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Ghica, Dan R.; Alyahya, Khulood

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Latent semantic analysis of game models using LSTM

Dan R. Ghica*

University of Birmingham, UK

Khulood Alyahya*

University of Exeter, UK
King Saud University, Riyadh, KSA

*Corresponding author

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Dan R. Ghica*

University of Birmingham, UK

Khulood Alyahya*

University of Exeter, UK
King Saud University, Riyadh, KSA

Abstract

We are proposing a method for identifying whether the observed behaviour of a function at an interface is consistent with the typical behaviour of a particular programming language. This is a challenging problem with significant potential applications such as in security (intrusion detection) or compiler optimisation (profiling). To represent behaviour we use game semantics, a powerful method of semantic analysis for programming languages. It gives mathematically accurate models ('fully abstract') for a wide variety of programming languages. Game-semantic models are combinatorial characterisations of all possible interactions between a term and its syntactic context. Because such interactions can be concretely represented as sets of sequences, it is possible to ask whether they can be learned from examples. Concretely, we are using LSTM, a technique which proved effective in learning natural languages for automatic translation and text synthesis, to learn game-semantic models of sequential and concurrent versions of Idealised Algol (IA), which are algorithmically complex yet can be concisely described. We will measure how accurate the learned models are as a function of the degree of the term and the number of free variables involved. Finally, we will show how to use the learned model to perform latent semantic analysis between concurrent and sequential Idealised Algol.

*Corresponding author

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1. Programming languages and machine learning

Software systems often consist of many components interacting via APIs, which can be internal to the language (e.g. libraries, modules) or external (w.g. Web APIs). The final process in producing such a system is usually the "linking" of several object files into one (or several) binary executable(s). Since the linker does not have access to the source files it is a reasonable, and very difficult, question to ask whether the object code in those files originates from an assumed programming language via correct compilation. This is an important question to ask in many contexts: compiler correctness, compiler optimisation, tamper-proofing, intrusion detection, and more. In this paper we propose a simple black-box approach to answering this question, based on game semantics and machine learning.

Programming language semantics, the way we ascribe meaning to programming languages, comes in different flavours. There is the operational approach, which consists of a collection of effective syntactic transformations that describes the execution of the program in a machine-independent way (see [39] for a tutorial introduction). There is also the denotational approach, in which subprograms (terms) are interpreted, compositionally on syntax, as objects in a mathematical semantic domain (see [44] for an introduction). The two approaches are complementary, and both have been studied extensively. Most commonly, especially for most 'real life' programming languages, there is another, *ad hoc*, approach of specifying a language, through a compiler, often informally described in a 'standard'.

Relating operational and denotational models is a mathematically difficult but worthwhile endeavour. Term equality is operationally defined in a way which is almost unworkable in practice: contextual equivalence. By contrast, term equality in the denotational model is just equality of the denoted mathematical objects. As a result, denotational models are presumably handy in

applications where equality of terms is important, such as compiler optimisations. When contextual equivalence coincides with semantic equality the model is said to be *fully abstract*, a gold standard of precision for a denotational model. Constructing fully abstract denotational models even for relatively simple higher order (PCF [42]) or procedural (Algol [36]) languages turned out to be a difficult problem, extensively studied in the 1990s. Many interesting semantic developments emerged out of this concerted effort, including *game semantics*, a technique which finally gave the first such fully abstract models first for PCF [1] and Algol [3] then for many other programming languages [16].

Relating any mathematical (operational or denotational) model to the de facto 'model' which is the compiler is a much different proposition. Whereas constructing a compiler from a mathematical specification is an arduous but achievable task, what we want to consider is the converse question. Given a compiler, could we, at least in principle, construct a semantic model of the language? What is the right avenue of attack for this daunting problem?

A compiler is in some sense a formal specification. However, the compiler as a specification does not help us reason about basic properties of terms, such as contextual equivalence. How can we extract a more conventional kind of semantics? On the face of it, the question may seem preposterous at worst, unanswerable at best.

Operational semantics (OS), the workhorse of much applied programming language theory, seems an unsuitable candidate for this job. Much like structural models of natural language, the rules of OS have a syntactic intricacy which cannot be hoped to be reconstructed from behavioural observations. Noting that recent progress has been achieved in learning the structural semantics of natural languages [23], operational semantics of programming languages cannot take advantage of these methods. For example the basic beta-reduction rule present in some form in all functional languages requires a complex form of substitution which assumes the concepts of binder, free variable and alpha equivalence. Denotational semantics on the other hand seems a more plausible candidate because of its independence of syntax. A final key observation is that some de-

notational models can be mathematically elementary. This is true of trace-like models in general [14] and game semantic models [5] in particular. In fact one can think of game semantics as compositional trace models suitable for higher order programming languages. This seems to give us a foothold in attacking the problem. If a model can be specified simply as a set of traces subject to combinatorial constraints, perhaps such models can be machine-learned using techniques that proved successful in the learning of natural languages.

For tutorials and surveys of game semantics the reader is referred to the literature [4, 16]. The basic elements of a game semantics are *moves*, with a structure called an *arena*. Arenas are determined by the type signature of the term and consist of all the possible interactions (calls and returns) between a term and its context. Sequences of interactions are called *plays* and they characterise particular executions-in-context. Finally, terms are modelled by sets of plays called *strategies*, denoting all possible ways in which a term can interact with its context.

Certainly, not all interactions are possible, so plays are constrained by legality conditions. Conversely, strategies are subject to certain closure conditions, such as prefix-closure, stipulating that if certain plays are included so must be other ones. Because all features of a game semantic model are combinatorial properties of sequences (plays) or sets of sequences (strategies), using machine learning to identify them is no longer a preposterous proposition. The question certainly remains whether these properties can be learned and how accurately.

In this paper we present two sets of computational experiments focussing on the learnability of known game semantic models of two similar programming languages. We first look at the intrinsic learnability of the language, automatically creating models from positive examples of legal plays, tested against sets of plays which are slightly modified so that to become illegal. The second experiment uses the learned models to perform latent semantic analysis [30] on the two languages, attempting to determine the provenance of a set of legal plays. These experiments are repeated both for a precise and approximate representation of the game model.

To learn the model we use neural networks, more precisely *long short-term memory neural nets* [26] (LSTM), which proved to be highly successful in automated translation [45] and text synthesis [13]. The results are surprisingly good, with the trained net being able to reliably discriminate both between legal and illegal plays, and between legal plays from two slightly different programming languages. Moreover, the neural net had a standard architecture and, relative to LSTMs used in natural language processing, was quite small. Training converged rapidly, requiring relatively modest computational resources.

These positive results should be received with cautious optimism. Methodologically, a strong case can be made that game semantics gives a possible angle of attack on the machine-learning problem for programming languages, compared to operational or other denotational programming language semantics (e.g. domain-theoretic [2]). Moreover, it appears that the algorithmically complex combinatorial patterns which characterise the legality of game models are learnable enough to be able to reliably distinguish between legal plays and plays with small illegal irregularities and between plays belonging to slightly different languages.

Of course, the resulting model is opaque and cannot serve as a basis for true understanding of a language, but it could be the starting point of a deeper automation of certain programming language processes which require an effective, even if opaque, semantic model to distinguish between legal (possible) behaviour and illegal (impossible) behaviour. Activities such as testing, fuzzing, or compiler optimisations fall within this broad range.

A possible objection to our approach is that we generate training data sets from a known semantic model, whereas our stated initial problem referred to languages 'in the wild' which have no such models. To respond, there is no substantial difference between generated plays from a known model and collecting interaction traces from instrumented real code, using some form of time-stamped profiling – that is the consequence of the full abstraction result for the model. But this process is far more laborious than producing the traces from a known model. A known model has other advantages compared to using unknown code.

Models of IA, both sequential and concurrent, have been studied algorithmically and are known to be complex, so the learning problem is non-trivial [34, 19]. For a controlled experiment in learnability ours is a suitable methodology, and the results indicate that applying the technique to real languages has potential.

2. Idealised Algol (IA)

For the sake of a focussed presentation we shall look at two variations on the programming language Idealised Algol (IA) [43]. IA is suitable for this experiment for several reasons. To begin with, it is a family of well-studied programming languages having at their core an elegant fusion of functional and imperative programming. We will concentrate in particular on two members of this family, Abramsky and McCusker's version of sequential IA [3] and Ghica and Murawski's version of concurrent IA [20]. Both these languages have mathematically precise (*fully abstract*) game semantics which have an underlying common structure which makes it possible, but not trivial, to compare them. Finally, from a pragmatic point of view, the models themselves are elegant, can be presented concisely, and lend themselves well to computational experimentations.

IA has basic data types, such as integers and booleans, with which three kinds of ground data types are constructed: commands (unit), local variables (references) and expressions. Function types are uniformly created out of these ground types. The terms of the language are those common in functional (abstraction and application, recursion, if expressions, arithmetic and logic) and imperative (local variables, assignment, de-referencing, sequencing, iteration). A peculiarity of IA, which sets it apart from most commonly encountered programming languages is the fact that it uses a *call-by-name* mechanism for function application [41]. For technical reasons, the IA we study here allows side-effects in expressions and admits a general variable constructor in which reading and writing to a variable can be arbitrarily overloaded. Concurrent IA, as described here, uses the same types plus a new type for *binary semaphores*, along with new

terms for parallel execution of commands and semaphore manipulation. Both languages have the type system of the simply-typed lambda calculus, with all language constants definable as (possibly higher-order) constants.

2.1. Game Semantics

In game semantics the element of interaction between a term-in-context and the context is called a *move*. Interactions characterising any particular execution are called *plays*. All possible interactions with all possible contexts are called *strategies.*

Moves happen in *arenas*, mathematical structures which define the basic causal structures relating such actions.

Definition 1 (Arena)**.** *An arena A is a set M equipped with a function $\lambda : M \to \{\mathsf{o}, \mathsf{p}\} \times \{\mathsf{q}, \mathsf{a}\}$ assigning each move four possible* polarities *and a relation $\vdash \subseteq M \times M$ called* enabling.

The four polarities are opponent/proponent and question/answer.

Arenas are used to give interpretion to types.

Definition 2 (Base arenas)**.** *The arenas for unit and boolean are:*

$$\mathit{unit} : M = \{q, a\}, \qquad \lambda = \{(q, \mathsf{oq}), (a, \mathsf{pa})\}, \qquad \vdash = \{(q, a)\}$$

$$\mathit{bool} : M = \{q, t, f\}, \quad \lambda = \{(q, \mathsf{oq}), (t, \mathsf{pa}), (f, \mathsf{pa})\}, \quad \vdash = \{(q, t), (q, f)\}.$$

The significance of the *question* q is that a computation is initiated and of the *answer* a (or, respectively t or f) is that a result is produced. The enabling relation establishes that the answer must be justified by the asking of the question. The interpretation of the opponent/proponent polarity is that 'proponent' moves are initiated by the term whereas 'opponent' moves by the context. As we can see, for computation at base types the computation is initiated via questions asked by the opponent, i.e. the context, and terminated via answers provided by the proponent, i.e. the term.

Definition 3 (Initial move)**.** *The set of moves without an enabler are called initial moves.*

In both arenas above the set of initial moves is $I = \{q\}$. Arenas with multiple initial moves correspond to product formation, where the two initial questions correspond to computing the two projections.

From the basic arenas, *composite* arenas may be created, for example for function type $A \Rightarrow B$ from arenas for A and B is defined as

Definition 4 (Composite arena)**.**

$$\begin{aligned} A &= \langle M_A, \lambda_A, \vdash_A \rangle \\ B &= \langle M_B, \lambda_B, \vdash_B \rangle \\ A \Rightarrow B &= \langle M_A \uplus M_B, \lambda_A^* \uplus \lambda_B, \vdash_A \uplus \vdash_B \uplus (I_B \times I_A) \rangle, \end{aligned}$$

The function λ^* is only λ but with the o and p polarities reversed. The significance of this polarity reversal is that in the case of arguments to a function the term/context polarity of the interaction becomes reversed. Enabling for function arenas relates not only moves in the two component arenas, but also each initial move in the argument A to each initial move in the return type B, indicating that arguments may be invoked only after the function as a whole has started executing.

For a term in context with type judgement $x_1 : A_1, \ldots, x_k : A_k \vdash M : A$, the arena in which it is interpreted is $A_1 \Rightarrow \cdots \Rightarrow A_k \Rightarrow A$.

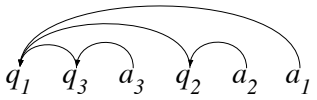
An interaction corresponding to an execution run of a term-in-context is called a *play*, and it is a sequence of *pointed moves* subject to correctness conditions which will be discussed later. A pointed move is an arena-move equipped with two *names* (in the sense of [40]), the first one representing its 'address' in the sequence and the second one is 'the pointer', i.e. the address of an enabling arena-move which occurs earlier in the sequence [15].

Example 1. *The typical play in the interpretation of sequential composition* $seq : unit_3 \to unit_2 \to unit_1$ *is*

$$q_1 n_1 \star \cdot q_3 n_2 n_1 \cdot a_3 n_3 n_2 \cdot q_2 n_4 n_1 \cdot a_2 n_5 n_4 \cdot a_1 n_6 n_1.$$

This sequence of actions is explained as follows: start computation (q_1), ask first argument (q_3, justified by initial question n_1), receive result (a_3, justified by

preceding question), ask second argument (q_2, justified also by initial question n_1), receive result (a_2, justified by preceding question), indicate termination (a_1, justified by initial question n_1). Pointers are usually represented diagrammatically by drawing an edge between moves with equal pointer names, then eliding the pointer names:



Because the actual correctness conditions for plays are language-specific we will present them separately for sequential and concurrent IA, bearing in mind that everything up to this point is shared by the two.

2.2. Plays for sequential IA

Given a justified sequence s in an arena A the notion of player and opponent *view* are defined by induction as follows:

Definition 5 (View)**.**

$$
\begin{aligned}
\mathit{pview}\ (\epsilon) &= \epsilon \\
\mathit{pview}\ (s \cdot mnn') &= \mathit{pview}(s) \cdot mnn' && \text{when } (\pi_1 \circ \lambda)(m) = \mathsf{p} \\
\mathit{pview}(s \cdot mn_1n_2 \cdot s' \cdot m'n_1'n_1) &= \mathit{pview}(s) \cdot mn_1n_2 \cdot m'n_1'n_1 && \text{when } (\pi_1 \circ \lambda)(m') = \mathsf{o} \\
\mathit{pview}\ (s \cdot mn\star) &= mn\star
\end{aligned}
$$

$$
\begin{aligned}
\mathit{oview}\ (\epsilon) &= \epsilon \\
\mathit{oview}\ (s \cdot mnn') &= \mathit{oview}(s) \cdot mnn' && \text{when } (\pi_1 \circ \lambda)(m) = \mathsf{o} \\
\mathit{oview}(s \cdot mn_1n_2 \cdot s' \cdot m'n_1'n_1) &= \mathit{oview}(s) \cdot mn_1n_2 \cdot m'n_1'n_1 && \text{when } (\pi_1 \circ \lambda)(m') = \mathsf{p}
\end{aligned}
$$

The view of a sequence is related to the stack discipline of computation in sequential IA, where certain actions, although present in the interaction traces, are temporarily 'hidden' by other actions.

Legal plays are sequences subject to certain combinatorial conditions which capture the extent of possible behaviour in a given language.

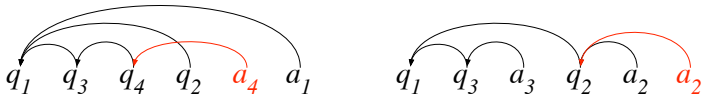
Definition 6 (IA-legal play)**.** *A justified sequence is an IA-legal play if it is:*

alternating: *the proponent/opponent polarities of consecutive moves are different,*

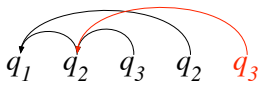
well-bracketed: *only the most recently unanswered question in a sequence can be answered,*

P- and O-visible: *a proponent (opponent, respectively) move must have a justifier in the proponent (opponent, respectively) view of the preceding sequence*

Violating the alternation condition means that successive p-moves or o-moves occur. The bracketing condition can be violated when questions are answered in the wrong order or multiple times:



Finally, the sequence below shows a violation of the visibility condition, for opponent:



2.3. Plays for Idealised Concurrent Algol (ICA)

It is a general, and somewhat surprising, feature of game semantics that richer languages have simpler models. This is not as strange as it seems, because the more features a language has the more unrestricted its interaction with the context can be. In fact it is possible to think of 'omnipotent' contexts in which the interactions are not constrained combinatorially [22]. When sequential IA is enriched with parallelism, the alternation constraint disappears and bracketing and visibility are relaxed to the following, more general constraints:

Definition 7 (ICA-legal play)**.** *A justified sequence is an ICA-legal play if it is:*

forking: *In any sequence* $s \cdot qn_1n'_1 \cdot s' \cdot mn_2n_1$ *the question* q *must be pending,*

joining: *In any sequence* $s \cdot qn_1n'_1 \cdot s' \cdot an_2n_1$ *all questions justified by* q *must be answered.*

The idea is that a 'live thread' signified by a pending question can start new threads (justify new questions) so long as it is not terminated (answered). Conversely, a thread can be terminated (the question can be answered) only after the threads it has started have also terminated. The simplest sequences that violate **fork** and **join**, respectively, are:

Example 2. *The typical play in the interpretation of parallel composition* $par : unit_3 \to unit_2 \to unit_1$ *is*

$$q_1n_1 \star \cdot q_3n_2n_1 \cdot q_2n_4n_1 \cdot a_3n_3n_2 \cdot a_2n_5n_4 \cdot a_1n_6n_1.$$

This sequence of actions is interpreted as: start computation (q_1), ask first argument (q_3, justified by initial question n_1), immediately ask second argument (q_2, justified also by initial question n_1), receive the results in some order (a_3, justified by n_1, and a_2, justified by n_4), indicate termination (a_1, justified by initial question n_1). The play is represented diagrammatically as in Fig. 1:

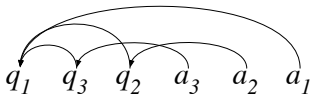


Figure 1: Legal play with pointers

2.4. Strategies

As we mentioned, plays characterise an interaction between term and context occurring in a particular run. In order to characterise the term we take the set of all such possible interactions, noting that they are also characterised by various *closure conditions*. They all share prefix-closure as a common feature, typical to all trace-like models. In the case of sequential IA, the strategies are required to be *deterministic* whereas in the case of concurrent IA they must be closed under certain permutations of moves in plays. It is strategies which give the fully abstract model of the language.

We are not going to give the detailed definitions here because we shall focus on the learning of plays, rather than strategies. Learning strategies seems a more difficult proposition, which we will shall leave for future work.

2.5. Algorithmic considerations

Compared to the complexity of the syntax, the formal rules describing legality of behaviours in the language in terms of combinatorial properties of pointer sequences are remarkably succinct: just three rules for sequential IA (alternation, bracketing, and visibility) and two for concurrent IA (fork and join). A reasonable question to ask is whether these sets of sequences, taken as formal languages, are computationally complex or simple.

It turns out that the answer depends on the order of the arena, where ground type is order 0 and an arena $A \Rightarrow B$ is the maximum between the order of A plus one and the order of B. Plays in sequential IA defined in arenas of order up to 2 are regular languages, definable in terms of finite state automata [18], and for arenas of order up to 3 they are context-free languages, definable in terms of push-down automata [38]. Beyond this, strategies form undecidable languages [35]. In the case of concurrent IA, games in arenas of order 2 or more are undecidable [21].

Of course, the results above refer to questions of language equivalence. Checking whether a (finite) sequence is a valid play in the models of sequential

or concurrent IA is always decidable. But the results above suggest that the problem of learning such models is computationally challenging.

3. Learnability of IA models

We will evaluate the learnability of sequential and concurrent IA using latent semantic analysis. First, a type signature is chosen, which determines an arena. Then a neural network is trained with random plays of the arena so that the level of *perplexity* exhibited by the model is minimised. Using perplexity as a measure of accuracy is common in natural language processing. Given a probability model Q, one can evaluate it by how well it predicts a separate test sample $x_1, \ldots, x_N$ from a sample P.

Definition 8 (Perplexity)**.** *The perplexity of the model is defined by:*

$$\Psi = 2^{-\frac{1}{N}\sum_{i=1}^{N} log_2 Q(x_i)}.$$

The concept of perplexity of a probability distribution is the established measure of quality for a natural language model, and it is the exponentiation of the (cross)entropy of the model. The reasons for using perplexity rather than entropy are largely related to the history and culture of the discipline.

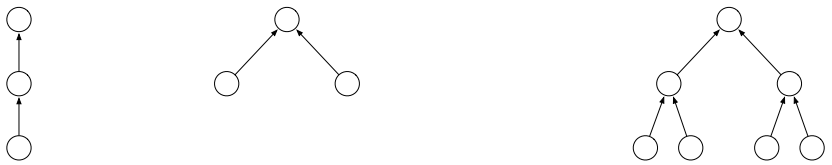
Better models have lower perplexity, as they are less 'perplexed' by the sample. In natural language processing, the perplexity of large corpora (1 million words) is of around 250 (per word). The exponent in the definition of perplexity (the *cross-entropy*) indicates how many bits are required to represent the sequence in the word. For high quality natural language corpora, the cross-entropy is around 8 bits/word or 1.75 bits/letter [9]. The models are validated by computing the perplexity of a model against a different random sample of correct plays coming from the same language, over the same arena. A successful learning model will exhibit similar perplexities between the training set and the validation set.

The accuracy of the learned model is then tested in two ways. The first test is to expose the model to a new sample, coming from the same programming

language but perturbed using several single-character edits (insertions, deletions or substitutions) applied randomly to each sequence. The number of such edits is known as "Levenshtein distance". This results in a set of plays at a small normalised Levenshtein distance from the correct plays which were used for training. Concretely, we use a distance of up to 0.1, e.g. 5 random edits applied to a sequence of length 50. In order for the test to be successful we expect to see a significant increase in perplexity between the test set as compared against the validation and training sets.

The second test is to expose the model to a sample of correct plays coming from the other language, i.e. testing the sequential model against concurrent plays and *vice versa*. Noting that each sequential program is a particular (degenerate) form of a concurrent program, we expect the concurrently-trained neural network to exhibit similar levels of perplexity when exposed to the test data set and the validation data set, but we expect the sequentially-trained program to exhibit greater perplexity when exposed to the test data set — since obviously there are concurrent plays which have no sequential counter-part.

Game models are determined by the arena in which they happen. As discussed in Sec. 2.5, the order of the arena has a significant impact on the algorithmic complexity of the model. We would expect games in low-order arenas to be faster to learn than games in higher-order arenas, but it is difficult to guess the effect the arena shape has over the accuracy of the model. As a consequence we examine both 'narrow' and 'wide' arenas. If we visualise and arena as a tree, the order is the height. The width of the arena corresponds to the number of arguments a function takes, and determines the number of distinct moves in its vocabulary of symbols. Below we show several arenas, depicted as trees:



The arenas above have types $(unit \Rightarrow unit) \Rightarrow unit$ (order 2, width 1), $unit \Rightarrow unit \Rightarrow unit$ (order 1, width 2) and, respectively, $(unit \Rightarrow unit) \Rightarrow (unit \Rightarrow unit) \Rightarrow unit$ (order 2, width 2).

The total number of moves is of the order $O(w^o)$ where w is the width of the arena and o the order of the arena. From the point of view of the term modelled, the width corresponds to the number of free variables in the term or the number of arguments a function might have, whereas the height is the order of the type of the term. We will conduct the experiment on arenas of orders 1 to 3. Going beyond 3 seems rather irrelevant as functions of order 4 or higher are rarely used in practice. We will conduct the experiment on arenas of width 1, i.e. functions taking one argument, in order to emphasise the complexity of the model as caused by higher-order features, and on arenas of width 5, i.e. functions with relatively large numbers of arguments. The two will be contrasted and compared.

The corpora of plays we are creating consists of plays which are abstracted in two ways. The first simplification is that we replace all ground types with the *unit* type. Indeed, in the legality rules for plays, sequential or concurrent, values play no role, and they can be safely abstracted by a generic notion of answer-move. This is important, because the presence of integers in plays would explode the vocabulary of moves beyond what is manageable. The second simplification is eliding the pointer information and focussing on sequences of moves only. By eliminating pointers we make the model easier to learn, but we simultaneously make it less powerful since the pointer information is lost. The reason for removing the pointers is similar to that of removing values: they are usually represented by integers, and the presence of integers in traces may increase too much both the size of the vocabulary and the length of the sequences. However, eliding pointer information is a common abstraction in games-based static analysis of programs, so studying its impact on learnability seems relevant [17].

The length of random plays used in learning, validation and testing is at most 50. The size of the corpus of random sequences used for learning is 10,000 and 100,000, and the size of the corpora used for validation and testing are of

10,000 sequences. These parameters are arbitrary and not too important. Since the sequences are generated there are no limits on maximal sequence length or corpus size. Keeping in mind that the length of a play represents the number of function-argument interactions, a size of 50 seems generous. The number of plays in the corpora only impacts accuracy (which is already very good, as it will be seen) and the duration of the training process (which is reasonable, as it will be seen). For learning we use LSTMs, briefly described in Sec. 5.1. The details of the implementation and the (hyper)parameters of the model are discussed in the next section.

3.1. Latent semantic analysis of plays with justification pointers

Pointers raise an additional problem in that the concrete representation matters, and it may clutter the learning process with extraneous information. For example, the sequence $qnn' \cdot an'n'$ (diagrammatically, ) can be concretely represented, using integers for names, as $q01a12$, but also equivalently as $q10a03$. For learning we must consider a different representation which is neither the original, based on absolute sequence indices [27], nor the representation based on using names for pointers [15].

Absolute indices are not invariant under concatenation, so the same combinatorial patterns occurring earlier or later in the sequence will involve different numerical values, which is generally difficult to learn. Using names seems even harder because, via alpha equivalence, any play can have many equivalent but distinct representations. Whether alpha equivalence can be learned is an interesting but different question. A representation which seems suitable is to use relative indices to indicate the offsets of the pointers, in the stile of de Bruijn indices. So the sequence above will be concretely represented as $q0q1$. The more complex play in Fig. 1 is represented as $q_1 0 q_3 1 q_2 2 a_3 2 a_2 2 a_1 5$.

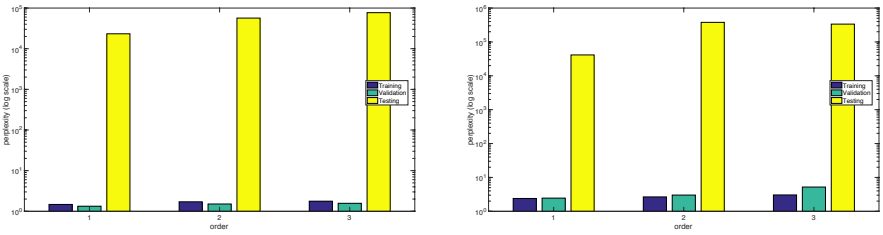
The computational experiments consist of creating LSTM models, via learning, from sets of plays belonging to given arenas, from the two languages, concurrent and sequential variants of IA. We call the former *concurrent pointer models* (CPMs) and the latter *sequential pointer models* (SPMs).

We are then using the learned models to perform latent semantic analysis by measuring the perplexity of the model for a new set of plays generated by both concurrent and sequential variants, in the same arena. We use independently generated random sets of plays from the same model to *validate* the data and random sets of plays for a different model to *test* the data.

Figs. 2-3 show the perplexity of using the model trained on sequential pointer plays to analyse concurrent pointer plays as the third bar in each diagram. Indeed, the latent semantic analysis is conclusive. The validation data, consisting of a different set of random plays from the sequential model, elicits the same perplexity from the model as the training data, indicating concordance of the languages, and more than 4 orders of magnitude lower than the testing data. There is no obvious benefit, or indeed drawback in terms of precision, from increasing the size of the corpora tenfold, from 10k to 100k.

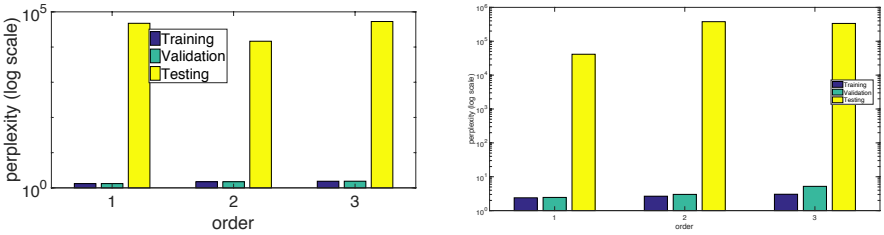
Conversely, as seen in Figs. 4-5, we may use the concurrently-trained model to analyse sequential plays. Since behaviourally sequential plays can be always found as a subset among the concurrent plays, we expect the perplexity differences between validation and testing to be significantly smaller, and indeed they are. However, there is an observable difference between the two, with testing perplexity being up to 100 times higher than validation perplexity.

How can this be explained if the testing set is a subset of the training set? A possible explanation is that the distribution of sequential plays among the concurrent plays is relatively rare. The number of possible interleavings of a play in the concurrent language grows very fast, faster than exponentially in the length of the sequence. Out of all possible interleavings, precisely one is the sequential interleaving. This means that the exposure of the neural network during training to sequential plays is likely to be negligible, which is consistent with a higher perplexity for the relatively rare sequential plays. Should we interpret this as a failure of the latent semantic analysis? The answer to this question depends on our aim. From the point of view of a formal language analysis we can consider this as a failure because, formally, sequential plays are included in concurrent plays. However, if we are interested in sequentiality as an



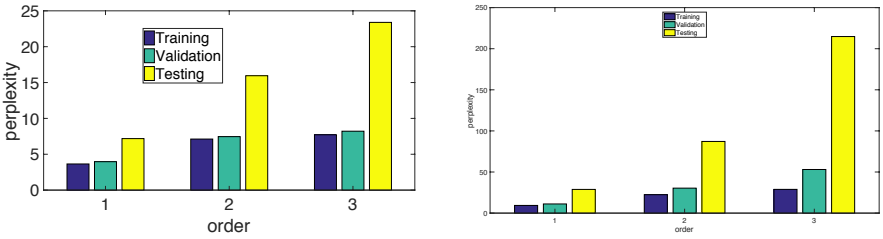
(a) Width 1, $perp < 10^5$ (b) Width 5, $perp < 10^6$

Figure 2: Latent semantic analysis of concurrent plays in SPMs (10k plays)



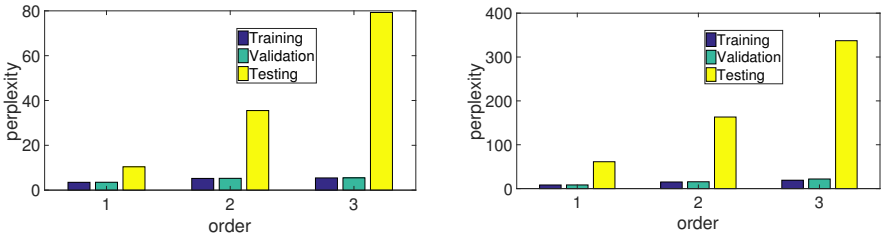
(a) Width 1, $perp < 10^5$ (b) Width 5, $perp < 10^6$

Figure 3: Latent semantic analysis of concurrent plays SPMs (100k plays)



(a) Width 1, $perp < 25$ (b) Width 5, $perp < 250$

Figure 4: Latent semantic analysis of sequential plays in CPMs (10k plays)



(a) Width 1, $perp < 80$ (b) Width 5, $perp < 400$

Figure 5: Latent semantic analysis of sequential plays CPMs (100k plays)

idiomatic concurrency then the result is a surprising success, and a success that would be difficult to achieve using more conventional automata-based methods. Because just like British English (cf. sequential programming) is a dialect of English (cf. concurrent programming), it does not mean that British English text is not, or should be not, recognisable as such.

3.2. Latent semantic analysis of pointer-free plays

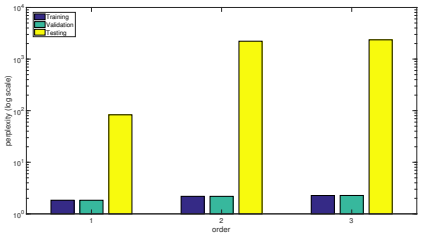
We have run the same experiment, but this time we used an abstracted representation of plays in which the pointer information has been deleted. We call these models *sequential pointer-free models* (SPFMs) and, respectively, *concurrent pointer-free models* (CPFMs).

Fig. 6 and 7 show the perplexity (third bar) of testing a sequential model on concurrent plays over the same arena. In this case the evidence is overwhelming, from 2-5 orders of magnitude in perplexity increase. The extra training provided by using 100k samples is not significant. We conclude that some precision has been lost, since the difference in perplexity between testing data and validation data is now smaller. However, even reduced, the perplexity of the testing data can result in an unambiguous classification.

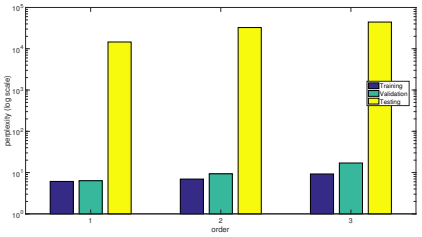
We again use the concurrent model to test sequential plays (Fig. 8-9), this time for pointer-free representation. The experiment shows, as expected the concurrent model cannot identify sequential plays, since all sequential plays can be again found in the concurrent model. As in the case of the pointer model, the perplexity of the test set is not as low as that in the validation set. However, in comparison to the pointer model (Figs. 4-5) the differences in perplexity are negligible, indicating that the pointer information was useful in identifying the concurrent behaviour as idiomatic.

3.3. Detecting perturbations in pointer-free plays

In the case of the pointer-free representation we further test the robustness of the learned model by using test plays from the same model, but intentionally randomly perturbed at a fixed normalised Levenshtein distance ($\delta \leq 0.1$). The

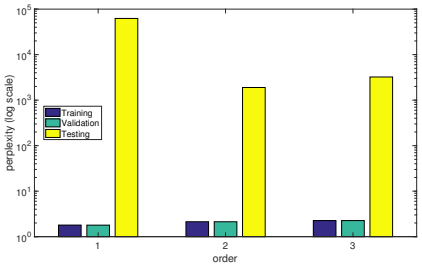


(a) Width 1, $perp < 10^4$

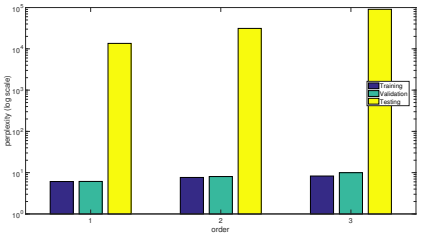


(b) Width 5, $perp < 10^5$

Figure 6: Latent semantic analysis of concurrent plays in SPFMs (10,000 plays)

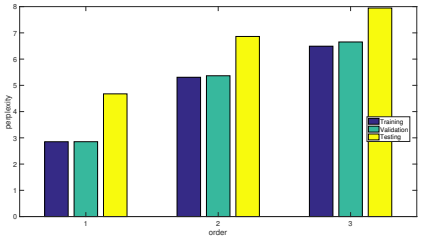


(a) Width 1, $perp < 10^5$

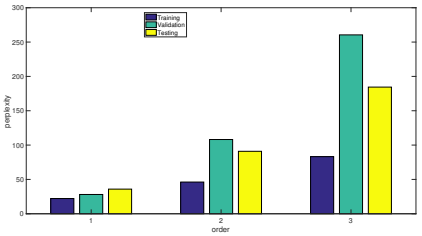


(b) Width 5, $perp < 10^5$

Figure 7: Latent semantic analysis of concurrent plays in SPFMs (100,000 plays)

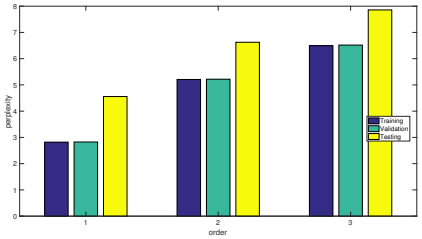


(a) Width 1, $perp < 8$

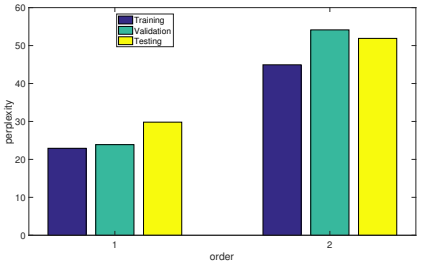


(b) Width 5, $perp < 300$

Figure 8: Latent semantic analysis of sequential plays in CPFMs (10,000 plays)



(a) Width 1, $perp < 8$



(b) Width 5, $perp < 60$

Figure 9: Latent semantic analysis of sequential plays in CPFMs (100,000 plays)

results are in Fig. 10 for sequential models trained on 10K samples and in Fig. 11 on models trained on 100K samples, and in Fig. 12-13 for concurrent models. Each bar chart contains arenas of order 1-3. Where data is missing is because our hardware computational resources (memory) could not cope with the size of the model.

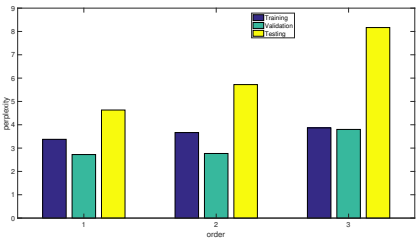
We note that in both the case of sequential and concurrent models the model is significantly more accurate when learned from 100k samples, rather than 10k samples. In absolute terms, the perplexity of the 100k samples models ranges from single digits to just over 50. However, the absolute perplexity is not relevant in latent semantic analysis, just the relative difference in perplexity between training, validation, and test data. Even in the cases with the weakest discrimination (Fig. 13) the perplexity of the test data is almost 2 times larger than that of the validation data.

3.4. Implementation notes

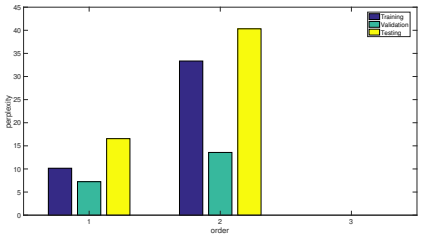
We are using the standard implementation of LSTM distributed with TensorFlow[1]. The model uses an LSTM cell which processes moves sequentially, computing probabilities for possible values of the next move in the sequence. The memory state is initially zeroes, updated after each word. Ideally, in a recurrent neural net (RNN), the output depends on arbitrarily distant inputs. However, this makes the training process computationally intractable, so it is common in practice to 'unfold' the net a fixed number of steps; in the concrete case of our model this value is 20. The inputs are represented using a dense embedding. This is considered undesirable for text but it is demanded here by the large size of the symbol set [6]. The loss function for the model is the sample perplexity, discussed earlier. To increase the expressive power of the model, two LSTMs are layered, each containing 200 nodes. This is considered a small LSTM model.

The training cycle consists of several (13) cycles of training ("epochs"), al-

[1] `https://github.com/tensorflow/models`

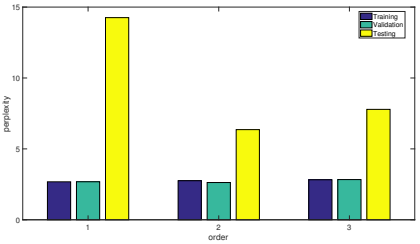


(a) Width 1, $perp < 9$

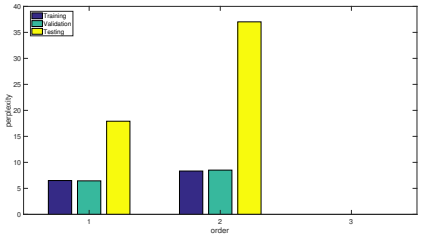


(b) Width 5, $perp < 45$

Figure 10: Latent semantic analysis of perturbed SPFMs (10,000 plays)

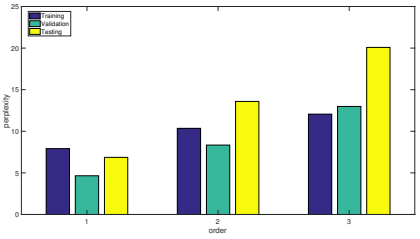


(a) Width 1, $perp < 15$

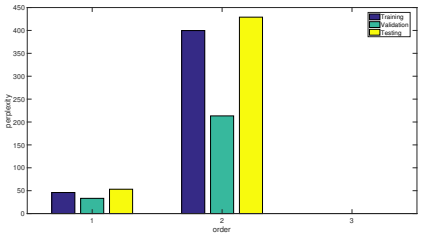


(b) Width 5, $perp < 40$

Figure 11: Latent semantic analysis of perturbed SPFMs (100,000 plays)

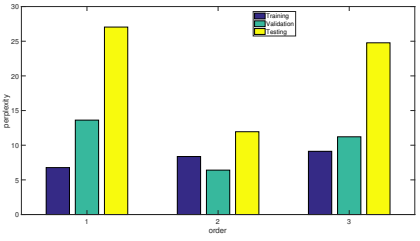


(a) Width 1, $perp < 55$

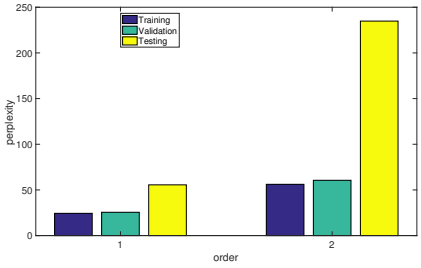


(b) Width 5, $perp < 450$

Figure 12: Latent semantic analysis of perturbed CPFMs (10,000 plays)



(a) Width 1, $perp < 30$



(b) Width 5, $perp < 250$

Figure 13: Latent semantic analysis of perturbed CPFMs (100,000 plays)

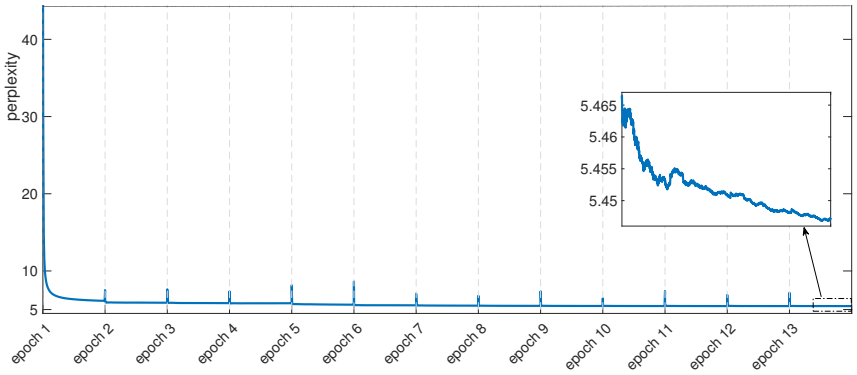


Figure 14: Training convergence of the model on the perturbed sequential plays of order 2 and width 5. The perplexity is shown after processing each mini-batch (size 20) for all training epochs. The learning rate is fixed for the first 4 epochs at 1, and then reduced by a factor of 2 at each subsequent epoch reaching 0.002 in epoch 13.

though in almost all cases except the largest arenas, the model converges after only 1-2 epochs. Further training leads to little or no improvement in the model, as seen in Fig. 14, which is a typical example. The experiments were carried out on a mid-range CUDA device, GeForce GTX 960. The training cycle for each model was around one hour.

4. The challenge of nominal features

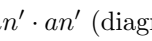
In this section we will give some negative results, less successful experiments attempting to apply neural network learning to nominal patterns.

One of the most difficult conceptual, mathematical, and algorithmic features of games are justification pointers. We examined both the learnability of plays with pointers, and of plays with pointers abstracted away. In the case of plays with pointers we chose a novel representation in which the pointer indicates the offset between the justifier and the justified moves. This representation would be awkward for mathematical proofs but it seemed appealing for learning as it is translation invariant. This means that a particular sub-sequence would have the same representation whether it occurs earlier or later in a sequence, which would not be the case if pointers were absolute indices, as used in the original

Hyland-Ong paper [27]. From a mathematical point of view, however, the most convenient representation of pointers is using *atoms*, in the context of *nominal set theory* [15].

In nominal set theory the essential property is *equivariance*, which is closure over uniform changes of atoms, which represent names. A strategy is in this setting an equivariant set of sequences, which means it is closed under name permutations.

Definition 9 (Equivariant strategy)**.** *Let $\pi \cdot p$ as the* permutation action *of a bijection $\pi : \mathbb{A} \to \mathbb{A}$ on a sequence p. For any play $p \in \sigma$, $\pi \cdot p \in \sigma$ is also a play.*

Names can be concretely represented by any discrete set such as natural numbers or strings. For the purpose of learning we will approximate it with a large finite set, let us say natural numbers less than some N. This means that even the simplest plays, $qnn' \cdot an'$ (diagrammatically, ) can have $N \times N$ distinct representations, for any $n \neq n' \leq N$. This seems exceedingly demanding for the powers of generalisation of a neural net. And indeed, the results are order of magnitude worse if using the nominal representation. For example, in the simple case of sequential games of order 1 and width 1 the perplexity of the model increases from sub-unitary to 22.5.

The poor learnability of the nominal representation inspired us to ask a related question which, in some sense, represents a lowering of the bar. Pointer-plays are complex combinatorial structures. However, can much simpler equivariant structures be learned? We fixed on the pattern $abab \in \mathbb{A}^4$ as a short, fixed, simple such pattern. This problem seems both easier, as the pattern is short and fixed, and harder, as the pattern is purely nominal. As it turns out the nominal challenge dominates the combinatorial simplification. On the same neural architecture, with the same parameters as above, and a set of names fixed at size $N = 10^5$ the performance of the net was very poor, as indicated in Fig. 15. We can see that the evolution of the perplexity is non-monotonic, which usually indicates an unusually rugged landscape of the loss function with

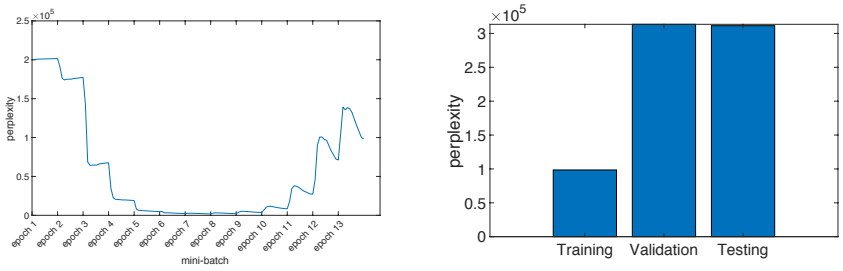


Figure 15: Equivariant pattern *abab*

many local extrema, and an extreme high perplexity (3×10^5) for validation an testing in contrast to the training set, which indicates a memorisation of the training set rather than genuine learning.

The poor performance of the neural net to learn equivariant patterns was confirmed by independent experiments [46] carried out using a feed-forward neural network using various hidden-layer configurations attempting to learn the same equivariant pattern *abab*. When the set of atoms was large enough ($N > 2^{10}$) to prevent the network from simply memorising all instances of the pattern the precision of recognition dropped under 60%, little better than random guessing.

The difficulty of learning equivariant patterns is perhaps best illustrated by an even simpler experiment[2]. Using the same settings (a feed-forward 4-layer network with 6 and respectively 2 neurons in the middle layer, activated using the hyperbolic tangent function) we can compare the success of learning a line segment in a finite two-dimensional space versus a partition of the same space. A line is a basic equivariant pattern, a representation of $\{(x, x) \mid x \in [-1, 1]\}$ whereas a partition $[0, 1] \times [-1, 1]$ is not. The results are seen in Fig. 16, which shows both the training data and the resulting model as a classification of the entire input space. The partition is learned almost perfectly, whereas only a

[2]Using ConvNetJS, `https://cs.stanford.edu/people/karpathy/convnetjs/`

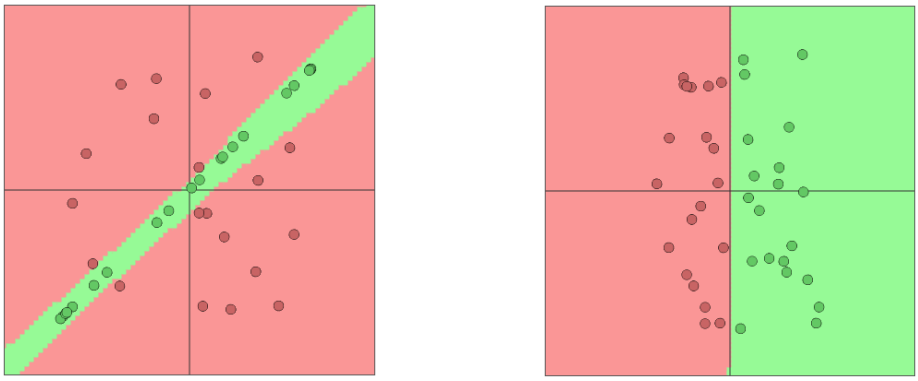


Figure 16: Attempting to learning a line vs. a partition

gross approximation of the line is produced. For the discretisation used by the model, the ideal line has a width of 0.25% of the size of the input space, whereas the rough approximation in the figure has error $7.29\% \leq \epsilon \leq 17.51\%$ relative to the size of the input space. In contrast, for the partition the error at the boundary is mostly within the discretisation margin.

It is of course difficult to conclusively assert that a particular feature is not learnable by a neural network, particularly as they inhabit an infinitely large space of configurations. This is not to say that other ML techniques cannot prove successful at learning nominal features, and in fact the symmetries of nominal languages can be used to make the learning more effective [28]. What we can say is only that the same methods that produce remarkably good results for non-nominal representations fail to produce similar results on nominal (equivariant) representations.

5. Conclusion, related and further work

5.1. Recurrent neural nets

A *perceptron* is a simple computational element from a vector of real numbers to real numbers, which behaves like a weighted sum of the input composed with a step function. A perceptron is *trained* by adjusting the weights and the threshold values so that it fits a given set of examples. A *feed-forward neural*

network (FFN) is essentially a directed acyclic graph in which each node is a perceptron. The most common graph topology for a FNN consists of several *layers* of perceptrons so that each output from any given layer is connected to all inputs of the subsequent layer. A FFN is trained using *back-propagation*, which is a family of gradient-descent algorithms for adjusting the weights and thresholds of the perceptrons to match a given training data set.

Traditional FFNs have been successfully applied to many machine learning problems, however when it comes to the task of sequence-learning, the architecture of an FFN suffers from two main limitations: it cannot readily handle inputs of arbitrary length and it does not explicitly model time [31]. Further more, FFN models that implement some form of a sliding context window to implicitly capture the time dependency between the inputs cannot sufficiently model the time since the range of the captured dependency is limited by the size of the window [8, 12].

Recurrent neural networks (RNN), unlike FNNs, allow for the presence of cycles in their underlying topology. This creates memory-like effects in the network which allow dynamic temporal behaviour. The way in which the RNN is topologically structured is connected to both its expressiveness and the training algorithms. As a result of these compromises, RNN architectures can be very diverse.

Unlike FNNs, recurrent neural networks (RNNs) can readily handle inputs of arbitrary length and can model the temporal patterns present in sequential data. On the other hand, the expressive power of an RNN grows exponentially with the number of its hidden nodes while the growth of the training complexity is maintained to a polynomial (at most quadratic growth) [31]. In most of sequence learning tasks, an RNN or a variant of it is usually the state-of-the-art method. RNNs have been applied not only to learning natural languages, but also artificially generated languages of algorithmic patterns, and proved themselves to be more effective than other methods [29].

The addition of the recurrent edges to the architecture of RNNs gives them great expressive powers, however, they also introduced the 'vanishing and ex-

ploding gradient' problem which occurs while training the network when the errors are back-propagated across many time steps [7]. The Long Short-Term Memory (LSTM) is a crucial variant of RNNs that was introduced by [26] specifically to address this problem. Unlike conventional nets in which the weights have the role of an implicit and quite rudimentary memory, LSTMs have explicit memory cells in their architecture, used to store gradient information for training. The architecture of the LSTM is quite sophisticated and a detailed presentation is beyond the scope here, but accessible tutorials are available [37].

5.2. Machine learning for programming languages

In this exercise we have intentionally used a particularly simple, off-the-shelf, LSTM-based algorithm for latent semantic analysis. All the parameters of the computational experiments were fixed in advance and were not tweaked to improve results. The results were in general excellent. We noted that pointer models, which have more structure, tended to be more amenable to learning than pointer-free models, and also that sequential models, which have more structure, tend to be more recognisable than concurrent models. Investigating whether more structured languages are more learnable is a general feature of LSTM-based language learning would be an interesting exercise for the future. We also note that in general 10k samples was enough, except for detecting intentional perturbations in pointer-free models when models constructed with 100k samples where good, whereas models constructed with 10k samples where unsatisfactory. We note that our convergence criterion was fixed (13 epochs) and it did not take into account residual learning rates. Examining the training logs suggests that training for these models had high residual learning rates, thus extra epochs might have helped. It would be interesting to re-evaluate this study by changing the stopping criterion by considering the evolution in time of the learning rate rather than fixing the number of epochs.

As in all optimisation work many parameters can be tweaked in search of improvement, but doing that would detract from the main point of our paper, which is that *the game model is a representation of the semantics of program-*

ming languages which is amenable to machine learning via LSTMs. Details notwithstanding, we find this fact alone quite remarkable.

Using machine learning for programming language semantics is largely new and unexplored terrain, even though heuristic search techniques such as genetic algorithms have been applied to software engineering problems [24]. This is a well researched area which is related but complementary to our interest. Primarily, search-based software engineering (SBSE) is a collection of syntactic techniques, which rely on manipulation of code, usually as a syntax tree, to extract information about the code, to manipulate the code, or to detect patterns in the code (common bugs, anti-patterns, etc.). There is a significant area of overlap between the aims and techniques of SBSE and other heuristic-heavy programming-language analyses and manipulation such as refactoring, slicing, test-generation, or verification. By contrast, semantic models are independent of syntax. In fact the kind of analysis we have proposed here ignores syntax and relies directly on program behaviour instead. Indeed, latent semantic analysis of code when the source code is available is trivial: one can merely scan it for occurrences of terms associated with concurrency, such as parallel execution or semaphores. The problem becomes more interesting when the source code is not available: given a piece of compiled code, e.g. a module or a library, can we determine whether it originates in one language or another just by examining the way it interacts with its calling context? Our analysis shows that at least sometimes the answer is positive.

There are some obvious limitations to our approach. First of all we looked for distinctions in plays rather in strategies, just because learning strategies (potentially infinite sets of plays) seems significantly harder than learning the plays themselves. But there are semantic differences between languages which are only reflected at the level of the strategy. For example PCF [27], sequential IA [3] and non-deterministic IA [25] have the same notion of legality on plays but differ at the level of strategies. PCF requires *innocent strategies*, sequential IA *deterministic strategies*, and non-deterministic IA *non-deterministic strategies*. Moreover, the formulation of these distinctions requires both pointers and

answer-values, information which we abstract away from our modelling. Using our set-up these distinctions are lost. Capturing such subtle distinctions would require a different approach.

5.3. Future work

However, by and large the results of our experiment are very encouraging. The quality of the models is high, as evidenced by their robust discriminatory powers, and the required computational resources modest. These results make us optimistic about using this methodology on practical programming languages as encountered 'in the wild'. The process of creating a corpora of training traces in the absence of a model is of course different. We need a large code base, a compiler, and a way to instrument the interface between a part of the code taken to be as 'the term' and the rest of the program taken to be 'the context'. Much like a profiler, the instrumentation should record how, in any execution, the term interacts with the context via its free variables (function or method calls and returns).

What is interesting is that a model of code obtained from real code 'in the wild' will learn not only what is 'legal' behaviour but also what is 'idiomatic' behaviour – patterns of behaviour which are specific to the code-base used for learning. Depending on the quality of the model this can have some possibly interesting applications. Note that this same phenomenon appears in the case of machine-learning natural language from corpora [10], except that in the case of programming languages the idiomatic aspects are more likely to be seen as embodiments of de-facto practices rather than problematic biases.

For example, the model can be used for novelty detection [32, 33] in order to augment code inspections: instead of merely studying the code syntactically, the behaviour of code-in-context can be analysed for conformance with the existing body of code. Syntax-independent novelty analysis can have other well-known applications, for example to security. Unexpected or unusual patterns of interactions are, for example, typical for attempts to compromise the integrity of a system.

A recogniser running in reverse is a generator, and generating valid traces – especially idiomatic valid traces – is a possibly interesting way of automating the testing of functional interfaces. Generating random data for automating testing is a well-understood process [11]. However, generating random functional behaviour is a much more complicated proposition, and syntactic approaches do not seem equally promising.

Semantic-directed techniques, in particular using models that are both compositional and operational such as trace semantics or game semantics, have been advocated for a long time but did not make as deep inroads as expected in the practice of programming. A pragmatic disadvantage is that semantic models can be mathematically demanding, but this is not even the main problem. The main difficulty is that on the balance they are both difficult to construct and brittle, in the sense that small changes to the language can require a total rethinking of its semantic model. Moreover, most languages are not syntactically (and semantically) self-contained because they interact with other languages via mechanisms such as foreign function interfaces. Machine learning, if effective on real languages, solves both these problems. It hides the mathematical complexity of the model behind the automated learning, and it can derive models out of existing code-bases, capturing not only a sense of what is legal but also what is idiomatic (for engineering, but also cultural reasons) in a particular language. In the end, as it tends to be the case with machine learning, the resulting model may be opaque and uninformative but it may end up being effective enough for practical purposes.

Finally, our research has incidentally discovered an unexpected limitation of neural nets in learning equivariant patterns, i.e. patterns closed under permutation. Even a simple equivariant pattern such as *abab* was beyond the ability of a LSTM to generalise, even from thousands of examples, whereas a human subject will only need a handful of examples to infer the pattern in sequences such as *John, Mary, John, Mary* or *Tony, Dave, Tony, Dave* or *Foo, Bar, Foo, Bar* is. Since equivariance is of critical importance in understanding *names*, either in the context of programming languages (variables) or natural languages (proper

names) we think we have identified a significant new challenge for neural nets, and machine learning in general. This should be investigated more deeply.

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[1] Samson Abramsky, Radha Jagadeesan & Pasquale Malacaria (2000): *Full Abstraction for PCF*. *Inf. Comput.* 163(2), pp. 409–470, doi:10.1006/inco.2000.2930.

[2] Samson Abramsky & Achim Jung (1994): *Domain theory*. *Handbook of logic in computer science* 3, pp. 1–168.

[3] Samson Abramsky & Guy McCusker (1996): *Linearity, Sharing and State: a fully abstract game semantics for Idealized Algol with active expressions*. *Electr. Notes Theor. Comput. Sci.* 3, pp. 2–14, doi:10.1016/S1571-0661(05)80398-6.

[4] Samson Abramsky & Guy McCusker (1999): *Game semantics*. In: *Computational logic*, Springer, pp. 1–55. doi:10.1007/978-3-642-58622-4_1.

[5] Samson Abramsky et al. (1997): *Semantics of interaction: an introduction to game semantics*. In A. Pitts and P. Dybjeer (Eds.) Semantics and Logics of Computation 1–33. Cambridge University Press, doi:10.1017/CBO9780511526619.002.

[6] Marco Baroni, Georgiana Dinu & Germán Kruszewski (2014): *Don't count, predict! A systematic comparison of context-counting vs. context-predicting semantic vectors*. In: *Proceedings of the 52nd Annual Meeting of the Association for Computational Linguistics, ACL 2014, June 22-27, 2014,*

Baltimore, MD, USA, Volume 1: Long Papers, pp. 238–247, Available at http://anthology.aclweb.org/P/P14/P14-1023.pdf.

[7] Y. Bengio, P. Simard & P. Frasconi (1994): *Learning long-term dependencies with gradient descent is difficult. IEEE Transactions on Neural Networks* 5(2), pp. 157–166, doi:10.1109/72.279181.

[8] Yoshua Bengio, Réjean Ducharme, Pascal Vincent & Christian Jauvin (2003): *A neural probabilistic language model. Journal of machine learning research* 3(Feb), pp. 1137–1155, Available at http://www.jmlr.org/papers/v3/bengio03a.html.

[9] Peter F. Brown, Stephen Della Pietra, Vincent J. Della Pietra, Jennifer C. Lai & Robert L. Mercer (1992): *An Estimate of an Upper Bound for the Entropy of English. Computational Linguistics* 18(1), pp. 31–40.

[10] Aylin Caliskan, Joanna J. Bryson & Arvind Narayanan (2017): *Semantics derived automatically from language corpora contain human-like biases. Science* 356(6334), pp. 183–186, doi:10.1126/science.aal4230.

[11] Koen Claessen & John Hughes (2000): *QuickCheck: a lightweight tool for random testing of Haskell programs.* In: *Proceedings of the Fifth ACM SIGPLAN International Conference on Functional Programming (ICFP '00), Montreal, Canada, September 18-21, 2000.*, pp. 268–279, doi:10.1145/351240.351266.

[12] Wim De Mulder, Steven Bethard & Marie-Francine Moens (2015): *A Survey on the Application of Recurrent Neural Networks to Statistical Language Modeling. Comput. Speech Lang.* 30(1), pp. 61–98, doi:10.1016/j.csl.2014.09.005.

[13] Yuchen Fan, Yao Qian, Feng-Long Xie & Frank K Soong (2014): *TTS synthesis with bidirectional LSTM based recurrent neural networks.* In: *Fifteenth Annual Conference of the International Speech Communication Association*, pp. 1964–1968.

[14] Nissim Francez, C. A. R. Hoare, Daniel J. Lehmann & Willem P. de Roever (1979): *Semantics of Nondeterminism, Concurrency, and Communication*. *J. Comput. Syst. Sci.* 19(3), pp. 290–308, doi:10.1016/0022-0000(79)90006-0.

[15] Murdoch Gabbay & Dan R. Ghica (2012): *Game Semantics in the Nominal Model*. *Electr. Notes Theor. Comput. Sci.* 286, pp. 173–189, doi:10.1016/j.entcs.2012.08.012.

[16] Dan R. Ghica (2009): *Applications of Game Semantics: From Program Analysis to Hardware Synthesis*. In: *Proceedings of the 24th Annual IEEE Symposium on Logic in Computer Science, LICS 2009, 11-14 August 2009, Los Angeles, CA, USA*, pp. 17–26, doi:10.1109/LICS.2009.26.

[17] Dan R. Ghica & Adam Bakewell (2009): *Clipping: A Semantics-Directed Syntactic Approximation*. In: *Proceedings of the 24th Annual IEEE Symposium on Logic in Computer Science, LICS 2009, 11-14 August 2009, Los Angeles, CA, USA*, pp. 189–198, doi:10.1109/LICS.2009.25.

[18] Dan R. Ghica & Guy McCusker (2003): *The regular-language semantics of second-order idealized A_{LGOL}*. *Theor. Comput. Sci.* 309(1-3), pp. 469–502, doi:10.1016/S0304-3975(03)00315-3.

[19] Dan R. Ghica & Andrzej S. Murawski (2006): *Compositional Model Extraction for Higher-Order Concurrent Programs*. In: *Tools and Algorithms for the Construction and Analysis of Systems, 12th International Conference, TACAS 2006 Held as Part of the Joint European Conferences on Theory and Practice of Software, ETAPS 2006, Vienna, Austria, March 25 - April 2, 2006, Proceedings*, pp. 303–317, doi:10.1007/11691372_20.

[20] Dan R. Ghica & Andrzej S. Murawski (2008): *Angelic semantics of fine-grained concurrency*. *Ann. Pure Appl. Logic* 151(2-3), pp. 89–114, doi:10.1016/j.apal.2007.10.005.

[21] Dan R. Ghica, Andrzej S. Murawski & C.-H. Luke Ong (2006): *Syntactic control of concurrency. Theor. Comput. Sci.* 350(2-3), pp. 234–251, doi:10.1016/j.tcs.2005.10.032.

[22] Dan R. Ghica & Nikos Tzevelekos (2012): *A System-Level Game Semantics. Electr. Notes Theor. Comput. Sci.* 286, pp. 191–211, doi:10.1016/j.entcs.2012.08.013.

[23] Edward Grefenstette, Georgiana Dinu, Yao-Zhong Zhang, Mehrnoosh Sadrzadeh & Marco Baroni (2013): *Multi-Step Regression Learning for Compositional Distributional Semantics.* In: *Proceedings of the 10th International Conference on Computational Semantics, IWCS 2013, March 19-22, 2013, University of Potsdam, Potsdam, Germany*, pp. 131–142, Available at arXiv:1301.6939.

[24] Mark Harman, S. Afshin Mansouri & Yuanyuan Zhang (2012): *Search-based software engineering: Trends, techniques and applications. ACM Comput. Surv.* 45(1), pp. 11:1–11:61, doi:10.1145/2379776.2379787.

[25] Russell Harmer & Guy McCusker (1999): *A Fully Abstract Game Semantics for Finite Nondeterminism.* In: *14th Annual IEEE Symposium on Logic in Computer Science, Trento, Italy, July 2-5, 1999*, pp. 422–430, doi:10.1109/LICS.1999.782637.

[26] Sepp Hochreiter & Jürgen Schmidhuber (1997): *Long Short-Term Memory. Neural Computation* 9(8), pp. 1735–1780, doi:10.1162/neco.1997.9.8.1735.

[27] J. M. E. Hyland & C.-H. Luke Ong (2000): *On Full Abstraction for PCF: I, II, and III. Inf. Comput.* 163(2), pp. 285–408, doi:10.1006/inco.2000.2917.

[28] Malte Isberner & Falk Howar & Bernhard Steffen (2013): *Learning register automata: from languages to program structures. Machine Learning* 96(1-2), pp. 65-98, doi:10.1007/s10994-013-5419-7.

[29] Armand Joulin & Tomas Mikolov (2015): *Inferring algorithmic patterns with stack-augmented recurrent nets.* In: *Advances in neural information processing systems*, pp. 190–198.

[30] Thomas K Landauer (2006): *Latent semantic analysis.* Wiley Online Library, doi:10.1002/0470018860.s00561.

[31] Zachary C Lipton, John Berkowitz & Charles Elkan (2015): *A critical review of recurrent neural networks for sequence learning.* Available at arXiv:1506.00019.

[32] Markos Markou & Sameer Singh (2003): *Novelty detection: a review - part 1: statistical approaches.* *Signal Processing* 83(12), pp. 2481–2497, doi:10.1016/j.sigpro.2003.07.018.

[33] Markos Markou & Sameer Singh (2003): *Novelty detection: a review - part 2: : neural network based approaches.* *Signal Processing* 83(12), pp. 2499–2521, doi:10.1016/j.sigpro.2003.07.019.

[34] Andrzej S. Murawski (2005): *About the undecidability of program equivalence in finitary languages with state.* *ACM Trans. Comput. Log.* 6(4), pp. 701–726, doi:10.1145/1094622.1094626.

[35] Andrzej S. Murawski & Igor Walukiewicz (2008): *Third-order Idealized Algol with iteration is decidable.* *Theor. Comput. Sci.* 390(2-3), pp. 214–229, doi:10.1016/j.tcs.2007.09.022.

[36] Peter O'Hearn & Robert Tennent (2013): *ALGOL-like Languages.* Springer Science & Business Media.

[37] Christopher Olah: *Understanding LSTM Networks.* Available at http://colah.github.io/posts/2015-08-Understanding-LSTMs/.

[38] C.-H. Luke Ong (2002): *Observational Equivalence of 3rd-Order Idealized Algol is Decidable.* In: *17th IEEE Symposium on Logic in Computer Science*

(LICS 2002), 22-25 July 2002, Copenhagen, Denmark, Proceedings, pp. 245–256, doi:10.1109/LICS.2002.1029833.

[39] Andrew M. Pitts (2000): *Operational Semantics and Program Equivalence*. In: *Applied Semantics, International Summer School, APPSEM 2000, Caminha, Portugal, September 9-15, 2000, Advanced Lectures*, pp. 378–412, doi:10.1007/3-540-45699-6_8.

[40] Andrew M Pitts (2013): *Nominal sets: Names and symmetry in computer science*. Cambridge University Press, doi:10.1017/CBO9781139084673.

[41] Gordon D. Plotkin (1975): *Call-by-Name, Call-by-Value and the lambda-Calculus*. *Theor. Comput. Sci.* 1(2), pp. 125–159, doi:10.1016/0304-3975(75)90017-1.

[42] Gordon D. Plotkin (1977): *LCF Considered as a Programming Language*. *Theor. Comput. Sci.* 5(3), pp. 223–255, doi:10.1016/0304-3975(77)90044-5.

[43] John C. Reynolds (1997): *The Essence of Algol*, pp. 67–88. Birkhäuser Boston, Boston, MA, doi:10.1007/978-1-4612-4118-8_4.

[44] Robert D. Tennent (1976): *The denotational semantics of programming languages*. *Communications of the ACM* 19(8), pp. 437–453, doi:10.1145/360303.360308.

[45] Yonghui Wu, Mike Schuster, Zhifeng Chen, Quoc V Le, Mohammad Norouzi, Wolfgang Macherey, Maxim Krikun, Yuan Cao, Qin Gao, Klaus Macherey et al. (2016): *Google's Neural Machine Translation System: Bridging the Gap between Human and Machine Translation*. arXiv preprint *arXiv:1609.08144*. Available at arXiv:1609.08144.

[46] Liangye Yu, (2018): *On the accuracy of machine learning for equivariant patterns.* MSc. Thesis. University of Birmingham.