiased, then the regression implies that $\alpha = 0$ and $\beta = 1$. The p-values are in parentheses.

Panel B reports the forecast efficiency regression estimates for below the median (α^-, β^-) and for above the median (α^+, β^+). In this framework, if the mean and the variance forecast of the asset return comovements are unbiased, then the regression implies that $\alpha = 0$ and $\beta = 1$. The findings indicate that the non-regime switching models provide biased out-of-sample forecasts. The p-values are in parentheses.

Table 4: Economic value of forecasting asset return comovements

	MCC Strategy			Dynamic Strategy				
	Mean	Std. Dev	SR	Mean	Std. Dev	SR	WTP	p-value
$\gamma = 5$								
0.5%	17.31	18.59	0.90	24.87	22.55	0.99	0.19	0.089
1.0%	16.68	17.16	0.86	22.73	21.45	0.94	0.35	0.070
1.5%	16.57	15.73	0.85	21.80	20.91	0.91	0.40	0.053
2.0%	15.20	15.39	0.83	22.05	22.09	0.86	0.55	0.045
2.5%	14.34	14.48	0.77	21.57	21.00	0.84	0.85	0.024
3.0%	14.29	12.74	0.78	22.09	21.89	0.84	1.21	0.001
3.5%	13.45	12.12	0.76	20.92	21.29	0.80	1.65	0.001
$\gamma = 10$								
0.5%	10.63	9.89	0.90	20.15	18.50	0.98	0.11	0.090
1.0%	9.71	8.78	0.87	20.63	19.58	0.92	0.33	0.063
1.5%	8.40	7.68	0.84	20.55	20.13	0.91	0.39	0.058
2.0%	7.85	6.54	0.83	18.52	18.93	0.86	0.50	0.047
2.5%	7.08	5.35	0.81	16.86	16.80	0.86	0.52	0.039
3.0%	6.36	3.98	0.77	16.97	18.40	0.82	0.68	0.015
3.5%	5.63	2.68	0.79	14.76	17.74	0.80	0.79	0.007
$\gamma = 15$								
0.5%	6.18	5.95	0.88	17.93	16.59	0.98	0.10	0.092
1.0%	5.79	5.21	0.85	15.86	15.17	0.93	0.31	0.077
1.5%	5.03	3.85	0.83	14.79	14.76	0.90	0.32	0.076
2.0%	4.53	2.80	0.84	13.66	14.91	0.86	0.46	0.040
2.5%	3.67	1.42	0.80	11.86	15.35	0.86	0.48	0.037
3.0%	3.61	0.79	0.79	9.22	13.69	0.82	0.55	0.022
3.5%	3.56	0.13	0.78	7.23	30.16	0.81	0.60	0.010

Note: The table compares the performance of the MCC strategy and the dynamic strategy. The portfolio formation is done using the period 1987 to 2002 and the investment period is from 2003 to 2012. The annualized mean, standard deviation and the Sharpe ratios are reported for both strategies. It is clear that the dynamic strategy yields higher returns and is more volatile than the MCC strategy. However, the Sharpe ratios are higher for the dynamic strategy, suggesting that investors are better rewarded for their risky portfolios. The investors are assumed to have power utility function and constant relative risk aversion represented as coefficient γ. The Willingness-to-pay (WTP) certainty-equivalence measure computes the maximum fee (f) an investor is willing to pay for holding a dynamic strategy over the other strategy. The last column reports the bootstrapped p-values of the hypothesis: $H_{null}: WTP \leq 0$. The hypothesis is rejected for all the cases at 10, 5 or 1 percent significance levels. The findings show that the dynamic strategy outperforms the MCC strategy.
*, **, *** represents significance at 10, 5 and 1 percent levels

Table 5: Specification tests for state variables and MSSV models

Specification Tests for State variable Models

State Variables	Univariate Test mean	lag 1[a]	lag 2[a]	lag 3[a]	lag 4[a]	lag 5[a]	Excess Skewness	Excess Kurtosis	Variance	Covariance Test
Rf	0.999	0.738	0.723	0.871	0.587	0.688	0.170	0.370	0.742	0.900
O	0.999	0.061	0.169	0.311	0.462	0.515	0.380	0.250	0.096	0.970
I	0.876	0.830	0.514	0.681	0.752	0.817	0.280	0.430	0.828	0.976
Ra	0.999	0.553	0.621	0.667	0.714	0.726	0.270	0.400	0.530	0.999
Ou	0.999	0.259	0.324	0.382	0.438	0.496	0.390	0.400	0.202	0.999
Iu	0.149	0.981	0.988	0.995	0.442	0.195	0.180	0.220	0.535	0.936
Lr	0.999	0.522	0.755	0.879	0.934	0.96	0.220	0.270	0.936	0.999
Ds	0.282	0.907	0.981	0.105	0.512	0.323	0.340	0.260	0.929	0.982
Ts	0.999	0.694	0.781	0.839	0.895	0.934	0.150	0.280	0.935	0.999
Vp	0.999	0.927	0.877	0.353	0.501	0.639	0.310	0.220	0.198	0.999
Dr	0.999	0.934	0.953	0.947	0.967	0.973	0.160	0.320	0.825	0.999

Note: The table reports the specification tests for the state variables and the MSSV models that examine the determinants of the asset return comovements. Panel A presents the Monte-Carlo p-value estimates of the univariate and covariance tests for the 11 state variables - the risk free rate (rf_t), output gap (o_t), inflation (i_t), risk aversion (ra_t), output uncertainty (ou_t), inflation uncertainty (iu_t), bond market illiquidity (ds_t), equity market illiquidity (lr_t), variance premium (vp_t), term spread (ts_t) and the depth of recession (dr_t). The p-values are reported for zero mean, serial correlation for up to five lags, zero excess skewness, zero excess kurtosis, and constant variance. The covariance test reports the Monte-Carlo p-values of zero covariance of the factor shocks of one state variable with the factor shocks of the other state variables. The results indicate that the state variable models are correctly specified, providing consistent outcomes that adequately accommodate the dynamics of the state variables and the determinants of the return comovements.

Table 6: Quantile regressions and factor contributions to model performance

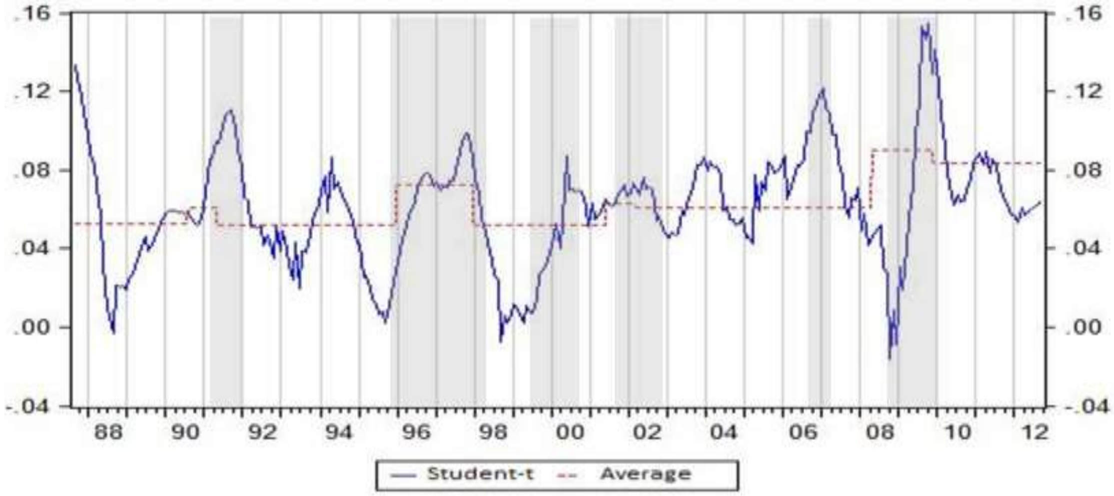
Factors	Variables	Quantile Regression (θ)						OLS
		0.01	0.10	0.25	0.50	0.75	0.99	Regression
Macroeconomic variables	RF	-1.150**	-0.862	-0.781	-0.873	-0.708	0.511	-0.085***
		(0.022)	(0.192)	(0.162)	(0.144)	(0.121)	(0.559)	(0.001)
	O	-0.348	-0.632	-0.095	-0.007	-0.062	-0.048	-0.087
		(0.545)	(0.287)	(0.144)	(0.067)	(0.183)	(0.319)	(0.194)
	I	0.883**	0.729***	0.587***	0.782***	0.502***	0.351***	0.772***
		(0.018)	(0.003)	(0.000)	(0.001)	(0.001)	(0.000)	(0.001)
	RA	0.051	0.075	0.076	0.032**	0.032**	0.085***	0.064**
		(0.428)	(0.205)	(0.266)	(0.036)	(0.031)	(0.001)	(0.026)
Non-Macroeconomic variables	OU	-0.185	-0.132	-0.172*	0.163*	0.169**	0.432***	0.324**
		(0.156)	(0.189)	(0.092)	(0.065)	(0.011)	(0.004)	(0.038)
	IU	2.931***	1.847***	1.591***	2.300**	-2.156***	-4.994***	-2.453***
		(0.006)	(0.001)	(0.000)	(0.035)	(0.000)	(0.000)	(0.000)
	LR	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		(0.672)	(0.789)	(0.841)	(0.811)	(0.500)	(0.805)	(0.774)
	DS	-3.101***	-3.427***	-4.781***	2.872***	3.580***	6.930***	2.713***
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	TS	0.011	-0.106	-0.219	-0.131	-0.223	0.072	-0.245
		(0.956)	(0.636)	(0.339)	(0.457)	(0.178)	(0.810)	(0.136)
	VP	0.033	0.186	0.099	-0.192	-0.004	-0.091	-0.057
		(0.874)	(0.401)	(0.658)	(0.247)	(0.982)	(0.790)	(0.653)
	BS	0.004	0.001	0.006	0.003	0.001	-0.011**	0.002
		(0.556)	(0.906)	(0.234)	(0.421)	(0.805)	(0.022)	(0.581)
	Constant	0.247	0.307	0.245	-0.107	-0.223	-0.223	-0.266*
		(0.357)	(0.195)	(0.370)	(0.623)	(0.319)	(0.491)	(0.072)
R^2Measure		0.576	0.538	0.466	0.458	0.583	0.645	0.623
	JDS Mean	0.007	0.022	0.046	0.061	0.079	0.154	0.063

[a]Note: The table reports quantile regression estimates at θ (denotes the quartiles for which the relation between the dependence structures and the explanatory variables is estimated). The lower θ values represent economic expansion regime (DSLS) and the higher θ values represent economic contraction regime (DSHS). Amongst the macroeconomic state variables, RF is the risk free rate, O is output gap, I is inflation and RA is risk aversion. In the set of non-macro factors OU is output uncertainty, IU is inflation uncertainty, LR is the measure of equity illiquidity, DS is the bond illiquidity measure, TS is the term spread, VP is the variance premium and DR is the depth of recession.

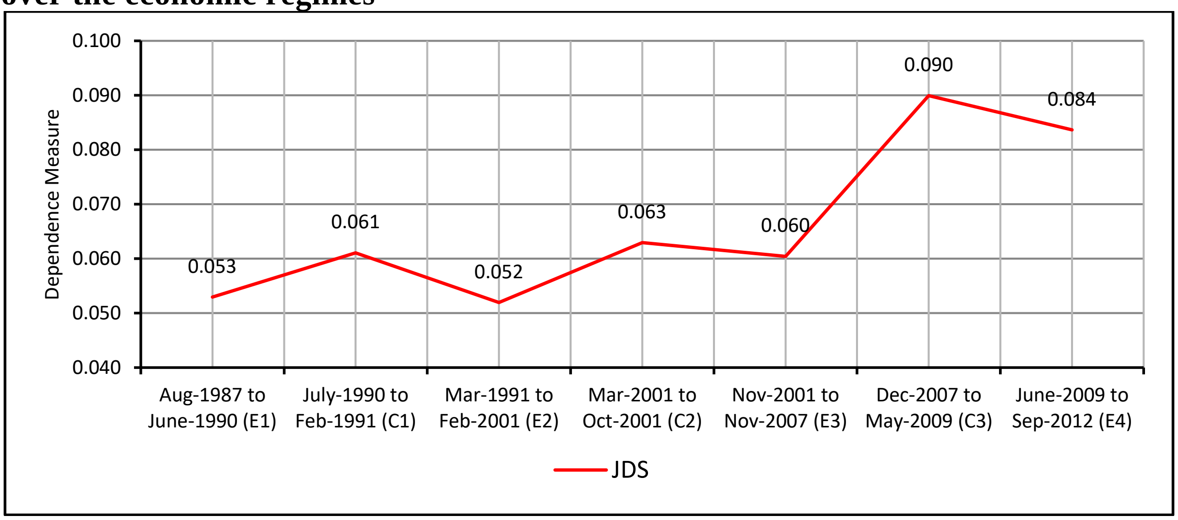
* corresponds to 10 percent level, ** corresponds to 5 percent significance level and *** corresponds to one percent significance level.

Figure 1: Characteristics of multi-asset return comovements

Panel A: The time path of multi-asset return comovements



Panel B: The average multi-asset return comovements over the economic regimes



Note: The figure shows the joint dependence structure (JDS) of multi-asset return comovements of the three different asset classes. The period of analysis is from the fourth quarter 1987 to the fourth quarter 2012. The shaded portion in Panel A represents the economic contraction regime. Panel B presents the average dependence measure for the whole sample. It is clear that the dependence measure increases post sub-prime crisis. In panel B the various economic expansion (E) and economic contraction (C) correspond to the economic cycles as dated by NBER. The periods are described in Appendix C.

Appendix A

Estimation filter for the MSSV model

The Kalman filter employed for projection is an iterative process. It forecasts the state variable at $'t+1'$ period and updates it when z_t is observable (see eq. (12)-(16)). For deriving the filtering equations we denote

$g_{t|t-1}^{(m,n)} = E\left[g_t \left| S_t = m, S_{t-1} = n, \psi_{t-1}\right.\right]$, $p_{t|t-1}^{m,n} = E\left[(g_t - g_{t|t-1}^{m,n}) \left| S_t = m, S_{t-1} = n, \psi_{t-1}\right.\right]$,

$g_{t|t-1}^{m} = E\left[g_t \left| S_t = m, \psi_{t-1}\right.\right]$ and $p_{t|t-1}^{m} = E\left[(g_t - g_{t|t-1}^{m})^2 \left| S_t = m, \psi_{t-1}\right.\right]$.

Following Smith (2002), we first forecast log-volatility and then update the previous forecasted estimate. The sequential steps are:

Step 1: The log-volatility is forecast using

$$g_{t|t-1}^{m,n} = \omega_m + \varphi_m g_{t-1|t-1}^{n} \quad \text{(A- 1)}$$

$$p_{t|t-1}^{m,n} = \varphi_m^2 p_{t-1|t-1}^{n} + \sigma_{\eta n}^2 \quad \text{(A- 2)}$$

Step 2: The forecasted estimate is updated using

$$g_{t|t}^{m.n} = g_{t|t-1}^{m.n} + p_{t|t-1}^{m,n}\left(p_{t|t-1}^{m,n} + \frac{\pi^2}{2}\right)^{-1}\left(z_t - z_{t|t-1}^{m,n}\right) \quad \text{(A- 3)}$$

$$p_{t|t}^{m.n} = p_{t|t-1}^{m.n} - p_{t|t-1}^{m,n}\left(p_{t|t-1}^{m,n} + \frac{\pi^2}{2}\right)^{-1} p_{t|t-1}^{m,n} \quad \text{(A- 4)}$$

The conditional densities are computed using the following equation

$$f\left(z_t \left| S_t = m, S_{t-1} = n, \psi_{t-1}\right.\right) = \frac{1}{\sqrt{2\pi\left(p_{t|t-1}^{m,n} + \frac{\pi^2}{2}\right)}} - \exp\left(\frac{-\left(z_t - z_{t|t-1}^{m,n}\right)^2}{2\left(p_{t|t-1}^{m,n} + \frac{\pi^2}{2}\right)}\right)^{-1} p_{t|t-1}^{m,n} \quad \text{(A- 5)}$$

It can be noted that the above procedures makes our process exclusively path dependent. Hence, to remove the path dependence we rely on Kim (1994) as stated in Smith (2002). We compute the conditional expectation of the log-volatility forecast by taking the weighted average output of the previous iteration using the formulations stated below.

$$g_{t|t}^{m} = \frac{\sum_{n=1}^{N} \Pr\left[S_t = m, S_{t-1} = n \middle| \psi_t\right] g_{t|t}^{m,n}}{\Pr\left[S_t = m \middle| \psi_t\right]} \tag{A- 6}$$

$$p_{t|t}^{m} = \frac{\sum_{n=1}^{N} \Pr\left[S_t = m, S_{t-1} = n \middle| \psi_t\right] \left(p_{t|t}^{m,n} + (g_{t|t}^{n} - g_{t|t}^{m,n})^2\right)}{\Pr\left[S_t = m \middle| \psi_t\right]} \tag{A- 7}$$

We calculate the regime probabilities based on Smith's (2002) modification of Hamilton's (1989) filter. First, we estimate the regime probabilities using

$$\Pr\left[S_t = m, S_{t-1} = n \middle| \psi_{t-1}\right] = \Pr\left[S_t = m \middle| S_{t-1} = n\right] \times \Pr\left[S_{t-1} = m \middle| \psi_{t-1}\right] \tag{A- 8}$$

The term $\Pr\left[S_{t-1} = m \middle| \psi_{t-1}\right]$ in the equation (A- 8) is the previous iteration filter output. Next we calibrate the joint density using

$$f\left(z_t, S_t = m, S_{t-1} = n \middle| \psi_{t-1}\right) = f\left(z_t \middle| S_t = m, S_{t-1} = n, \psi_{t-1}\right) \times \Pr\left[S_{t-1} = m \middle| \psi_{t-1}\right] \tag{A- 9}$$

where $f\left(z_t, S_t = m, S_{t-1} = n \middle| \psi_{t-1}\right)$ is defined previously in equation (A- 5). In step three we integrate the regimes to calculate the unconditional density as given in equation (A- 10) and then we update the probability of the regimes in state $'t'$ using equation (A- 11).

$$f\left(z_t \middle| \psi_{t-1}\right) = \sum_{m=1}^{M} \sum_{n=1}^{N} f\left(z_t \middle| S_t = m, S_{t-1} = n, \psi_{t-1}\right) \tag{A- 10}$$

$$\Pr\left[S_t = m, S_{t-1} = m \middle| \psi_{t-1}\right] = \frac{f\left(z_t \middle| S_t = m, S_{t-1} = n, \psi_{t-1}\right)}{f\left(z_t, \middle| \psi_{t-1}\right)} \tag{A- 11}$$

Appendix B

Data Description: All variables are defined for US data unless stated otherwise

Output Gap (o_t): Gross Domestic Product (GDP) is the measure of output. The gap is the percentage difference between the output and its quadratic trend.

Expected Output Gap (g_e): It is estimated as:

$$E_t[g_e] = E_t\left[\frac{GDP_t}{GDP_t}\left(\frac{GDP_{t+1}}{GDP_{t+1}^{qt}} - 1\right)\right] = GDP_t \frac{E_t\left[\frac{GDP_{t+1}}{GDP_t}\right]}{GDP_{t+1}^{qt}}$$

where GDP_t is the level of real GDP at time *t* and GDP^{qt} is the quadratic trend value of real GDP. To measure $E_t\left[\frac{GDP_{t+1}}{GDP_t}\right]$, the median of the survey response from Survey of Professional Forecasters (SPF) is used when available.

Output Uncertainty (ou_t): Mean of SPF's real output volatility.

Inflation (i_t measured as (π)): Log difference of the Consumer Price Index (CPI) for all items for all urban consumers

Expected Inflation (π_e): The yield of Treasury Inflation Protected Securities (TIPS) note subtracted from the yield of ten-year US Treasury note.

Inflation Uncertainty (iu_t measured as (π_u)): Estimated as the absolute measure of $(\pi_e - \pi)/\pi$, where π_e is *Expected Inflation* and π is *Inflation.*

Risk Aversion Factor (ra_t): The measure of the risk aversion factor is based on external habit specifications of Campbell and Cochrane (1995) taken from Baele et al. (2010).

Nominal Risk-free Rate (R_f): Three-month US Treasury bill rate.

Stock Market Illiquidity (lr_t): Capitalization-based zero daily returns across all listed firms.

Bond Market Illiquidity (ds_t): Bid-ask spreads across all securities, i.e. one month , three months, and one, two, three, five, seven, ten, twenty and thirty years of maturity.

Variance Premium (vp): The difference between ex-post realized variance and variance swap rate.

Term Spread (ts_t): Difference between ten-year and three-month Treasury bill yields. This will serve as a proxy for short-term economic conditions.

Depth of recession (dr_t): Based on Lee and Wang's (2012) estimate of business cycle proxy.

Appendix C

Turning Points in the Business Cycle

Turning Point	Date	Expansion (E)/Contraction (C)	Months in Phase
0	8/1987	E1	35
1	7/1990	C1	8
2	3/1991	E2	120
3	3/2001	C2	8
4	11/2001	E3	73
5	12/2007	C3	18
6	6/2009	E4	40

Notes: The turning points of the business cycle are based on the NBER-official dates of troughs and peaks (NBER, 2013). The sample period is from the fourth quarter of 1987 to the fourth quarter of 2012, yielding 302 monthly observations. Each month in the sample is divided into an expansionary phase or a contractionary phase, based on the turning point. The expansionary period has 268 months and the contractionary period has 34 months.

Appendix D: Copula robustness checks

Here, we establish the estimation procedure for examining the cross-sectional dependence of the asset return comovements. We use conditional time-varying copula models to capture the dynamic non-linear relationships and the asymmetries the asset returns. Another motivation for using copula models is that they allow for the separation of marginal distributions. The copula approach constructs multivariate distribution functions that avoid the limitations of the normality assumptions (Philippas and Siriopoulos, 2013). We estimate the copula models using inversion method by substituting the marginal densities of the asset returns.

For two random variables, in our case the marginal distributions of the asset returns, we characterize the conditional distribution function as

$$H_{XY|K}(x, y \mid k) = C((F_{X|K}(x \mid k), F_{Y|K}(y \mid k)) = C(u, v) \quad \textbf{(1)}$$

where, $H_{XY|K}(x, y \mid K)$ is the conditional distribution function and $F_{X|K}(x \mid K)$ and $F_{Y|K}(y \mid K)$ are the marginal distributions given a conditioning vector K. In the above equation $(x, y \mid K) = k$ and υ is the support of k for all $k \in \upsilon$ and $(x, y) \in \overline{R} \times \overline{R}$. In equation (1), u and v are the realizations of $U \equiv F_{X|K}(x \mid k)$ and $V \equiv F_{Y|K}(y \mid k)$ given $K = k$. U and V are the conditional probability integrals of the asset return marginal distributions, X and Y. To allow for upper and lower tail dependence we characterize our copula function (modified Joe-Clayton copula) as

$$C_{MJC}(u, v; \tau^U, \tau^L) = \frac{1}{2}\left(C_{JC}(u, v; \tau^U, \tau^L) + C_{JC}(1-u, 1-v; \tau^U, \tau^L) + u + v - 1\right) \quad \textbf{(2)}$$

where the C_{JC} , the Joe-Clayton (JC) copula (Joe, 1997). The use of such copula framework to capture the tail asymmetries has been proposed in wide range of studies (see Patton, 2012).

(3)

$$C_{JC}(u,v;\tau^U,\tau^L) = 1 - \left(1 - \left\{\left[1-(1-u)^k\right]^{-\gamma} + \left[1-(1-v)^k\right]^{-\gamma} - 1\right\}^{-\frac{1}{\gamma}}\right)^{\frac{1}{k}}$$

where $k = \dfrac{1}{\log_2(2-\tau^U)}$, $\gamma = \dfrac{1}{\log_2(\tau^L)}$ and $\tau^U, \tau^L \in (0,1)$. Alternatively, the JC copula is the Laplace transformation of the Clayton copula.