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Research paper

A primer on the pricing of electric energy options in Brazil via mean-reverting stochastic processes

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HIGHLIGHTS

- The CKLS model was applied to find the best SDE for Brazilian energy prices.
- The parameters were estimated using the quasi-maximum likelihood method.
- Monte Carlo simulation was performed to price basic and exotic energy options.
- Call and put options were priced for different maturities.

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ABSTRACT

Pricing option contracts on electricity remains methodologically challenging, with a lack of clearly defined and robust methods. In particular, little is known about pricing options in Brazilian energy markets, despite their economic significance. Using weekly price data (R$/MWh) on four electrical subsystems from the Chamber for Commercialization of Electrical Energy, we estimate models to price Brazilian electricity energy options. This paper has three objectives: (i) to identify the occurrence of change-points (regime-switching) in time series of Brazilian energy spot prices; (ii) to determine the best Stochastic Differential Equation (SDE) with which to model Brazilian energy spot prices and (iii) to price five types of options used to manage electricity price risk in Brazil. We show that the change-point occurred between 2002 and 2018. During this period, the long-run marginal cost of production was the most affected. Furthermore, we find that the Ornstein–Uhlenbeck/Vasicek stochastic process and resulting SDE best explains electricity prices in Brazil, even with the occurrence of structural changes. Finally, our results indicate that Asian-style options are the least costly option contracts to manage electricity price risk in Brazil.

1. Introduction

The initial model of the Brazilian electricity sector was revised in 2004. Since then, consumers and distributors in Brazil can buy or sell electricity contracts in two ways. The first, Regulated Contracting Environment (RCE), is mainly used by power distributors who provide electricity to households, who are captive consumers. The second, Free Contracting Environment (FCE), allows industries and large corporations to negotiate their electricity purchases directly from electricity suppliers by means of bilateral contracts. Both of these contract environments are regulated by the National Agency of Electrical Energy (ANEEL), and managed by the Chamber for Commercialization of Electrical Energy (CCEE) (Souza and Legey, 2010; Rego and Parente, 2013; Rego, 2013).

By 2016, the FCE market turnover had witnessed a jump from the prevailing average of 45 GW to 120 GW. To address this important growth, a clearinghouse was set up in Brazil to facilitate the negotiation of physical and financial contracts to buy or sell electricity. The clearinghouse was created by two companies, Brix and BBCE, and was expected to contribute to financing the market expansion by increasing competition and lowering energy costs (Freire, 2016). Its main purpose was to introduce electricity energy derivatives, which would be financially settled by means of electronic trading platforms. The latter would attract banks and investment firms, increasing the liquidity of the electricity contracts (Costa, 2016a,b).

However, the clearinghouse has yet to introduce the energy derivative contracts in question. Part of the challenge is the difficulty of accurately pricing these instruments. Thus, there is a need

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for theoretical and empirical research to develop and estimate stochastic models to allow market participants to price energy derivatives in Brazil. To achieve this, there are two key steps (Geman, 2005; Iacus, 2011). First, one must determine the best model to describe the stochastic process for electricity spot prices. Second, the option contract, its payoff, and its characteristics must be defined and related to the spot price process through the solution of a stochastic differential equation (SDE), enabling the option to be priced accurately. Unfortunately, the existing literature has not yet provided robust models for energy contracts in Brazil.

The exception is Simões et al. (2011), who simulate swing options for a hypothetical derivatives market relating to the FCE. To describe the price stochastic process, they use an Ornstein–Uhlenbeck (or Vasicek) model without jumps. Options are priced using a trinomial multi-layer tree. However, their study does not consider: (i) the occurrence of a change-point (or regime-switching) in the time series of energy spot prices; (ii) the use of other stochastic differential equations that could better model energy spot prices; or (iii) the pricing of other, popular options such as European, American or Asian.

Thus, the contribution of this paper is to present a primer for the estimation of prices of call and put option prices of five different types (European, American, Asian, Lookback-floating strike, and Lookback-fixed strike) using the most appropriate stochastic processes to describe the underlying asset prices. To the best of our knowledge, this study is the first to make these contributions. Our results have the potential to be useful to many different stakeholders in energy markets, to help them make better and more informed decisions, and to improve market efficiency when the instruments will be formally introduced.

A contribution to theory is that our results provide evidence that the options are best described as being written on mean-reverting stochastic processes, rather than geometric Brownian motion as is often assumed in the literature.

The remainder of this paper is organized as follows: Section 2 describes the models, methodology, and results for the stochastic processes that are the best suited to describe the time series behavior of electricity spot prices in the FCE market. Section 3 provides evidence for the models, methodology, and results concerning the pricing of the option contracts in question. Section 4 summarizes the main findings of this study and suggests potential, related research questions for future research.

2. Definition of stochastic price models for the FCE market

2.1. Estimation of the models

The first step is to determine the most appropriate stochastic process to describe the time series behavior of spot prices. It is useful to consider the CKLS model, which is very general and nests a wide range of possible models we wish to consider (Chan et al., 1992). Although it was introduced to describe the stochastic behavior of short-term interest rates, the CKLS model has been found useful in many financial market settings (Iacus, 2008, 2011; Geman and Shih, 2009; Iacus and Yoshida, 2018). We define the model as:

$$dX_t = \underbrace{(\theta_1 - \theta_2 X_t)\, dt}_{Drift} + \underbrace{\theta_3 X_t^{\theta_4} dW_t}_{Diffusion} \tag{1}$$

where the parameters θ_1, θ_2, θ_3 and θ_4 must be estimated so that the curve fits the data appropriately. dW_t is a Wiener process. Table 1 presents eight drift–diffusion models nested in the CKLS model under different parametric specifications. In all cases, $\theta_3 > 0$, because it describes the standard deviation on the diffusion coefficient (Chan et al., 1992; Iacus, 2008, 2011; Iacus and Yoshida, 2018).

For our sampled data $X_n = (X_{ti})_{i=0,\dots,n}$, with $t_i = i\Delta_n$, $\Delta_n \to 0$ as $n \to \infty$. The estimation method used is a quasi-maximum likelihood estimator (QMLE) which approximates the true likelihood for multidimensional diffusions (Iacus, 2011; Brouste et al., 2014; Iacus and Yoshida, 2018). We define QMLE as:

$$l_n(X_n, \beta) = -\frac{1}{2}\sum_{i=1}^{n}\left\{\log\det\left(\sum_{i-1}(\beta_1)\right) + \frac{1}{\Delta_n}\sum_{i-1}^{-1}(\beta_1)\right.$$
$$\left.\left[(\Delta X_i - \Delta_n a_{i-1}(\beta_2))^{\otimes 2}\right]\right\} \tag{2}$$

where $\beta = (\beta_1, \beta_2)$, $\Delta X_i = X_{ti} - X_{ti-1}$, $\Sigma_i(\beta_1) = \Sigma(\beta_1, X_{ti})$, $a_i(\beta_2) = a(X_{ti}, \beta_2)$, $\Sigma = b^{\otimes 2}$, $A^{\otimes 2} = AA^T$ and A^{-1} the inverse of A, $A[B] = tr(AB)$. The drift parameters are represented by β_2 while the diffusion parameters are represented by β_1. The QMLE of β is an estimator that satisfies Eq. (3).

$$\hat{\beta} = \underset{\beta}{\operatorname{argmax}}\; l_n(X_n, \beta) \tag{3}$$

In order to get consistent estimators that are stationary and ergodic for the process X, we have $n\Delta_n = T \to \infty$; $\Delta_n \to 0$; $n \to \infty$, T being the terminal value of the rescaled sample, having the same size as of n.

As there are no contractual mechanisms to trade futures or options on electricity in Brazil markets, the parameter estimation of the model can be inefficient. This is true especially for weekly data, as the feature of jumps with a very high speed of mean reversion can be distorted. Thus, as in Simões et al. (2011), we do not include jump processes in the drift–diffusion models.

Given the changes in the markets, we must pay attention to change-point analysis. This involves identifying the date at which the parameters of the stochastic model change due to exogenous factors. The most important parameter in this setting, as in financial markets more generally, is changes in the volatility process, i.e. the diffusion term in the model above. Indeed, the drift term is typically treated as unknown or as a nuisance term in the statistical model. However, if $T \to \infty$, then it is possible to estimate it consistently (Iacus, 2008, 2011).

Here, we consider the change-point problem for volatility as a one-dimensional Itô process, formalized as follows (Iacus and Yoshida, 2012, 2018; Brouste et al., 2014):

$$X_t = \begin{cases} X_0 + \int_0^t a_s ds + \int_0^t b(X_s, \theta_0^*) dW_s & for\ t \in [0, \tau^*) \\ X_{\tau^*} + \int_{\tau^*}^t a_s ds + \int_{\tau^*}^t b(X_s, \theta_1^*) dW_s & for\ t \in [\tau^*, T] \end{cases} \tag{4}$$

where the change-point τ^* is not known and needs to be estimated, along with θ_0^* and θ_1^*, using the observations sampled from the path of X. The drift term of Eq. (1) is represented by a_s while the diffusion term is represented by $b(X_s, \theta)$. We estimate the parameters using the quasi-maximum likelihood approach as reported in Iacus and Yoshida (2012). Considering $\Delta_i X = X_{ti} - X_{ti-1}$, we have

$$\Phi_n(t; \theta_0, \theta_1) = \sum_{i=1}^{[nt/T]} G_i(\theta_0) + \sum_{i=[nt/T]+1}^{n} G_i(\theta_1) \tag{5}$$

$$G_i(\theta) = \log\det(X_{ti-1}, \theta) + \Delta_n^{-1}(\Delta_i X)' S(X_{ti-1}, \theta)^{-1}(\Delta_i X) \tag{6}$$

and $S = b^{\otimes 2}$. Eq. (5) is the change-point statistic. Assume that there exists an estimator $\hat{\theta}_k$ for each θ_k, k = 0, 1. In case θ_k^* are known, we define $\hat{\theta}_k$ just as $\hat{\theta}_k = \theta_k^*$. Thus, the change-point estimator of τ^* is

$$\hat{\tau} = \underset{t \triangleright [0,T]}{\operatorname{argmin}} \Phi_n(t; \hat{\theta}_0, \hat{\theta}_1) \tag{7}$$

In practice, the initial values of θ are unknown, and we must specify some preliminary estimators for them. A useful

Table 1
The drift–diffusion models set of CKLS stochastic processes.
Source: Chan et al. (1992), Iacus (2008, 2011) and Iacus and Yoshida (2018).

Stochastic process	θ_1	θ_2	θ_4	Mean reverting	Drift–Diffusion model
1. Merton	Any	0	0	No	$dX_t = \theta_1 dt + \theta_3 dW_t$
2. Vasicek	Any	Any	0	Yes	$dX_t = (\theta_1 - \theta_2 X_t)\, dt + \theta_3 dW_t$
3. CIR-SR	Any	Any	1/2	Yes	$dX_t = (\theta_1 - \theta_2 X_t)\, dt + \theta_3 X_t^{1/2} dW_t$
4. Dothan	0	0	1	No	$dX_t = \theta_3 X_t dW_t$
5. GBM or B&S	0	Any	1	Yes	$dX_t = -\theta_2 X_t dt + \theta_3 X_t dW_t$
6. Brennan–Schwartz	Any	Any	1	Yes	$dX_t = (\theta_1 - \theta_2 X_t)\, dt + \theta_3 X_t dW_t$
7. CIR-VR	0	0	3/2	No	$dX_t = \theta_3 X_t^{3/2} dW_t$
8. CEV	0	Any	Any	Yes	$dX_t = -\theta_2 X_t dt + \theta_3 X_t^{\theta_4} dW_t$

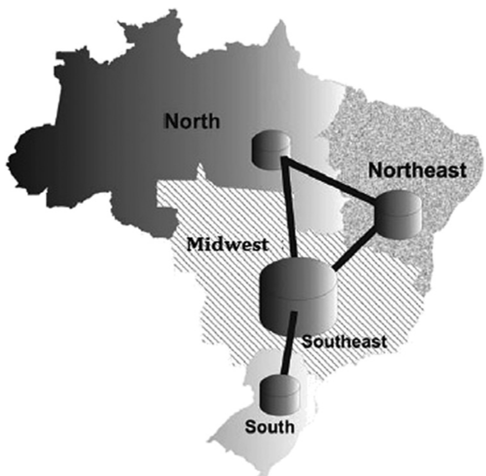

Fig. 1. The Brazilian electrical system.
Source: Souza and Legey (2010).

approach is the two-stage change-point estimation method (Iacus and Yoshida, 2012, 2018). The aim is to take a small subset of observations at the very beginning and the end of the time series to get initial values of the parameters θ, estimate a change-point, and then improve the estimation of θ using the information about the change-point.

2.2. Data collection, model setup, and descriptive statistics

We collect data from 8 March 2002 to 27 July 2018 on Prices for Settlement of Differences (PSD) (R$/MWh), which are short-run market weekly spot prices for electricity in Brazil. The source is the CCEE website.[1] This data collection yields 857 observations for the heavy market in four Brazilian subsystems: Southeast–Midwest (SE), South (S), Northeast (NE) and North (N) (see Fig. 1). We have chosen this market because it represents the time of day where electricity consumption is the highest and when spikes may occur. This feature is a stylized fact of electricity prices (Geman, 2005; Weron, 2009).

The prices are denominated in the Brazilian currency (R$). It could be misleading to convert prices to US dollars because they would reflect an additional source of market risk, namely exchange rate fluctuations, and would not reflect the problem faced by market participants. For estimation purposes, we use the natural logarithm of PSD. To estimate the parameters of Eq. (1), the Δ_n was set up in 0.5 such that empirically $n\Delta_n = T \to \infty$; $\Delta_n \to 0$; $n \to \infty$ which allows us to obtain consistent estimators of the parameters from the data.

[1] https://www.ccee.org.br. (In Portuguese).

Table 2
Descriptive statistics for log returns PSD, heavy market (N = 856).

Variable	Southeast–Midwest	South	Northeast	North
Mean	0.0055	0.0056	0.0057	0.0057
SD	0.2803	0.3623	0.3118	0.3393
Skewness	−0.9020	0.0005	−1.3108	−0.6773
Kurtosis	8.9835	22.3944	16.1437	8.6121
Minimum	−1.9049	−2.8582	−2.6199	−1.9166
Maximum	1.3915	3.5590	1.9585	1.7966
Prob. Function	Cauchy	Cauchy	Cauchy	Cauchy
CvM *p*-value	0.0842	0.0787	0.0549	0.0537

To determine the change-point in the data, we divide the rescaled sample in two (857*0.5/2 ≅ 214), then add/subtract 10 units (204 and 224). Thus, each subsample contains 408 observations of the original sample. This step is justified by the QMLE algorithm's sensitivity to sample size, which affects the consistency of the estimated variance–covariance matrix. Lastly, we employ a four-stage change-point estimation approach to ensure the stability of τ^* during its estimation.

Table 2 shows the descriptive statistics of log returns and Fig. 2 displays the histograms and the kernel densities for the heavy energy market in all subsystems.

Log returns for the South subsystem presents the highest standard deviation (SD) or volatility, the largest amplitude (based on its minimum and maximum), and it is the most symmetrical and leptokurtic compared to the other subsystems. The remaining subsystems exhibit a slightly negative asymmetry, and they are also leptokurtic. This feature implies a higher probability of extreme events (compared to a Normal distribution) for the historical data (Geman, 2005; Weron, 2009).

To estimate the best-fitting probability distribution function (PDF), we estimate and test 10 different distributions: normal, lognormal, beta, gamma, exponential, logistic, uniform, t, Cauchy and Weibull. To test the null hypothesis that the data fits the distribution, we employ the two-sample Cramér–von Mises criterion (Anderson, 1962).

The results in Table 2 show that all log return series are best described by a Cauchy density, which is in the family of stable Paretian–Lévy distributions. Previous studies have shown that these distributions best describe the probability of extreme events in many equity and commodity price datasets, rather than the normal distribution (Mandelbrot, 1963, 1967; Fama, 1965; Jin, 2007; Kateregga et al., 2017).

2.3. Estimation results for the stochastic parameters

We first estimate the parameters in Eq. (1) without considering the existence of change-point. Table 3 shows the results of the estimated CKLS models, which are strictly positive and stationary since the inequality $2\theta_1 > \theta_3^2$ holds.

Comparing these results with the drift–diffusion models presented in Table 1, we see that the log PSD for the heavy market in the Southeast–Midwest and the South subsystems are well

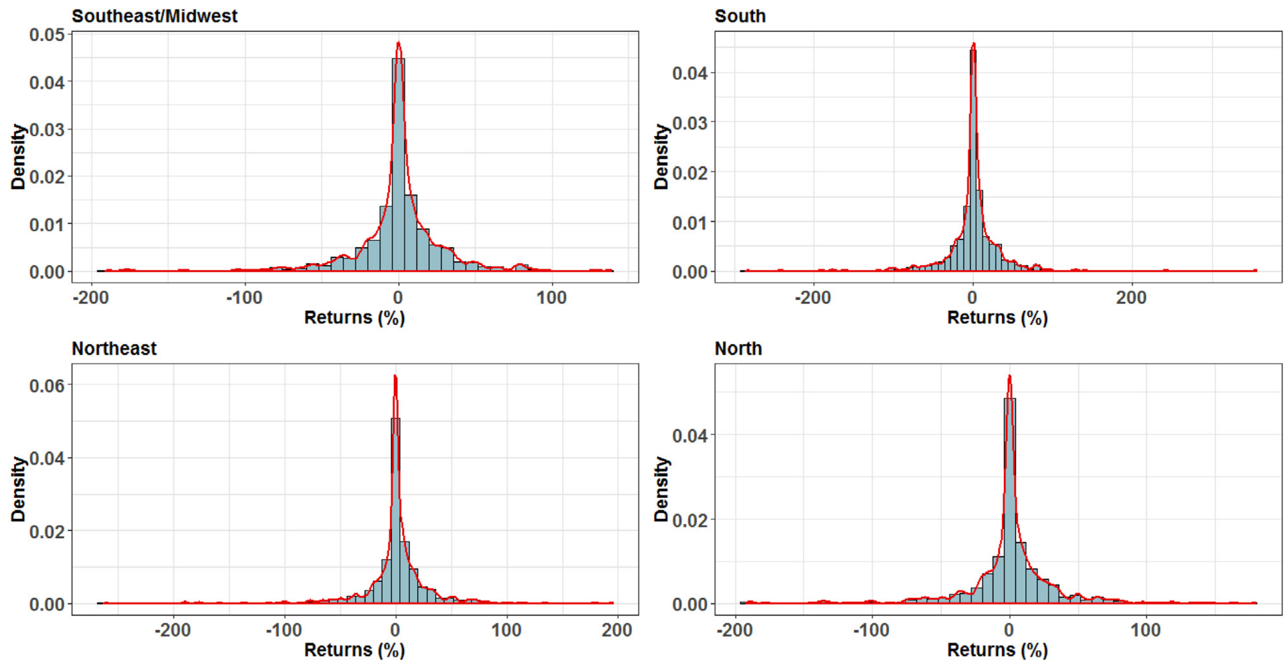

Fig. 2. Histograms and kernel densities for log returns PSD, heavy market.

Table 3
Results of CKLS parameters for log PSD (R$/MWh), heavy market, without change-point.

Parameters	Southeast–Midwest	South	Northeast	North
θ_3	0.396^+	0.512^+	0.306^+	0.365^+
θ_4	0.000	0.000	0.262^+	0.198^+
θ_1	0.208^+	0.332^+	0.218^+	0.278^+
θ_2	0.046^+	0.076^+	0.049^+	0.065^+
θ_3 t-statistic	7.518	8.846	8.323	7.740
θ_4t-statistic	0.000	0.000	3.091	2.145
θ_1t-statistic	3.251	4.405	3.598	3.978
θ_2t-statistic	3.228	4.099	3.387	3.847
μ (R$/MWh)	89.53	80.56	84.69	73.47

Note: Values with $^+$ are significant at 5%. $\mu = \exp(\theta_1/\theta_2)$.

described by an Ornstein–Uhlenbeck/Vasicek process (OU). This is consistent with Simões et al. (2011)'s findings that such prices follow a mean-reverting process with finite variance for all $t \geq 0$.

Moreover, the results show that the long-run equilibrium price, μ, also known as the long-run marginal cost of production (Dixit and Pindyck, 1994), increased from R$/MWh 41.87 (Simões et al., 2011) to R$/MWh 89.53 and R$/MWh 80.56 for the two subsystems, respectively. However, the log PSD for the heavy market in the Northeast and the North subsystems follow a CKLS mean-reverting process. Unlike the Vasicek process, where the conditional volatility of the changes in log PSD is constant, the CKLS model implies that the conditional volatility of the changes in log PSD is proportional to log PSD (Chan et al., 1992), i.e., there is a type of stochastic volatility. For these subsystems, the long-run equilibrium price, μ, is between the range of R$/MWh 84.69 and R$/MWh 73.47.

In addition, as $\theta_4 < 1$, the results imply that volatility and prices are inversely related in the Northeast and North subsystems. This finding provides evidence of a leverage effect, capturing a negative volatility skew. This phenomenon is typical of equity markets, but is less expected in commodity markets (Geman, 2005; Geman and Shih, 2009).

Lastly, for all subsystems, the speed of mean reversion (θ_2) is very slow, indicating that Brazilian electricity prices take a long time to revert to their equilibrium state.

Table 4
Change-point τ^* values for all Brazilian subsystems (N = 857).

Change-point τ^*	Southeast–Midwest	South	Northeast	North
Stage 1	201st	638th	520th	202nd
Stage 2	167th	638th	520th	202nd
Stage 3	161st	638th	520th	202nd
Stage 4	161st	638th	520th	202nd

Next, we estimate the change-point, τ^*, using Eq. (7) for all subsystems. The values presented in Table 4 indicate the position (date) in the historical time series where there is evidence of a change/variation in the parameters of the model (which is the most standard definition for a change-point, see e.g. (Iacus, 2011).

Apart from the Southeast–Midwest subsystem, all other subsystems were affected by a change-point in stage 1. The first subsystem required four stages to achieve it.

As described in the calendar, the most relevant change-points occurred in the first week of April 2005 for the Southeast–Midwest subsystem, in the third week of May 2014 for South subsystem, in the third week of February 2012 for the Northeast subsystem, and in the second week of January 2006 for the North subsystem. Among several possibilities, the occurrence of extreme climactic events is a plausible explanation for the timing of these change-points. Since 64% of the electricity produced in Brazil comes from hydroelectric plants, major weather-related events can adversely affect water reservoirs and as a result, electricity production and prices (Oliveira and Mandal, 2018).

After determining the most suitable change-points for the electricity price series, the parameters of Eq. (1) were re-estimated for the periods before ($t \leq \tau^*$) and after ($t > \tau^*$). The respective change-points are shown in Table 5

In this new framework, three specifications of Eq. (1) are found. For $t \leq \tau^*$, the Southeast–Midwest and South subsystems follow an Ornstein–Uhlenbeck/Vasicek process, the Northeast subsystem follows a Brennan–Schwartz process (no leverage effect), and the North subsystem follows a CKLS process (leverage effect), quite close to a CIR-SR process. On the other hand, for $t > \tau^*$, all subsystems follow an Ornstein–Uhlenbeck/Vasicek process.

Table 5
Results of CKLS parameters for log PSD (R$/MWh), heavy market, with change-point.

Parameters	Southeast–Midwest		South[a]		Northeast		North	
	$t \leq \tau^*$	$t > \tau^*$	$t \leq \tau^*$	$t > \tau^*$	$t \leq \tau^*$	$t > \tau^*$	$t \leq \tau^*$	$t > \tau^*$
θ_3	0.228^+	0.426^+	0.440^+	0.679	0.135^+	0.373^+	0.219^+	0.513^+
θ_4	0.000	0.000	0.000	0.000	1.004^+	0.000	0.472^+	0.000
θ_1	0.254^+	0.336^+	0.251^+	1.574^+	0.254^+	1.311^+	0.362^+	0.434^+
θ_2	0.092^+	0.070^+	0.061^+	0.293^+	0.073^+	0.231^+	0.130^+	0.093^+
θ_3 *t*-statistic	4.439	5.205	7.925	1.642	8.429	4.072	4.747	4.609
θ_4*t*-statistic	0.000	0.000	0.000	0.000	10.303	0.000	2.264	0.000
θ_1*t*-statistic	2.229	3.503	3.115	3.979	3.635	5.230	3.087	3.868
θ_2*t*-statistic	2.134	3.505	3.055	4.046	2.941	5.176	2.855	3.902
μ (R$/MWh)	15.70	119.76	61.75	214.48	32.26	294.85	16.13	107.88

[a]In the South subsystem, it was detected for $t > \tau^*$ that θ_3 is not significant, which is implausible since this sub-sample has a standard deviation, as seen in Fig. 3. This is a failure of the QMLE approach, which can occur when dealing with small samples [n = 219; T = 110]. Thus, the estimated value was taken into account since its *t*-statistic is very close to 10% of significance (author's note).
Note: Values with $^+$ are significant at 5%. $\mu = \exp(\theta_1/\theta_2)$.

Table 6
Comparison of the moments for log PSD (R$/MWh), heavy market.

MOMENTS			Southeast–Midwest	South	Northeast	North
Mean	$t \leq \tau^*$	Estimated	2.75^{OU}	4.12^{OU}	NA	2.78^{CIR-SR}
		Empirical	2.57	3.85	3.32	2.67
	$t > \tau^*$	Estimated	4.79^{OU}	5.37^{OU}	5.69^{OU}	4.68^{OU}
		Empirical	4.65	5.39	5.59	4.57
Variance	$t \leq \tau^*$	Estimated	0.28^{OU}	1.59^{OU}	NA	0.51^{CIR-SR}
		Empirical	0.35	1.54	1.06	0.47
	$t > \tau^*$	Estimated	1.29^{OU}	0.79^{OU}	0.30^{OU}	1.42^{OU}
		Empirical	1.31	0.81	0.42	1.43

These results suggest a type of convergence in the stochastic behavior of prices across markets after the change-point date.

Looking individually at each subsystem, note that the Southeast–Midwest and the South kept the same stochastic processes, with only parameter values changing after the change-point. However, for the Northeast and North subsystems, both the process and the parameters changed. In particular, the diffusion component changed, such that volatility went from stochastic to constant.

The long-run equilibrium price, μ, is most affected by the change-point, indicating that the marginal cost of production of electricity increased considerably in all subsystems. In proportion, the largest increase was in the Northeast subsystem (9.1 times), while the smallest was in the South subsystem (3.5 times).

Using the estimated parameter values reported in Table 5, we use Monte Carlo simulation (250,000 time series observations simulated before and after the change-point) to construct forecasting intervals for each subsystem, as shown in Fig. 3. From a *trajectory* standpoint, the forecasting intervals satisfactorily describe the behavior of the log PSD for all subsystems.

Table 6 compares (from a *statistical* standpoint[2]) the estimated and empirical mean and variance values of the historical data before and after the change-point (the results of which are quite close).

3. Pricing options for the FCE market

3.1. Payoff functions

We have shown that electricity prices in Brazil follow mean-reverting processes with change-points. Thus, it is not possible to use explicit formulas such as the Black–Scholes–Merton equation to compute option prices. Thus, we use instead a Monte Carlo approach for random variables, with the general price formula is presented in Eq. (8) (Geman, 2005; Iacus, 2011):

$$C(t, x) = e^{-r(T-t)}\mathbb{E}f(Z_T^{t,x}) \tag{8}$$

where r is the free-risk interest rate, T is the maturity, t is the initial time, and $f(Z_T^{t,x})$ is the payoff function of the random underlying asset, based on a Wiener process (dW_t). The Monte Carlo method works as follows: (i) simulate M copies of the random underlying asset to get $Z_T^{t,x}$ values, (ii) apply the payoff function $f(\cdot)$ to each $Z_T^{t,x}$ that is simulated, (iii) calculate the average $\mathbb{E}(\cdot)$ of these results, and iv) compute the present value by applying the discount factor $e^{-r(T-t)}$. Table 7 shows the equations used to calculate the price of a call/put option for five different types/styles.

A *European* option gives its holder the right (but not the obligation) to buy/sell the underlying asset on at some given date and for a predetermined price. An *American* option is a European option that can be exercised by its holder at any time at or prior to maturity. These two types of options are grouped as basic or standard.

An *Asian* option is a generic name for a class of options whose terminal payoff is based on average asset value over some period within the option contract's lifetime. A *Lookback* option has a payoff function that depends on the minimum or maximum price of the underlying asset during the option's lifetime. In particular, if it has a floating strike price, it will be always exercised by its holder at maturity.

These last three classes of options are grouped as exotic path dependent because of the payoff at expiry depends not only on the final prices of the underlying asset, but also on asset price variation over the option's lifetime.

[2] The moments in Table 6 belong to the stationary distribution or to the transition probability density function of the stochastic processes. For a Vasicek model (OU), the mean is θ_1/θ_2 and the variance is $\theta_3^2/2\theta_2$. For a CIR-SR model, the mean is θ_1/θ_2 and the variance is $\theta_1\theta_3^2/2\theta_2^2$. Other models such as CKLS or Brennan–Schwartz (NA) do not have such explicit expressions (Brouste, 2017).

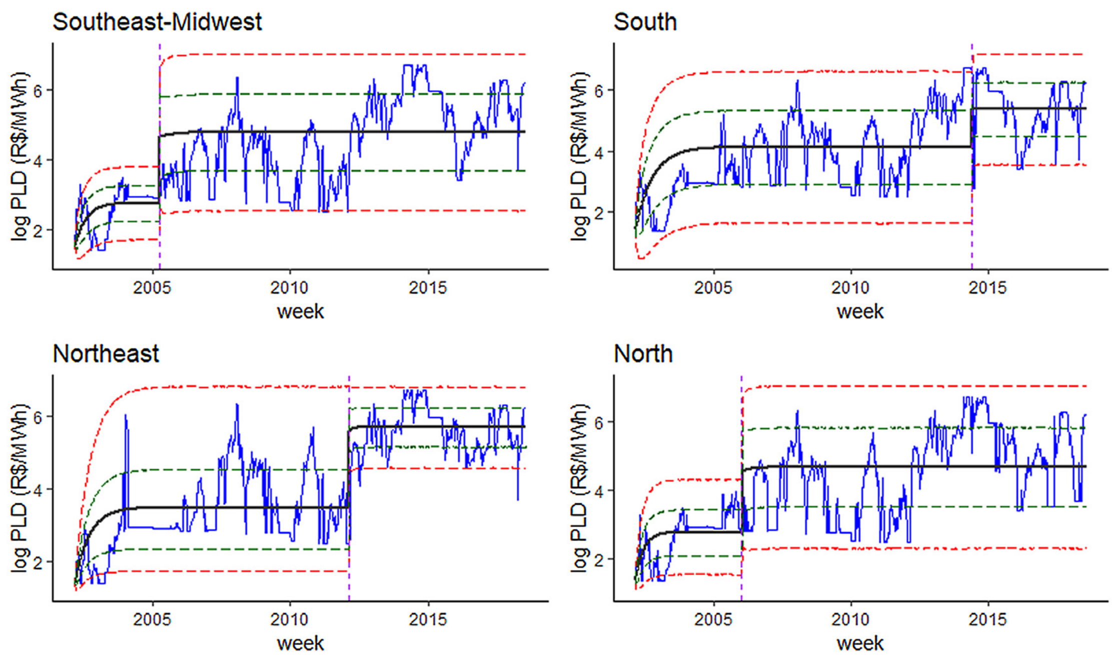

Fig. 3. Forecasting intervals for log PSD, heavy market. Legend. Blue lines: log PSD time series; black lines: historical averages; green dotted lines: 68% forecasting interval; red dashed lines: 95% forecasting interval; purple vertical lines: change-point.

Table 7
Functions for pricing basic and exotic options via Monte Carlo method (discrete formulas).
Source: Heynen and Kat (1995), Geman (2005), Musiela and Rutkowski (2005) and Iacus (2011).

Type of option	Call function - C(t, S)	Put function - P(t, S)
1. European	$e^{-rT}\mathbb{E}\max(S_T-K,0)$	$e^{-rT}\mathbb{E}\max(K-S_T,0)$
2. American	$\max_{i\in[0,T]}\mathbb{E}\left\{e^{-rt_i}max(S_i-K,0)\right\}$	$\max_{i\in[0,T]}\mathbb{E}\left\{e^{-rt_i}max(K-S_i,0)\right\}$
3. Asian	$e^{-rT}\mathbb{E}\max\left(\frac{1}{T+1}\sum_{i=0}^{T}S(t_i)-K,0\right)$	$e^{-rT}\mathbb{E}\max\left(K-\frac{1}{T+1}\sum_{i=0}^{T}S(t_i),0\right)$
4. Lookback fixed strike	$e^{-rT}\mathbb{E}\max\left(\max_{t\in[0,T]}S_t-K,0\right)$	$e^{-rT}\mathbb{E}\max\left(K-\min_{t\in[0,T]}S_t,0\right)$
5. Lookback floating strike	$e^{-rT}\mathbb{E}S_T-\min_{t\in[0,T]}S_t$	$e^{-rT}\mathbb{E}\max_{t\in[0,T]}S_t-S_T$

Note: The variables are: interest rate (r); maturity (T); underlying price (S); strike price (K); t = 0.

Table 8
Call and Put prices for European Options — PSD (R$/MWh).

Maturity (Months)	CALL				PUT			
	SE	S	NE	N	SE	S	NE	N
03	139.60	162.21	98.17	141.79	30.25	48.24	47.81	28.01
06	138.47	160.13	97.30	139.47	29.81	47.60	47.22	27.63
09	135.65	157.53	95.62	137.22	29.36	46.71	46.47	27.31
12	134.20	154.27	94.68	135.96	29.06	45.89	45.88	26.89

Table 9
Call and Put prices for American Options — PSD (R$/MWh).

Maturity (Months)	CALL				PUT			
	SE	S	NE	N	SE	S	NE	N
03	142.05	164.74	99.88	143.86	30.65	49.12	48.53	28.49
06	142.05	164.74	99.88	143.86	30.65	49.12	48.53	28.49
09	142.05	164.74	99.88	143.86	30.65	49.12	48.53	28.49
12	142.05	164.74	99.88	143.86	30.65	49.12	48.53	28.49

3.2. Monte Carlo simulation setup

To perform the Monte Carlo simulation, Eq. (8) is replaced with the parameters of Table 5, with the payoff functions of Table 7, and with the following variables configured:

- Risk-free interest rate (r): 6.4% per year or 0.1194% per week;
- Starting point (t): 27 July 2018 (t = 857). The initial value (S_0) was the simulated log PSD up to the starting point;
- Maturity (T) ∈ [870; 883; 896; 909], i.e., 13, 26, 39 and 52 weeks (or 3, 6, 9 and 12 months) ahead of the starting point;
- Number of simulations (M): 250,000 log PSD time series, to guarantee non-negativity and linearity of prices;
- Underlying price (S): the log PSD was transformed in PSD after the simulations;
- Strike price (K): simulated average PSD at maturity T, except for Lookback floating strike, which is the PSD_t minimum/maximum, $t \in [0, T]$;

The risk-free interest rate used is SELIC, which is applied in Brazilian public securities, and collected on 27 July 2018 from The Central Bank of Brazil.

3.3. Option price results

Tables 8–12 show the call and put prices for European, American, Asian, Lookback fixed strike, and Lookback floating strike options, respectively. We analyze several aggregated option properties in these tables make it easier to interpret the results for the Brazilian energy subsystems: Southeast/Midwest (SE), South (S), Northeast (NE) and North (N).

Considering option prices in relation to subsystem, the South subsystem tends to have the highest call and put option values,

Table 10
Call and Put prices for Asian Options — PSD (R$/MWh).

Maturity (Months)	CALL				PUT			
	SE	S	NE	N	SE	S	NE	N
03	135.61	140.24	82.92	136.41	25.99	26.09	32.25	22.79
06	130.62	128.79	73.09	130.72	22.41	16.49	23.95	18.93
09	126.08	121.95	67.49	125.56	19.46	11.10	18.35	15.88
12	121.69	116.83	63.15	121.45	17.16	7.93	14.76	13.40

Table 11
Call and Put prices for Lookback (fixed strike) Options — PSD (R$/MWh).

Maturity (Months)	CALL				PUT			
	SE	S	NE	N	SE	S	NE	N
03	322.06	613.22	302.81	364.14	52.51	123.57	119.10	51.45
06	436.68	868.75	410.88	506.21	62.72	146.82	147.48	62.08
09	530.99	1,051.98	485.94	622.79	69.49	156.22	161.35	68.85
12	611.29	1,189.68	540.47	723.17	74.60	160.73	169.45	73.11

Table 12
Call and Put prices for Lookback (floating strike) Options — PSD (R$/MWh).

Maturity (Months)	CALL				PUT			
	SE	S	NE	N	SE	S	NE	N
03	123.74	231.71	161.72	135.67	200.33	495.77	247.11	240.75
06	154.33	258.81	196.39	162.92	321.43	755.84	359.87	389.91
09	167.38	266.98	210.29	174.20	420.98	941.11	436.62	510.71
12	175.40	269.09	218.21	180.15	503.99	1,081.29	491.64	613.02

the Northeast subsystem tends to have the smallest call values, and the North subsystem tends to have the smallest put values.

Concerning the relationship between the call and put option values, the call price is higher than the put price (except for the Lookback option with a floating strike). This result means that if the strike price was the historical average PSD, it would be more expensive to exercise a call option than a put option in the FCE market context. On the other hand, for the Lookback option with a floating strike, the interpretation would be the opposite, given the variability of the strike price along with the maturity date.

Regarding the relationship between maturity and price described by a mean-reverting stochastic process, we note three cases. For European and Asian options, the price tends to decrease as the maturity increases. For both Lookback options, the opposite occurs: the price tends to increase as the maturity increases. Lastly, for American options, the price remains the same as the maturity increases.

In a mean-reverting stochastic process, one expects that, over the lifetime of a European option, the average payoff will be the same at each instant *t*, especially in an Ornstein–Uhlenbeck/ Vasicek process whose volatility is constant. Thus, the value of the option has an inverse relationship with maturity: a greater maturity implies a smaller option value and vice versa.

For Asian options, in a mean-reverting stochastic process, we would expect that, over its lifetime, the average payoff will decrease at each instant *t*, because its payoff function considers the average prices dependent on maturity, not the price itself. Therefore, the maturity has an inverse relationship with both the average payoff and the option price.

For American options, one would calculate the expected maximum present value of the payoff of a European option over its lifetime. However, since the expected payoff of a European option tends to be the same over its lifetime in a mean-reverting stochastic process, the highest expected value for an American option tends to occur in the early stages of its lifetime. Thus, no matter how much maturity increases, the value of the American option will remain the same.

For Lookback options, an explanation for the highest prices along their lifetimes can be related to the speed of mean reversion (θ_2), which is very slow, especially for prices that are above the historical mean. Therefore, the higher the maturity, the greater the chances of having higher maximum values and, thus, higher option prices, which is the case for the Lookback fixed strike call and the Lookback floating strike put options.

In relation to the price comparison between the aforementioned options, the Asian options had the smallest put values and nearly all of the smallest call values. On the other hand, the Lookback with fixed strike options had the largest call values, and the Lookback with floating strike options had the biggest put values. Thus, the results suggest that Asian options could be key instruments to manage price risk in commodities markets, especially in energy markets such as the market analyzed in this paper (Weron, 2008; Benth and Detering, 2015; Fanelli et al., 2016). Indeed, many indexes are defined as arithmetic averages of the underlying spot price. The purpose is to prevent momentary fluctuations from affecting transactions involving large exchanged quantities or volumes. In addition, Asian options have a lower volatility than other types of options, because it is based on an arithmetic mean. As a result, it will often be the least expensive option, assuming the same strike price, maturity, and free-risk interest rate (Geman, 2005).

4. Conclusion

In this study, we present a primer for the estimation of the prices of option contracts for electricity energy markets in Brazil, in anticipation of when the FCE's derivatives market becomes operational. To achieve this goal, we address three key issues which represent a significant contribution to the research literature on energy derivatives, in particular in emerging economies such as Brazil. First, we investigate the timing of a change-point (or regime-switching) in the time series behavior of energy spot prices which serve as underlying assets for option contracts. Second, we determine, from a general framework nesting several specifications, the most appropriate stochastic process to model energy spot prices. Third, we estimate the prices of five different option types using Monte Carlo simulation and the parameters estimated from the data.

Our results show that the change-points in the electricity prices in Brazil occur between 2002 and 2018, depending on the subsystem (region). These change-points significantly affect the long-run equilibrium price (or long-run marginal cost of production). We also find empirical support for log returns being described by distributions in the stable Paretian–Lévy family, indicating that the occurrence of extreme events in historical time series data is explained by heavy tails, and linked to the presence of structural changes.

We further find empirically that the Ornstein–Uhlenbeck/ Vasicek process remains the most suitable stochastic differential equation to explain the behavior of electricity prices in Brazil, even with the occurrence of structural changes. However, two other stochastic processes are empirically validated by our analysis. Indeed, we find evidence of a leverage effect in volatility, which is more commonly a feature of equity markets and not commodity markets.

Lastly, we show that Asian options are the least costly options contracts for electricity prices in Brazil. Moreover, our results indicate that maturity is inversely related with both the average payoff and the option price, based on the framework of a mean-reverting stochastic process.

For practitioners, our study provides an improved procedure for estimating the stochastic time series behavior of electricity prices in Brazil, in order to minimize pricing errors, and further to calculate the price of options by Monte Carlo simulation. A contribution to theory is that we show how mean-reverting stochastic processes, rather than geometric Brownian motion, better explain energy prices over time, with implications for option pricing. In addition, this study expands the scope of possibilities for examining option contracts on electricity prices in Brazil by focusing on price risk, as the Simões et al. (2011) only deal with volumetric risk.

Finally, this study proposes three avenues for future research on Brazilian electricity derivatives markets. First, identifying the causes of regime-switching in Brazilian electricity prices, as well as the existence of secondary change-points. Second, adding jumps to the stochastic models analyzed in this paper. Third, using alternative methods to estimate the parameters of the stochastic processes, such as the least absolute shrinkage and selection operator (LASSO), the generalized methods of moments (GMM), or the Kalman Filter.

Declaration of competing interest

None.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.egyr.2019.03.010.

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