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A reflection on analytical tornado-like vortex flow field models

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Abstract

Given the difficulties associated with undertaking full-scale measurements in tornadoes, recourse is often made to models. In this field, analytical models have, perhaps surprisingly, stood the test of time, with the Rankine, Burgers-Rott and Sullivan models frequently invoked to model the flow field of a tornado. These mathematical models are by their very nature, a simplification of what is a highly complex phenomenon. However, in many cases they have been represented as the ‘truth’ without the fundamental assumptions governing the model being either explored in detail or even acknowledged. This paper attempts to rectify this by giving detailed information about assumptions and limitations of each vortex model and critically assesses the ability (or otherwise) of the Rankine, Burgers-Rott, Sullivan, and the recently published Baker vortex model to simulate tornado-like flow. Comparisons are made to the flow field of a physically simulated tornado, which by its very nature is also a model, but arguably more realistic.

It was found that the vortex models are able to represent certain flow patterns at certain heights but fail, due to their simplifications, in replicating the entire three-dimensional flow structure obtained experimentally.

Keywords: Laboratory simulated tornado vortex; Analytical vortex models; Rankine vortex; Burgers-Rott vortex; Sullivan vortex; Baker vortex

1. Introduction

Within the Wind Engineering community, increasing attention is being paid to the effects of non-stationary, non-synoptic winds, i.e. tornadoes. The structure of full-scale tornadoes is highly complex, showing a three-dimensional flow field, instabilities, singularities and non-linear effects (e.g. Lewellen, 1993; Davies-Jones et al., 2001; Alexander and Wurman, 2008; Karstens et al., 2010). In order to understand the physical processes present in a tornado flow field, simplified models are needed, which reduce the degree of freedom present in full-scale observations, and therefore allow a detailed and statistically representative evaluation of velocity and pressure fields. In order to provide this type of datasets, the tornado-like flow field was modelled experimentally

40 and/or numerically by several authors such as Ward, 1972; Church et al., 1979; Lewellen et al.,
41 1997; Haan et al., 2008; Mishra et al., 2008; Natarajan, 2011; Sabareesh et al., 2012; Refan et al.,
42 2014; Gillmeier et al., 2016; Liu and Ishihara, 2016; Nolan et al., 2017 and Tang et al., 2017. An
43 attempt to analytically model the three-dimensional flow in the boundary layer of a tornado-like
44 vortex was made by Kuo (1971) by alternatingly solving the two nonlinear boundary-layer
45 equations for the radial and vertical distribution of velocities. The Bloor and Ingham vortex model
46 (1987) and the Vyas-Majdalani vortex model (Vyas et al., 2003) are exact inviscid solutions to the
47 Euler's equations in a confined conical and cylindrical domain, respectively. Xu and Hangan
48 (2009) analytically modelled an inviscid tornado-like vortex using a free narrow jet solution
49 combined with a modified Rankine vortex. However, it needs to be mentioned here that this
50 combined model is not an exact solution to the Navier-Stokes-Equations. Wood and White (2011)
51 presented a new parametric model of vortex tangential-wind profiles, which is based on the
52 Vatis model (Vatis et al., 1991) and is primarily designed to depict realistic-looking
53 tangential wind profiles observed in atmospheric vortices.

54
55 Despite this excellent work, the Rankine (Rankine, 1882), Burgers-Rott (Burgers, 1948; Rott,
56 1958) and Sullivan (Sullivan, 1959) vortex model are still the most commonly used vortex models
57 to replicate tornado-like flow behaviour. An overview of some of the before mentioned vortex
58 models can be found in e.g. Kilty (2005), Batterson et al. (2007) and Kim and Matsui (2017).
59 However, with the increasing interest in the simulation of tornado-like flows, these models have
60 (in some cases and with varying degrees of success) been invoked in order to describe some
61 elements of the flow field. The authors feel that it is worth reflecting on the fundamental
62 assumptions behind these models and bench marking their performance against measured data
63 obtained in controlled conditions. For that reason, this paper gives detailed information about the
64 derivation and simplifications of the Rankine, Burgers-Rott and Sullivan vortex model. In addition
65 to the Rankine, Burgers-Rott and Sullivan vortex models, the recently published vortex model by
66 Baker and Sterling (2017), hereafter called 'Baker vortex model', is also included in the analysis.

67
68 Section two of this paper provides detailed information about the derivation and simplifications of
69 the above mentioned vortex models, while section three outlines the experimental methodology
70 used to assess the model suitability. The results of the model benchmarking can be found in section
71 four, with the main conclusions presented in section five.

72
73

74 **2. Existing vortex models**

75 76 *2.1 Flow field notation*

77
78 In what follows, a cylindrical coordinate system has been adopted as illustrated in figure 1. In
79 figure 1, r , z and θ are the radial distance, vertical distance and circumferential angle, respectively.

80 Thus, u_r , u_z and u_θ represent the radial, vertical and circumferential components of velocity. For
 81 the sake of simplicity the flow is considered to be incompressible for all models and a density of
 82 air of $\rho = 1.21\text{kg/m}^3$ is assumed for all calculations. In this section, a brief description of the
 83 different vortex models examined in this paper is provided, together with the underlying
 84 assumptions.

85
 86 Using the aforementioned notation, the continuity equation (Eq. 1) and radial (Eq. 2),
 87 circumferential (Eq. 3) and vertical (Eq. 4) components of the Navier-Stokes-Equations (NSE) can
 88 be expressed as:

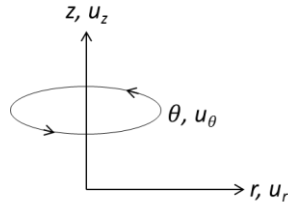


Figure 1: Flow field notation.

89
 90
 91
 92

$$\underbrace{\frac{1}{r} \frac{\partial(ru_r)}{\partial r}}_1 + \underbrace{\frac{1}{r} \frac{\partial u_\theta}{\partial \theta}}_2 + \underbrace{\frac{\partial u_z}{\partial z}}_3 = 0 \quad (1)$$

93
 94
 95

$$\begin{aligned} & \underbrace{\frac{\partial u_r}{\partial t}}_{R1} + \underbrace{u_r \frac{\partial u_r}{\partial r}}_{R2} + \underbrace{\frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta}}_{R3} - \underbrace{\frac{u_\theta^2}{r}}_{R4} + \underbrace{u_z \frac{\partial u_r}{\partial z}}_{R5} = \\ & - \underbrace{\frac{1}{\rho} \frac{\partial p}{\partial r}}_{R6} + \underbrace{g_r}_{R7} + \nu \left(\underbrace{\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right)}_{R8} - \underbrace{\frac{u_r}{r^2}}_{R9} + \underbrace{\frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2}}_{R10} - \underbrace{\frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta}}_{R11} + \underbrace{\frac{\partial^2 u_r}{\partial z^2}}_{R12} \right) \end{aligned} \quad (2)$$

96
 97
 98

$$\begin{aligned} & \underbrace{\frac{\partial u_\theta}{\partial t}}_{C1} + \underbrace{u_r \frac{\partial u_\theta}{\partial r}}_{C2} + \underbrace{\frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta}}_{C3} + \underbrace{\frac{u_r u_\theta}{r}}_{C4} + \underbrace{u_z \frac{\partial u_\theta}{\partial z}}_{C5} = \\ & - \underbrace{\frac{1}{\rho} \frac{\partial p}{\partial \theta}}_{C6} + \underbrace{g_\theta}_{C7} + \nu \left(\underbrace{\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_\theta}{\partial r} \right)}_{C8} - \underbrace{\frac{u_\theta}{r^2}}_{C9} + \underbrace{\frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2}}_{C10} - \underbrace{\frac{2}{r^2} \frac{\partial u_r}{\partial \theta}}_{C11} + \underbrace{\frac{\partial^2 u_\theta}{\partial z^2}}_{C12} \right) \end{aligned} \quad (3)$$

99
 100
 101

$$\begin{aligned} & \underbrace{\frac{\partial u_z}{\partial t}}_{Z1} + \underbrace{u_r \frac{\partial u_z}{\partial r}}_{Z2} + \underbrace{\frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta}}_{Z3} + \underbrace{u_z \frac{\partial u_z}{\partial z}}_{Z4} = \\ & - \underbrace{\frac{1}{\rho} \frac{\partial p}{\partial z}}_{Z5} + \underbrace{g_z}_{Z6} + \nu \left(\underbrace{\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right)}_{Z7} + \underbrace{\frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2}}_{Z8} + \underbrace{\frac{\partial^2 u_z}{\partial z^2}}_{Z9} \right) \end{aligned} \quad (4)$$

102
 103

104 Here ρ is the density of the fluid, t is the time, p is the static pressure and \vec{g} is the gravity vector in
 105 its different components. The different terms in equations (1 – 4) have been labelled since, as will
 106 be demonstrated below, it is possible to derive the majority of the analytical models by
 107 disregarding different terms.

108
 109 *2.2 Rankine vortex model*

110
 111 The Rankine model has been adopted by a number of researchers (e.g. Hoecker, 1960; Church et
 112 al., 1979; Winn et al., 1999; Wurman and Gill, 2000; Brown and Wood, 2004; Lee et al., 2004;
 113 Mishra et al., 2008; Bech et al., 2009; Hashemi Tari et al., 2010; Wood and Brown, 2011; Refan
 114 and Hangan, 2016; Tang et al., 2017) to model tornado-like flow behaviour. The following
 115 assumptions are made in the derivation of the Rankine vortex model:

- 116
- 117 • *The flow field is one-dimensional and as such equations (2) and (4) can be disregarded.*
- 118 • *The flow field is steady state, i.e., term R1 can be taken as zero.*
- 119 • *The flow is inviscid ($\mu=0$), i.e., terms R8 - R12 can be neglected.*
- 120 • *Body forces can be neglected, i.e., ($\vec{g}=0$).*
- 121

122 These assumptions reduce the NSE to the cyclostrophic equation (Eq. 5).

123

$$124 \quad \frac{dp(r)}{dr} = \rho \frac{u_{\theta}(r)^2}{r} \quad (5)$$

125

126 The Rankine model also assumes that the flow consists of two separate flow regions. In the first
 127 region, the core region (i.e., $r < R$, where R is the core radius, which is defined as the radial distance
 128 from the vortex centre at which the circumferential velocity component is maximal), the flow is
 129 assumed to have a constant vorticity and is considered to be similar to that of a solid body. In the
 130 second region, ($r > R$) it is assumed that the flow can be described by a potential flow field
 131 (incompressible, inviscid and irrotational) (Alekseenko et al., 2007) and is inversely proportional
 132 to the radial distance. These assumptions enable the circumferential velocity component to be
 133 modelled via an expression of the form:

134

$$135 \quad \overline{u_{\theta}}(\bar{r}) = \begin{cases} \bar{r} & \text{for } (\bar{r} < 1) \\ \frac{1}{\bar{r}} & \text{for } (\bar{r} > 1) \end{cases} \quad (6)$$

136

137 where $\overline{u_{\theta}}$ is the normalised circumferential velocity component ($= u_{\theta}/u_{\theta,max}$, where $u_{\theta,max}$ is
 138 the maximum value of u_{θ}) and \bar{r} is the radial distance normalised by the core radius, R . In equation
 139 6, a discontinuity occurs at $\bar{r} = 1$. In order to avoid this, the model is occasionally modified as
 140 shown in equation 6.1. However, the most commonly used form is shown in equation 6 and hence
 141 will be used in what follows.

142

143

$$\overline{u_\theta}(\bar{r}) = \frac{2\bar{r}}{(1+\bar{r}^2)} \quad (6.1)$$

144

145 Combining equation (6) with equation (5) and integrating, yields an expression for the normalised
146 pressure distribution of the Rankine vortex model (Eq. 7):

147

$$\bar{p}(\bar{r}) = \begin{cases} \overline{p_{(r=0)}} + \frac{1}{2} \left(\frac{r}{R}\right)^2 & \text{for } (\bar{r} < 1) \\ \overline{p_{r \rightarrow \infty}} - \frac{1}{2} \left(\frac{R}{r}\right)^2 & \text{for } (\bar{r} > 1) \end{cases} \quad (7)$$

149

150 where $\bar{p}(\bar{r})$ is the normalised pressure ($= p(r)/\rho u_{\theta,max}^2$), $\overline{p_{r \rightarrow \infty}}$ is the normalised static
151 pressure, which is unaffected by the vortex and $\overline{p_{(r=0)}}$ is the static pressure at the vortex centre.

152

153 2.3 Burgers-Rott vortex model

154

155 The Burgers-Rott model has been adopted by a number of authors (e.g. Winn et al., 1999; Brown
156 and Wood, 2004; Lee et al., 2004, Kosiba and Wurman, 2010; Wood and Brown, 2011; Wurman
157 et al., 2013) to model tornado-like flow behaviour. Explicit in the derivation of the model are the
158 following assumptions:

159

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177

The above assumptions reduce equations (1 – 4) to the following simplified versions:

$$\frac{1}{r} \frac{\partial(ru_r)}{\partial r} + \frac{\partial u_z}{\partial z} = 0 \quad (1^*)$$

$$u_r \frac{\partial u_r}{\partial r} - \frac{u_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) - \frac{u_r}{r^2} \right) \quad (2^*)$$

178
$$u_r \frac{\partial u_\theta}{\partial r} + \frac{u_r u_\theta}{r} = \nu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_\theta}{\partial r} \right) - \frac{u_\theta}{r^2} \right) \quad (3^*)$$

179
180
$$u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} \quad (4^*)$$

181
182 Now, the Burgers-Rott model acknowledges that the flow within a tornado-like structure is likely
183 to be subject to changing levels of vorticity, which in turn will have implications for the associated
184 pressure field. Thus, it is assumed that the vertical velocity component changes with respect to
185 height and the following relationship is adopted:

186
187
$$\overline{u_z}(\bar{z}) = 2\bar{a}\bar{z} \quad (8)$$

188
189 where $\overline{u_z}(\bar{z})$ is the normalised vertical velocity ($= u_z(z)/u_{\theta,max}$), \bar{z} is the normalised vertical
190 height ($= z/R$) and \bar{a} is a constant, whose magnitude purports to account for the strength of vortex
191 stretching. It is also assumed that \bar{a} is related to the viscous dissipation, ν , via an expression of the
192 form:

193
$$\bar{a} = \frac{2\nu}{Ru_{\theta,max}} \quad (8.1)$$

194
195 Equation (8.1) implies that the viscous dissipation, ν , continuously removes kinetic energy from
196 the flow, which is continuously introduced by vortex stretching. Using equation (8) and integrating
197 the simplified continuity equation (Eq. 1*), an expression for the normalised radial velocity
198 component, $\overline{u_r}$, can be obtained (Eq. 9).

199
200
$$\overline{u_r}(\bar{r}) = -\bar{a}\bar{r} \quad (9)$$

201
202 Using equations (8) and (9), and solving the simplified NSE in the circumferential direction (Eq.
203 3*), an expression for the normalised circumferential velocity component, $\overline{u_\theta}$, can be found (Eq.
204 10).

205
206
$$\overline{u_\theta}(\bar{r}) = \frac{1}{\bar{r}} (1 - \exp(-\bar{r}^2)) \quad (10)$$

207
208 It is perhaps worth noting that $\overline{u_z}$ and $\overline{u_r}$ increase to infinity as $\bar{z} \rightarrow \infty$ and $\bar{r} \rightarrow \infty$, respectively,
209 which, it is suggested, may not be representative of a tornado-like flow structure.

210
211 The pressure distribution of the Burgers-Rott vortex model can be obtained by solving the
212 simplified NSE (Eq. 2* and Eq. 4*) using the model velocities (Eq. 8, Eq. 9, and Eq. 10). This
213 leads to the following equation for the normalised pressure distribution (Eq. 11).

214
215
$$\overline{p_{Burgers}}(\bar{r}, \bar{z}) = \bar{p}(0,0) + \int_0^{\bar{r}} \frac{\overline{u_\theta}(\bar{r}')^2}{\bar{r}'} d\bar{r}' - \frac{\bar{a}^2}{2} (\bar{r}^2 + 4\bar{z}^2) \quad (11)$$

216

217 2.4 Sullivan vortex model

218

219 The Sullivan model has also been adopted by a few researchers (e.g. Winn et al., 1999; Wood and
220 Brown, 2011) to model tornado-like flow behaviour. The assumptions for this vortex model are:

221

222 • *The flow field is steady state, i.e., terms $R1$, $C1$ and $Z1$ are taken as zero.*

223 • *The viscosity is considered to be constant throughout the entire flow field.*

224 • *Body forces can be neglected, i.e., $(\vec{g}=0)$.*

225 • *The circumferential velocity component is assumed to be solely dependent on the radial
226 distance ($u_\theta = u_\theta(r)$).*

227 • *The vertical velocity component is assumed to be only dependent on radial and vertical
228 distances. The dependence on the vertical distance is linear ($u_z = u_z(r, z)$ and $u_z \propto z$).*

229 • *As a result of the last two assumptions, the radial velocity component is solely dependent
230 on the radial distance ($u_r = u_r(r)$).*

231 • *The static pressure is assumed to be solely dependent on radial and vertical distances ($p =$
232 $p(r, z)$).*

233

234 The above assumptions reduce the continuity equation (1), radial and circumferential components
235 of the NSE (2 – 3) to simplified versions shown in equations (1* – 3*). For the vertical component
236 of the NSE (4) the following simplified versions is obtained.

237

$$238 \quad u_z \frac{\partial u_z}{\partial z} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) \right) \quad (4^{**})$$

239

240 One main difference of the Sullivan model compared to the Burgers-Rott model lies in the
241 complexity of the model solution. While the Burgers-Rott vortex model only allows single-celled
242 vortices to be generated, the Sullivan model potentially enables solutions for single and two-celled
243 vortices to be obtained; this is obtained via the use of a shape parameter, b (Eq. 12 and Eq. 13).
244 The effect of this parameter on the tornado-like flow field will be discussed in detail in section 2.7.
245 Unless stated otherwise, $b = 3$. The required vortex stretching is generated by suction at relatively
246 large heights and is achieved by a non-linear increase of the vertical velocity component with
247 height, as illustrated in equation (12). The same normalisation used for the Burgers-Rott vortex
248 model is applied for the Sullivan vortex model.

249

$$250 \quad \overline{u_z}(\bar{r}, \bar{z}) = 2\bar{a}\bar{z}(1 - b \cdot \exp(-\bar{r}^2)) \quad (12)$$

251

252 Following the procedure described for the Burgers-Rott vortex model, expressions for $\overline{u_r}$ (Eq. 13)
253 and $\overline{u_\theta}$ (Eq. 14) can be obtained.

254

255
$$\overline{u_r}(\bar{r}) = -\bar{a}\bar{r} + \frac{2b\bar{v}}{\bar{r}}(1 - \exp(-\bar{r}^2)) \quad (13)$$

256

257
$$\overline{u_\theta}(\bar{r}) = \frac{1}{\bar{r}} \frac{H(x)}{H(\infty)} \quad (14)$$

258 with $x = \bar{r}^2$ and $H(x) = \int_0^x \exp(-x' + 3 \int_0^{x'} \frac{1}{x''} (1 - \exp(-x'')) dx'') dx'$

259

260 It is perhaps worth noting, that for $\bar{r} = 0$ and $\bar{z} \rightarrow \infty$ the magnitude of $\overline{u_z}$ increases to infinity.

261 Furthermore, also $\overline{u_r}$ increases to infinity for $\bar{r} \rightarrow \infty$ (Eq. 13). Similar to the Burgers-Rott model,

262 it is suggested that this behaviour may be physically unrealistic in a tornado-like structure.

263

264 The pressure distribution of the Sullivan vortex model can be obtained by solving the simplified

265 NSE (Eq. 2* and 4*) using the model velocities (Eq. 12 - 14). This leads to the following equation

266 for the normalised pressure distribution:

267

268
$$\overline{p_{Sullivan}}(\bar{r}, \bar{z}) = \overline{p_{Burgers}}(\bar{r}, \bar{z}) - \frac{18\bar{v}^2}{\bar{r}^2} (1 - \exp(-\bar{r}^2))^2 \quad (15)$$

269

270 *2.5 Baker vortex model*

271

272 Baker and Sterling (2017) developed a vortex model, which can reproduce the flow and pressure

273 characteristics of a single and two-celled vortex. In order to enable comparisons with the

274 aforementioned models, only the solution for the single-cell vortex with radial inflow and vertical

275 updraft is analysed in this paper. The following assumptions are made in the derivation of the

276 Baker vortex model:

- 277
- 278 • *The flow field is steady state, i.e., terms R1, C1 and Z1 are taken as zero.*
 - 279 • *The flow is inviscid, i.e., terms R8 - R12, C8 - C12 and Z7 - Z9 can be disregarded.*
 - 280 • *Body forces can be neglected, i.e., ($\vec{g}=0$).*
 - 281 • *The circumferential velocity component is assumed to be dependent on radial and vertical*
 - 282 *distance ($u_\theta = u_\theta(r, z)$).*
 - 283 • *The radial velocity component is assumed to be only dependent on radial and vertical*
 - 284 *distances ($u_r = u_r(r, z)$).*
 - 285 • *As a result of the last two assumptions, the vertical velocity component is solely dependent*
 - 286 *on radial and vertical distances ($u_z = u_z(r, z)$).*
 - 287 • *The static pressure is assumed to be solely dependent on radial and vertical distances ($p =$*
 - 288 *$p(r, z)$).*
- 289

290 These assumptions reduce the continuity equation (1) to the simplified version shown in equation

291 (1*) and the NSE (Eq. 2 - 4) to the following simplified versions:

292

293
$$u_r \frac{\partial u_r}{\partial r} - \frac{u_\theta^2}{r} + u_z \frac{\partial u_r}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} \quad (2^{***})$$

294
295
$$u_r \frac{\partial u_\theta}{\partial r} + \frac{u_r u_\theta}{r} + u_z \frac{\partial u_\theta}{\partial z} = 0 \quad (3^{***})$$

296
297
$$u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} \quad (4^{***})$$

298
299 In addition, the Baker model assumes that the radial velocity component takes the following form:
300

301
$$\overline{u_r}(\bar{r}, \bar{z}) = \frac{-4\bar{r}\bar{z}}{(1+\bar{r}^2)(1+\bar{z}^2)} \quad (16)$$

302
303 One of the potential advantages of this model compared to the Rankine, Burgers-Rott and Sullivan
304 vortex models is that the radial velocity component is assumed to show a more realistic behaviour
305 (Eq. 16), i.e., rather than increasing to infinity for large radial distances, a maximum value is
306 reached at $r = r_m$ which then falls to zero for $r = 0$ and $r = \infty$. In the vertical direction, the radial
307 velocity distribution shows an attempt to replicate the tornado boundary layer by assuming a
308 maximum in the radial velocity component at a known distance above the ground ($z = z_m$). For z
309 $= 0$ and $z = \infty$ the radial velocity falls to zero. Since the Baker vortex model focuses on the
310 distribution of the radial velocity component, different parameters are chosen for the normalisation
311 of velocities, radial and vertical distances. Velocities are normalised by the maximum radial
312 velocity ($u_r(r_m, z_m) = u_{r,max}$) and radial and vertical distances are normalised by r_m and z_m ,
313 respectively.

314
315 Using equation (16) and integrating the simplified continuity equation (1*), an expression for the
316 normalised vertical velocity component, $\overline{u_z}$, can be obtained as follows:
317

318
$$\overline{u_z}(\bar{r}, \bar{z}) = \frac{4\delta \ln(1+\bar{z}^2)}{(1+\bar{r}^2)^2} \quad (17)$$

319
320 where δ is the ratio z_m/r_m . Using equations (16) and (17), and solving the simplified NSE in the
321 circumferential direction (Eq. 3***), the following expression for the normalised circumferential
322 velocity component, $\overline{u_\theta}$, can be obtained:
323

324
$$\overline{u_\theta}(\bar{r}, \bar{z}) = \frac{K\bar{r}^{\gamma-1}[\ln(1+\bar{z}^2)]^{\gamma/2}}{(1+\bar{r}^2)^{\gamma/2}} \quad (18)$$

325
326 Here, γ is a shape parameter (an arbitrary real number which can be used to adjust the shape of the
327 circumferential velocity profile). K is a constant and related to Baker's definition of the swirl ratio,
328 i.e., $S_{Baker} = 0.347 K$. The swirl ratio in the Baker vortex model is defined as the ratio of $u_\theta(r_m, z_m)$
329 and $u_r(r_m, z_m)$. It is perhaps worth noting that $\overline{u_\theta}$ increases to infinity for $z \rightarrow \infty$. This increase is

330 assumed to be realistic for the lowest heights, relatively close to the surface, where surface
 331 roughness affects the vertical velocity distribution but becomes physically unrealistic for larger
 332 heights.

333

334 The pressure distribution of the Baker vortex model can be obtained by solving the simplified NSE
 335 (Eq. 2*** and Eq. 4***) using the model velocities (Eq. 16 – 18) and, assuming a shape parameter
 336 of $\gamma = 2$:

337

$$338 \quad \overline{p}_{Baker}(\bar{r}, \bar{z}) = -\frac{8\bar{r}^2\bar{z}}{(1+\bar{r}^2)^2(1+\bar{z}^2)^2} - \frac{4.15S_{Baker}^2(\ln(1+\bar{z}^2))^2}{(1+\bar{r}^2)} - \frac{4\ln(1+\bar{z}^2)(1-\bar{z}^2)}{(1+\bar{r}^2)^2(1+\bar{z}^2)^2} \quad (19)$$

339

340 It is worth noting that the surface pressure distribution equals zero for $z = 0$. This behaviour is
 341 physically unrealistic in a tornado-like structure and is discussed in Baker and Sterling (2017). It
 342 is assumed that pressure variations in the vertical direction can be neglected within the boundary
 343 layer ($z < z_m$) (Baker and Sterling, 2017) and consequently, it is assumed that $\overline{p}_{Baker}(\bar{r}, \bar{z} < 1) =$
 344 $\overline{p}_{Baker}(\bar{r}, \bar{z} = 1)$.

345

346 2.6 Circumferential velocity component

347

348 Unlike the Rankine, Burgers-Rott and Sullivan vortex models, the Baker model has a shape
 349 parameter, γ , which enables the shape of the circumferential velocity profile to be varied. The
 350 effect of this parameter on the circumferential velocity field is illustrated in table 1 and figure 2. It
 351 is perhaps worth noting that γ is related to \bar{r} in the following way ($\bar{r}^2 = \gamma - 1$) (Baker and
 352 Sterling, 2017). Consequently, for $\gamma \leq 1$, the circumferential velocity component becomes
 353 physically unreasonable. Thus, to ensure results, which describe the behaviour of a forced vortex
 354 at the centre, and a free vortex at larger radial distances, Baker and Sterling (2017) recommend
 355 setting $\gamma = 2$.

356

357 Table 1 illustrates the ratio of $u_{\theta,max}$ and R of the corresponding models compared to the Rankine
 358 vortex model with input parameters of $R = 10\text{m}$ and $u_{\theta,max} = 10\text{m/s}$. For the Baker vortex model
 359 a swirl ratio of $S_{Baker} = 1$ is assumed and readings for the maximum circumferential velocity
 360 component are taken at $z = z_m$. Table 1 shows that Burgers-Rott and Sullivan vortex models
 361 underestimate the actual input velocity by a factor of 0.64 and 0.32 , respectively, and overestimate
 362 the core radius position, R , by a factor of 1.12 and 2.29 respectively. For the Baker vortex model
 363 with a shape parameter of $\gamma = 2$, the radius at which \overline{u}_r attains a maximum, is identical to the core
 364 radius of the Baker vortex model and is also identical to the actual input core radius ($r_m = R_{Baker} =$
 365 R). Hence, the Rankine and Baker model results for $\gamma = 2$ in table 1 are identical. With increasing γ ,
 366 R_{Baker} increases and the magnitude of $u_{\theta,max,Baker}$ decreases (Table 1). For $\gamma = 3$, R and $u_{\theta,max}$ of
 367 Baker and Burgers-Rott vortex model are similar and show a decrease in $u_{\theta,max}$ by about
 368 $1/3 \times u_{\theta,max,Rankine}$ (Table 1). For $\gamma = 5$, R and $u_{\theta,max}$ of Baker and Sullivan vortex model are

369 similar and show a core radius, which is about $2 \times R_{Rankine}$ and a decrease in $u_{\theta,max}$ by about $2/3 \times$
 370 $u_{\theta,max,Rankine}$ (Table 1). Thus, in what follows, care has been taken to normalise, by the relevant
 371 model values of each vortex model as opposed to a standard value.

372

373 Table 1: Ratios of $u_{\theta,max}$ and R of the corresponding vortex models compared to the Rankine vortex model.

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	$\frac{u_{\theta,max,x}}{u_{\theta,max,Rankine}}$	$\frac{R_x}{R_{Rankine}}$
$x=Rankine$	1	1
$x=Burgers$	0.64	1.12
$x=Sullivan$	0.32	2.29
$x=Baker (\gamma=2, z=z_m)$	1	1
$x=Baker (\gamma=3, z=z_m)$	0.64	1.41
$x=Baker (\gamma=5, z=z_m)$	0.33	2.0

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Figure 2 shows that among the four vortex models, large differences in the circumferential velocity are found for $r < R_{model}$ between Sullivan and Baker vortex model with a shape parameter of $\gamma = 2$ (Figure 2). This is not surprising as these vortex models represent two entirely different tornado flow types. The Sullivan model represents a two-celled vortex whereas the Baker model shows a single-celled vortex structure. A single-celled vortex is defined as a relatively narrow column of rising and rotating air. With increasing rotational energy, a downdraft forms in the vortex centre and terminates aloft a stagnation point above which the vortex increases significantly in size (Trapp, 2000). This vortex transformation is known as ‘vortex breakdown’. With a further increase in rotational energy, the stagnation point lowers towards the ground. In a two-celled vortex, the downdraft reaches the ground and therefore decreases $\overline{u_{\theta}}$ close to $r = 0$ whereas the structure of the single-celled vortex shows a strong non-linear increase of $\overline{u_{\theta}}$ inside the vortex core ($r < R_{model}$) (Figure 2).

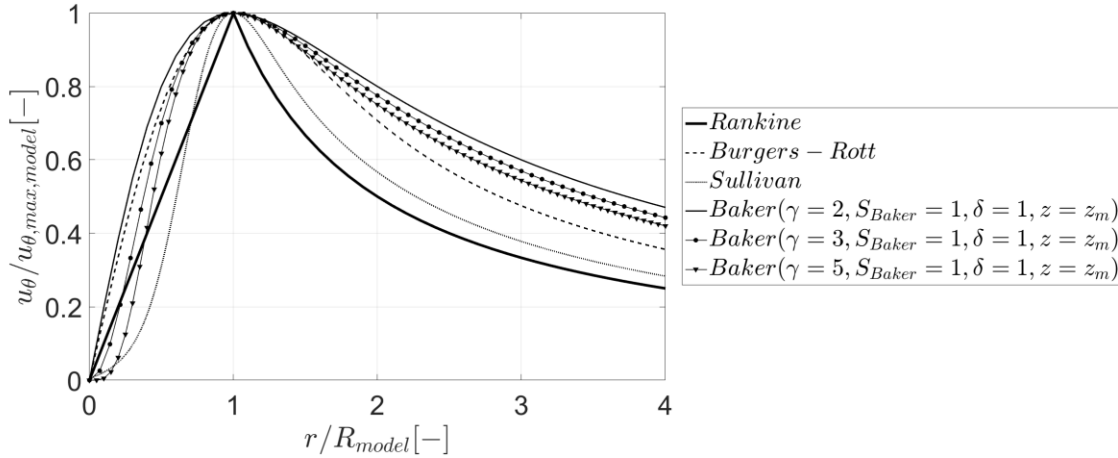


Figure 2: Circumferential velocity component of the Rankine, Burgers-Rott, Sullivan and Baker vortex model.

400
401
402

403 For the Baker model, close to the vortex core, $\overline{u_\theta}$ increases at a slower rate with increasing γ , and
 404 the circumferential velocity profile tends towards the shape of a two-celled vortex structure (Figure
 405 2) (compared to the Sullivan vortex model). For larger radial distances ($r > R_{model}$), $\overline{u_\theta}$ of the Baker
 406 vortex model decreases at a slower rate compared to the other three models and largest differences
 407 are found between Rankine and Baker vortex model ($\gamma = 2$). Differences in the circumferential
 408 velocity of all vortex models decrease as $r \rightarrow \infty$.

409

410 The effect of γ on the circumferential velocity component with height is shown in Figure 3. For
 411 relatively small vertical distances from the surface, $\overline{u_\theta}$ increases at a slower rate the larger γ is.
 412 With increasing vertical distance, this behaviour reverses and $\overline{u_\theta}$ increases faster with height as γ
 413 increases (Figure 3). Furthermore, figure 3 shows that independent from γ , $\overline{u_\theta}$ increases to infinity
 414 for $z \rightarrow \infty$, albeit at different rates. It needs to be mentioned that the Baker vortex model is the
 415 only model that takes a height dependence of u_θ into account. However, it does not represent the
 416 vertical profile of the circumferential velocity observed in simulated tornado-like vortices or full-
 417 scale. For instance, Tang et al. (2017) showed that u_θ increases rapidly in the lowest heights with
 418 the maximum circumferential velocity relatively close to the ground. With further increasing
 419 height $\overline{u_\theta}$ decreases and remains relatively uniform in even greater heights. Refan et al. (2017)
 420 showed that a similar behaviour was observed in five different full-scale tornadoes of different
 421 intensity and flow structure.

422

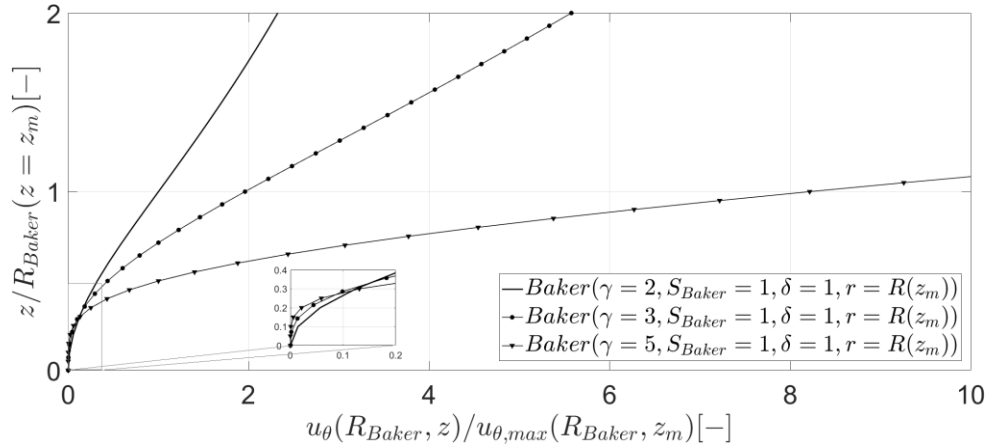


Figure 3: Circumferential velocity component of the Baker vortex model in different heights for different γ values.

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Another parameter that influences the shape of the circumferential velocity profile is the swirl ratio, S_{Baker} . The magnitude of u_θ increases as the value of S_{Baker} increases. The position of the core radius and the shape of the circumferential velocity profile are independent from the chosen swirl ratio. For that reason, the effect of S_{Baker} on u_θ cannot be seen when normalising u_θ with $u_{\theta,max,Baker}$ and radial distances with R_{Baker} .

431

2.7 Radial and vertical velocities and shape parameters

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433

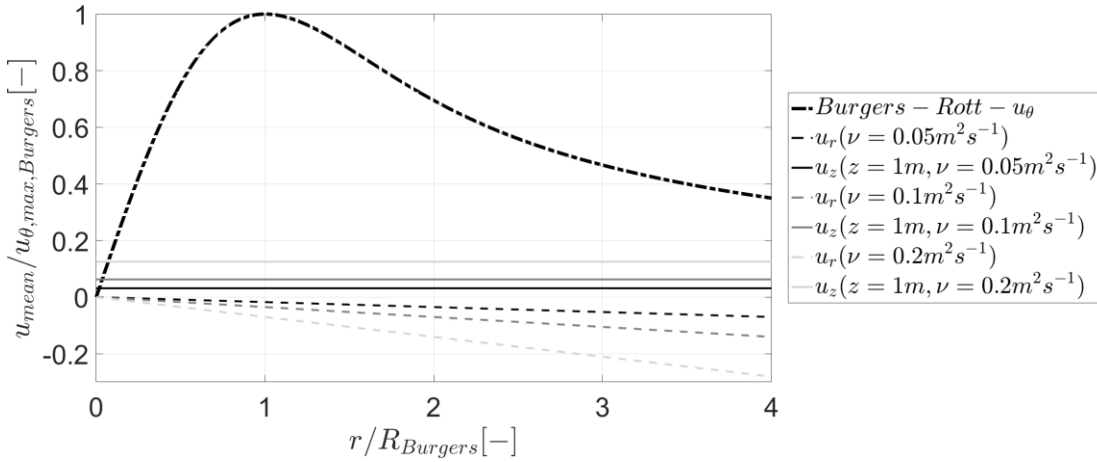
The Baker model is not the only model to employ a shape component. As shown in equation (8) and (12), both, the Burgers-Rott and Sullivan model also include a ‘constant’ to ‘correct’ the vertical velocity. In addition, both models have a ‘viscosity’ parameter explicitly included in the circumferential velocity component (which is not evident in equations (10) and (14) due to the normalisation adopted and assuming the relationship given in equation (8.1)). If it is assumed that the viscosity parameter corresponds to the kinematic viscosity of air (i.e., $\nu \sim 10^{-5} \text{ m}^2\text{s}^{-1}$ at 20 °C), then the calculated radial and vertical velocity components of the Burgers-Rott and Sullivan vortex models are small. Hence, to ensure reasonable magnitudes of the velocity components (Figures 4 and 5), the viscosity needs to be increased by several orders of magnitude (Davies-Jones and Kessler, 1974). Thus, in this context, the ‘viscosity’ parameter is essentially nothing more than a ‘simple’ shape parameter.

445

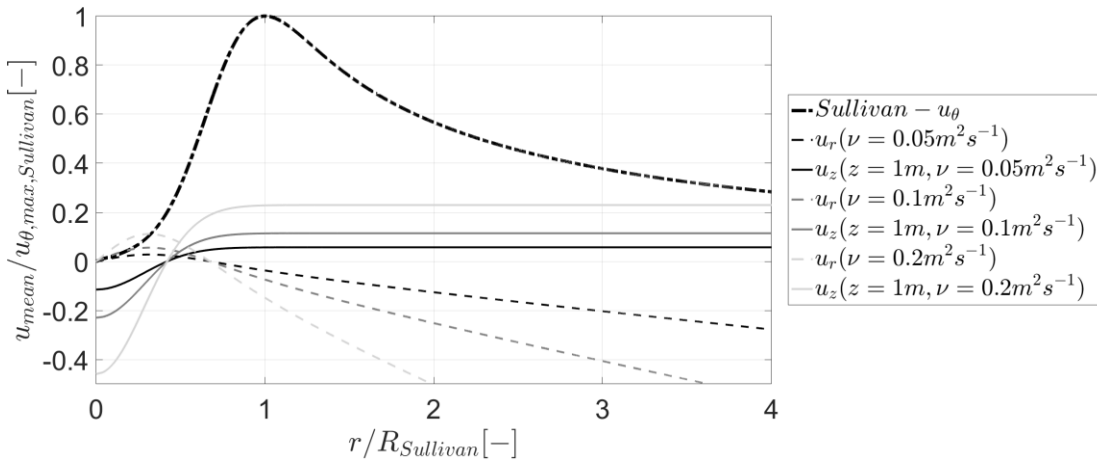
Figures 4 and 5 show the effect of the ‘viscosity’ parameter on the radial and vertical velocity distributions of the Burgers-Rott and Sullivan vortex models. For the Burgers-Rott vortex model, a larger ‘viscosity’ parameter results in larger vertical and radial velocities (Figure 4). Hence, the larger the value of ν becomes, the greater the radial inflow towards the vortex centre and the stronger the vertical updraft gets (Figure 4). For the Sullivan vortex model, an increase in ν results in larger negative vertical velocities for $r < R_{Sullivan}$, and larger positive vertical velocities for $r > R_{Sullivan}$ (Figure 5). For the radial velocity, the larger ν is, the stronger the radial outflow at $r <$

452

453 $R_{Sullivan}$, and the stronger the radial inflow for $r < R_{Sullivan}$ gets (Figure 5). The radial distance at
 454 which radial and vertical velocity components change sign is not affected by changes in the
 455 ‘viscosity’ parameter (Figure 5). This means that the size of the downdraft region close to $r = 0$ is
 456 independent of ν .
 457



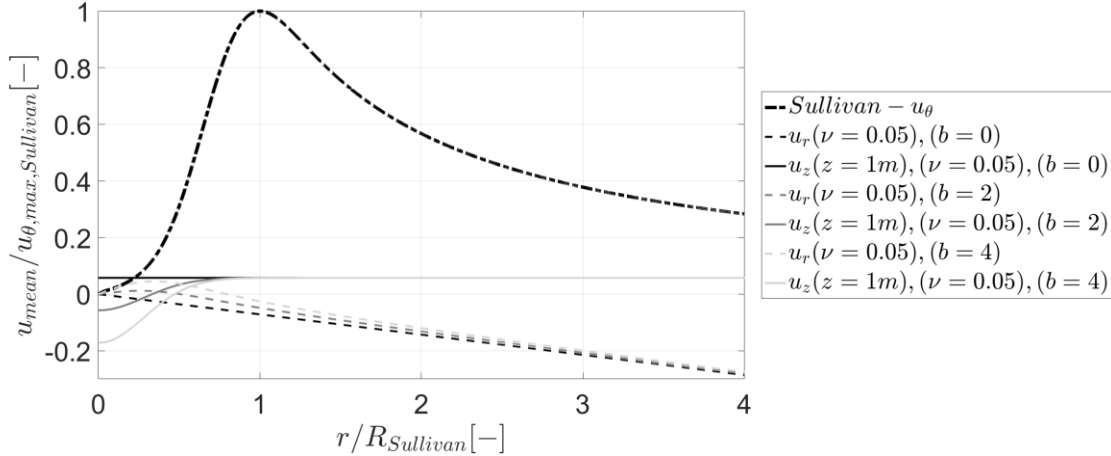
458
 459 Figure 4: Velocity components of the Burgers-Rott vortex model for different ‘viscosity’ values.



460
 461 Figure 5: Velocity components of the Sullivan vortex model for different ‘viscosity’ values.
 462

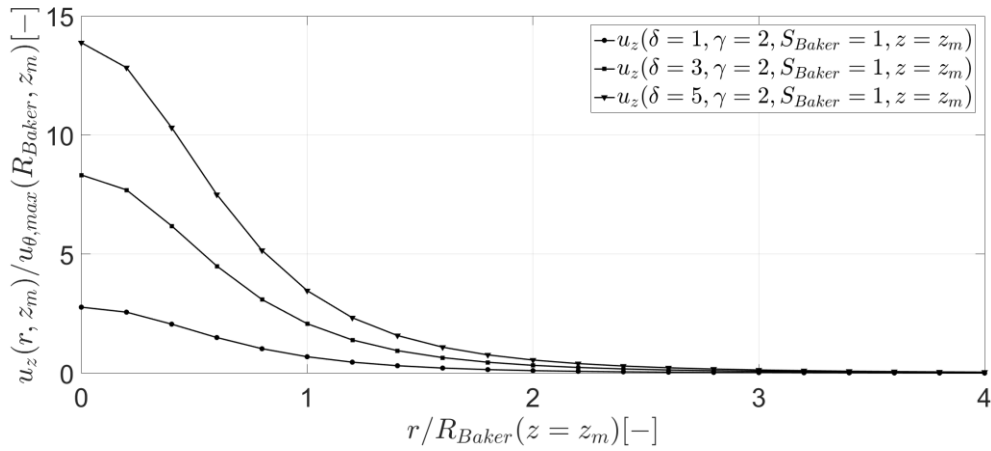
463 An additional shape parameter contained in the Sullivan vortex model is denoted as b , which
 464 influences the distribution of radial and vertical velocity components and can be adjusted to model
 465 solutions for single and two-celled vortices. Figure 6 illustrates the effect of b on the radial and
 466 vertical flow field of the Sullivan vortex model. For $b = 0$, radial and vertical velocity components
 467 of the Sullivan vortex model are identical to the solutions obtained from the Burgers-Rott vortex
 468 model. For $b > 1$, a two-celled structure can be obtained, which is indicated by negative vertical
 469 velocities close to the vortex centre. The greater the magnitude of b gets, the more negative the
 470 vertical velocities in the vortex centre become, and additionally, the further the downdraft region
 471 extends in the radial direction. For $r > R_{Sullivan}$, the vertical velocity converges to a value, which is
 472 independent of b but dependent on the height. Radial outflow velocities inside the vortex core are

473 larger for larger b values, and the larger b gets, the larger the region of radial outflow extends
 474 radially (Figure 6). Radial inflow velocities obtained with a lower b value increase slightly faster
 475 in magnitude close to the vortex core but converge for larger radial distances (Figure 6). Hence,
 476 differences of b are only significant inside the vortex core.

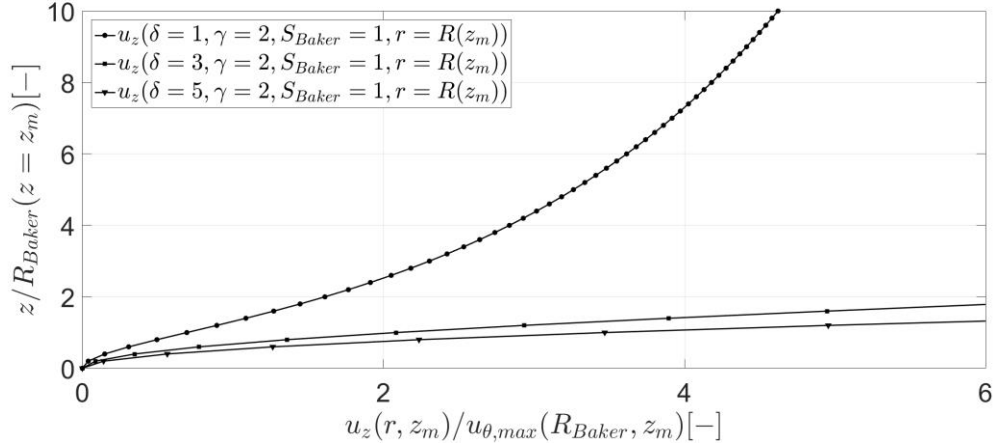


477
 478 Figure 6: Velocity components of the Sullivan model for different b values.

479
 480 The vertical velocity component of the Baker vortex model depends on the value of δ . The vertical
 481 velocity component at the vortex centre increases with increasing δ (Figure 7a). Additionally,
 482 figure 7b shows that the vertical velocity component increases faster with height as δ increases.



483 a)



484 b)

485 Figure 7: The effect of δ on the vertical velocity component of the Baker vortex model for different radial distances
 486 (a) and heights (b).
 487

488 2.8 The static surface pressure distribution

489

490 The surface pressure distribution of Rankine, Burgers-Rott and Sullivan vortex models are shown
 491 in figure 8 for the case of $\nu = 0$. This restriction implies that the surface pressure distributions of
 492 the Burgers-Rott and Sullivan vortex models are solely dependent on the circumferential velocity
 493 profile of the corresponding vortex model and equations (11) and (15) simplify to:

494

$$495 \quad \overline{p_{Burgers}}(\bar{r}, \bar{z}) = \overline{p_{Sullivan}}(\bar{r}, \bar{z}) = \bar{p}(0,0) + \int_0^{\bar{r}} \frac{\overline{u_{\theta}(\bar{r}')^2}}{\bar{r}'} d\bar{r}'. \quad (20)$$

496

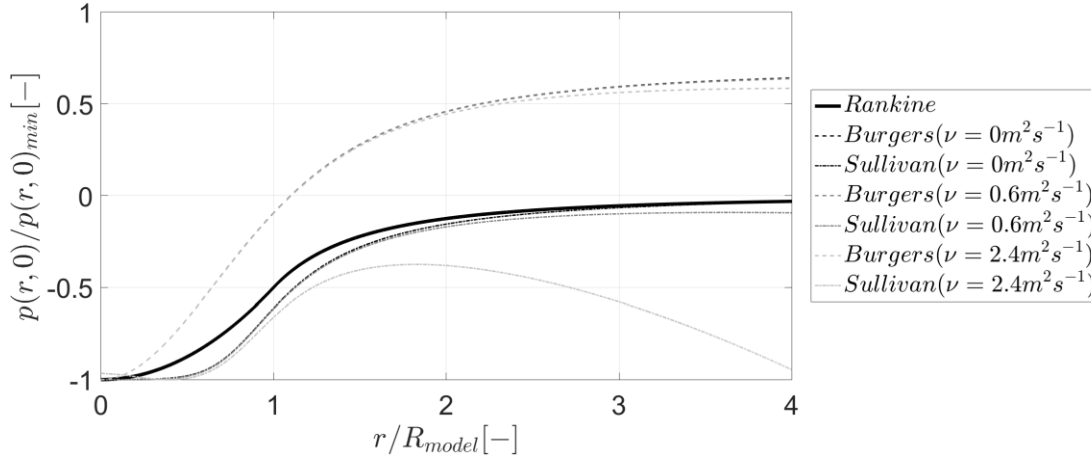
497 The last term on the right hand side gives the largest contribution to the entire pressure distribution
 498 of the Burgers-Rott and Sullivan vortex model. Thus, its value defines the magnitude of surface
 499 pressure increase from the vortex centre towards larger radial distances and is determined by the
 500 area under the circumferential velocity profile shown in figure 2. For that reason, this term is
 501 largest for the Burgers-Rott vortex model and results in what maybe a physically unrealistic
 502 surface pressure increase from the vortex centre (Figure 8). For the Sullivan vortex model, the
 503 magnitude of this term is of the same order as that one of the Rankine vortex model.

504

505 The entire pressure distribution of the Burgers-Rott and Sullivan vortex models, depend on the
 506 contribution of the circumferential, radial and vertical velocity (Eq. 11 and Eq. 15), and therefore
 507 is dependent on the ‘viscosity’ parameter. Also shown in Figure 8, is the effect of ν on the pressure
 508 distribution. The decrease in surface pressure with increasing radial distance originates from a
 509 combination of the vertical updraft and the potentially unrealistic increase in radial velocity, i.e.,
 510 $\frac{\bar{a}^2}{2} (\bar{r}^2 + 4\bar{z}^2)$. These terms are identical in Burgers-Rott and Sullivan vortex models, however, the
 511 different trends shown in figure 8 arise due to the normalisation, since $R_{Sullivan} > R_{Burgers-Rott}$.

512

513 The additional term in the surface pressure distribution of the Sullivan vortex model (i.e.,
 514 $-\frac{18\bar{v}^2}{\bar{r}^2}(1 - \exp(-\bar{r}^2))^2$) describes the effect of the non-linear behaviour of radial and vertical
 515 velocity components inside the vortex core on the pressure distribution. The downdraft close to
 516 the centre of the vortex increases the surface pressure distribution and places the minimum pressure
 517 at the radial position where vertical and radial velocity components are zero (Figure 8).



518 Figure 8: Surface pressure distribution of Rankine, Burgers-Rott and Sullivan vortex model for different ‘viscosity’
 519 values.
 520
 521

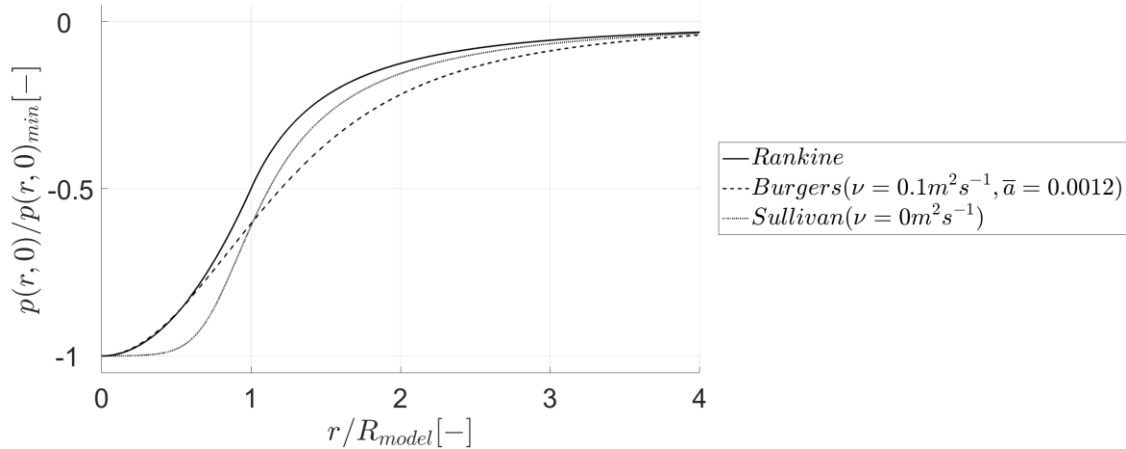
522 In order to enable a meaningful comparison of the surface pressure distributions, the ‘viscosity’
 523 parameter, ν , and the ‘vortex stretching’ parameters \bar{a} , need to be adjusted in the circumferential
 524 velocity and surface pressure distribution of the Burgers-Rott vortex model. This ensures that the
 525 increase in surface pressure is of similar order to that one of the Rankine and Sullivan vortex
 526 model. Thus, equation (10) needs to be modified by means of the relation given in equation (8.1)
 527 in the following way (Eq. 21) to obtain a circumferential velocity distribution for the Burgers-Rott
 528 vortex model, which is dependent on ν and \bar{a} (Eq. 21).
 529

$$530 \quad \bar{u}_\theta(\bar{r}) = \frac{1}{\bar{r}} \left(1 - \exp \left(-\frac{r\bar{a} u_{\theta,max}}{2\nu} \right)^2 \right) \quad (21)$$

531
 532 ν and \bar{a} need to be chosen independent from one another, which has the consequence that the input
 533 parameters, R and $u_{\theta,max}$ vary (see Eq. 8.1). Unquestionably, this is physically not consistent but
 534 seems to be the only way for the Burgers-Rott vortex model to generate a physically meaningful
 535 surface pressure increase with increasing radial distance.
 536

537 Figure 9 shows the surface pressure distribution for input parameters of $R = 10\text{m}$ and $u_{\theta,max} =$
 538 10m/s for the Rankine, Burgers-Rott and Sullivan models. It is worth noting that the ‘viscosity’
 539 and ‘stretching’ parameter differ for different input parameters. The two-celled structure of the
 540 Sullivan vortex, and hence, the decreased circumferential velocity component close to the vortex
 541 core result in a relatively flat pressure distribution close to the vortex centre (Figure 8). The surface

542 pressure distribution of the Burgers-Rott vortex increases at a faster rate close to the vortex centre
 543 due to a rapid increase in circumferential velocity in this region (Figure 8). The largest differences
 544 in the surface pressure distribution of the vortex models can be found inside the vortex core ($r <$
 545 R_{model}) and for radial distances around $r / R_{model} = 1.5$. This arises due to the relative differences in
 546 the magnitude of the circumferential velocity predicted by the analytical models (section 2.6).



547 Figure 9: Surface pressure distribution of Rankine, adjusted Burgers-Rott and Sullivan vortex model.

548
 549
 550 The static ‘surface pressure’ distribution and the effect of the swirl ratio on the shape of the
 551 distribution of the Baker vortex model is shown in figure 10. The ‘surface pressure’ distribution
 552 of the Baker vortex model falls to zero for $r \rightarrow \infty$, however, the ‘surface pressure’ minimum is not
 553 bounded for the Baker vortex model and decreases with increasing swirl ratio as shown in figure
 554 10 from $-1.99 \times \rho u_r(r_m, z_m)$ to $-17.94 \times \rho u_r(r_m, z_m)$ for $S_{Baker} = 1$ to 3 due to the term
 555
$$-\frac{4.15 S_{Baker}^2 (\ln(1+\bar{z}^2))^2}{(1+\bar{r}^2)}$$
 (compare equation (19)). The effect of this on the ‘surface pressure’
 556 distribution is masked in figure 10 due to normalising the pressure distribution additionally with
 557 $p(r, z_m)_{min}$ of the corresponding swirl ratio. This additional normalisation is applied to force all
 558 pressures to tend to -1 as r/R tends to 0. It is noted that some numerical and experimental data,
 559 e.g., Natarajan and Hangan (2012) and Haan et al. (2008), show that the surface pressure minimum
 560 decreases in magnitude with increasing swirl ratio which has been associated with a transition from
 561 a single-celled to a two-celled vortex. At present, although the Baker model is able to represent
 562 multiple cell tornadoes, this behaviour is not incorporated into the current paper.

563

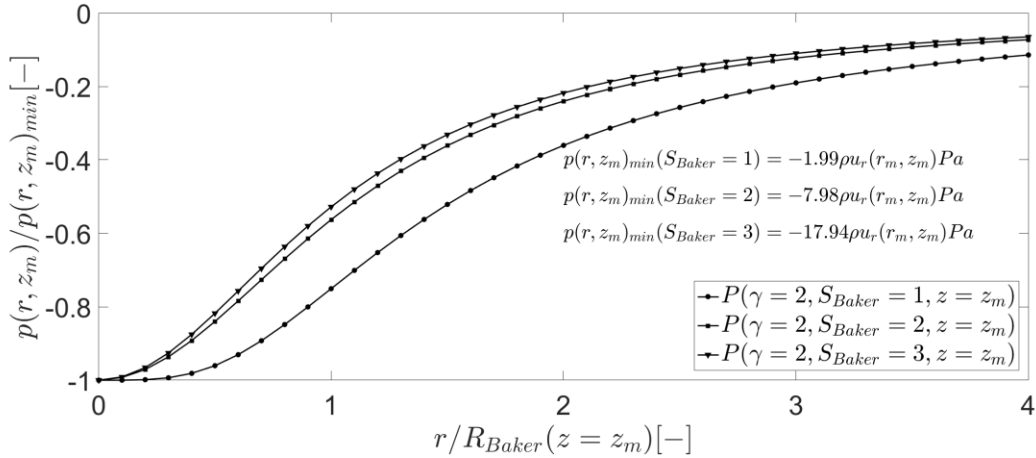


Figure 10: ‘Surface pressure’ distribution of the Baker vortex model for different swirl ratios (S_{Baker}).

564
565
566

567 The effect of the radial velocity profile on the ‘surface pressure’ is, not surprisingly, largest at the
568 core radius (R_{Baker}) since this is where the radial velocity is maximum for $\gamma = 2$. When this term is
569 normalised by the value of $p(r, z_m)_{min}$ for each corresponding swirl ratio, its magnitude changes and
570 results in a different distribution of relative pressure (Figure 10). As a consequence, figure 10
571 shows that with increasing S_{Baker} the rate of change of pressure with increasing radial distance
572 increases and hence, the smaller the vortex becomes. It needs to be mentioned here that this is only
573 the case because the core radius (R_{Baker}) remains constant even though the swirl ratio changes. Due
574 to the shape of the radial velocity component, differences in figure 10 are largest at $r / R_{Baker}(z =$
575 $z_m) = 1$. The last term in equation (19) represents the effect of vertical advection of radial velocity
576 on the ‘surface pressure’ distribution, i.e., $(-\frac{4\ln(1+\bar{z}^2)(1-\bar{z}^2)}{(1+\bar{r}^2)^2(1+\bar{z}^2)^2})$. When $z = z_m$, this term reduces to
577 zero.

578
579

3. Experimental methodology

580

581 In this section, the experimental methodology is presented, which was used to assess the ability
582 (or otherwise) of the introduced vortex models to simulate flow and pressure characteristics
583 obtained in a physical tornado-like vortex simulator.
584

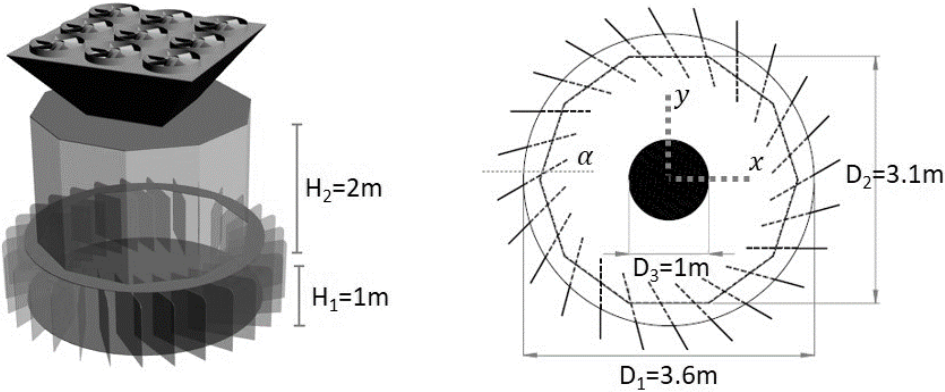
585

3.1 Tornado-like vortex simulator

586

587 For this analysis, the University of Birmingham (UoB) tornado-like vortex generator (3m× 3m),
588 which is based on the design by Ward (1972), is used (Figure 11). The generator consists of two
589 chambers, a convergence chamber with height H_1 and diameter D_1 and a convection chamber with
590 height H_2 and diameter D_2 . The aspect ratio (a) is defined as the ratio of updraft radius ($1/2 D_3$)
591 and convergence chamber height and therefore results in an aspect ratio of $a = 2$ for the
592 configuration shown in Figure 11. Angular momentum is introduced by guide vanes around the
593

594 convergence chamber, which can be set to different angles. By changing the guide vane angle, the
 595 vorticity in the flow can be altered and different vortex structures can be generated.
 596



597
 598 Figure 11: An illustration of the UoB tornado-like vortex generator.
 599

600 The kinematic and dynamic similarity of the generated vortex is controlled by the Reynolds
 601 number, Re , (Eq. 22) and the swirl ratio, S , (Eq. 23).
 602

603
$$Re = \frac{Q}{\nu 0.5D_3} \tag{22}$$

604
$$S = \frac{\tan(\alpha)}{2a} \tag{23}$$

605
 606 Here, Q is the flow rate through the simulator and ν is the kinematic viscosity of air. The guide
 607 vane angle, α , is the angle relative to the radial velocity component (Figure 11).
 608

609 **3.2 Measurement setup and data quality**

610
 611 Point velocity measurements were made with 100Hz using a Cobra Probe, which was mounted to
 612 a two-axis traverse system inside the simulator. This traverse system enabled the probe to be
 613 positioned with an accuracy of 1mm at nine heights (z) above the simulator's surface (0.01m,
 614 0.05m, 0.10m, 0.15m, 0.20m, 0.25m, 0.30m, 0.40m, 0.50m, 0.60m) with a radial spacing of
 615 0.025m from the centre of the simulator up to a radial distance of 0.60m. The size of the probe is
 616 less than 10^6 times smaller than the convergence chamber and the influence of the traverse system
 617 on the surface pressure measurements was found to be smaller than the experimental measurement
 618 uncertainty. The Cobra Probe can measure velocity data greater than 2m/s within a cone of
 619 influence of $\pm 45^\circ$. These limitations can have a direct influence on the measured data. For
 620 example, if the recorded data quality (percentage of velocity samples of a measured time series
 621 which are $>2\text{m/s}$ and have an angle of attack $<\pm 45^\circ$) is less than 100% then this can introduce a
 622 bias in the calculated velocity vector – the lower the data quality the greater the potential bias. To
 623 minimize the bias in time averaged velocities, only those positions with a data quality of greater
 624

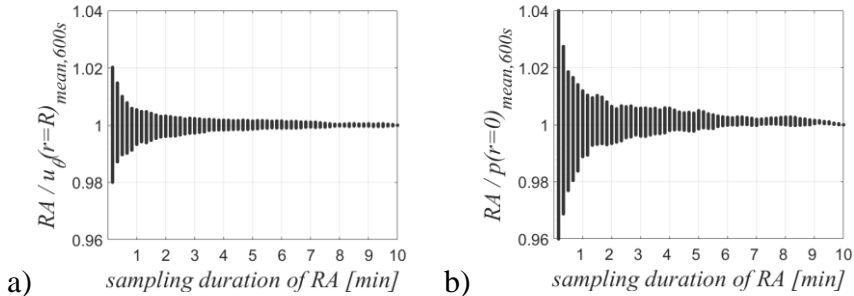
625 than 80% were accepted for further analysis. This threshold is assumed to provide a suitable
 626 compromise between data quality and quantity.

627
 628 The pressure distribution is measured with 100Hz on the ground plane along two perpendicular
 629 lines denoted x and y in figure 11. Surface pressure taps are distributed along these lines with a
 630 spacing of 0.05m from the simulator's centre up to a distance of 0.75m.

631
 632 Different types of uncertainties have to be distinguished. The experimental uncertainty is a
 633 combination of uncertainties due to measuring a finite time series (statistical uncertainty), human
 634 error such as probe and guide vane angle positioning (repeatability) and the uncertainty of the
 635 measurement device itself.

636
 637 *3.2.1 Statistical uncertainty*
 638

639 The statistical uncertainty is a measure of uncertainty of the time average with respect to the
 640 unsteadiness of the flow or surface pressure field. Therefore, it is highly important to verify that
 641 the time average of a statistically, stationary stochastic process (such as a tornado) converges
 642 against the mean value of all possible realisations within the chosen measurement duration. In
 643 order to address the statistical uncertainty for this study, convergence tests were undertaken. For
 644 the convergence tests, time series were measured for a duration of 600 seconds. Running averages
 645 (RA) with increasing sampling durations (from 10 seconds – 600 seconds) were calculated and
 646 shown as envelope on the y-axis for the corresponding averaging time. For example, figures 12a
 647 and 12b show the convergence tests of the circumferential velocity component and surface
 648 pressure at the core radius of the lowest measurement height and at the centre of the simulator,
 649 respectively for $S = 0.69$.



650
 651 Figure 12: Convergence test of circumferential velocity (a) and surface pressure (b) for $S = 0.69$.
 652

653 Figure 12 indicates that the uncertainty is reduced after a measurement duration of 80 seconds for
 654 velocities and 60 seconds for surface pressures. Hence, for this study, velocity and pressure
 655 measurements were conducted for a period of 80 and 60 seconds, respectively. The corresponding
 656 statistical uncertainties are shown in percentage of the time-average of the corresponding
 657 circumferential velocity component ($u_{\theta,mean,600s}$) and surface pressure ($p_{mean,600s}$) which was
 658 obtained after sampling for 600 seconds (Table 2).

659

660 Table 2: Statistical uncertainties of time-averaged velocity components and surface pressure.

	$S = 0.14$	$S = 0.30$	$S = 0.69$
	<i>Velocity</i>		
$u_{\theta}(r=R)_{mean,600s} [m/s]$	5.31	9.39	10.51
$u_{\theta}(r=R)_{uncertainty} [\%]$	± 1.94	± 1.02	± 0.54
$u_r(r=R)_{uncertainty} [\%]$	± 5.96	± 3.17	± 0.52
$w(r=R)_{uncertainty} [\%]$	± 1.21	± 0.55	± 0.25
	<i>Pressure</i>		
$p(r=0)_{mean,600s} [Nm^{-2}]$	-136.30	-224.22	-164.44
$p(r=0)_{uncertainty} [\%]$	± 26.75	± 5.94	± 1.16

661

662 The 80-second time average of circumferential and vertical velocity components can be
663 determined with an uncertainty of less than $\pm 2\%$ for all swirl ratios. Uncertainties of the radial
664 velocity components are slightly larger with about $\pm 6\%$, $\pm 3\%$ and $\pm 0.5\%$ for $S = 0.14$, $S = 0.30$
665 and $S = 0.69$, respectively.

666

667 For the surface pressure, 60 seconds of measurement time allow to determine the time-average to
668 about $\pm 27\%$, $\pm 6\%$ and $\pm 1\%$ for $S = 0.14$, $S = 0.30$ and $S = 0.69$, respectively. The uncertainty of
669 surface pressure measurements at the centre of the simulator seems to be dependent on the swirl
670 ratio, with lower uncertainties with increasing swirl ratio. This could be an indication for a more
671 unstable vortex movement for the lowest swirl ratio compared to larger S . This behaviour cannot
672 be observed in the uncertainties associated with the velocity measurements. A potential reason for
673 this could be that the vortex movement occurs relatively close to the vortex centre where surface
674 pressure gradients are relatively large, especially for the lowest swirl ratio, and for that reason,
675 core radii positions are not that strongly affected by this behaviour.

676

677 3.2.2 Repeatability

678

679 The repeatability is the degree to which repeated measurements under unchanged boundary
680 conditions show the same results. Surface pressure measurements and the radial profile of
681 velocities in the lowest measurement height were repeated for five times for each swirl ratio. The
682 measurement repeatability is analysed in form of a distribution of all possible differences of these
683 repetitions. For example, figures 13a and 13b show the measurement repeatability distribution of
684 circumferential velocity component and surface pressure for $S = 0.69$.

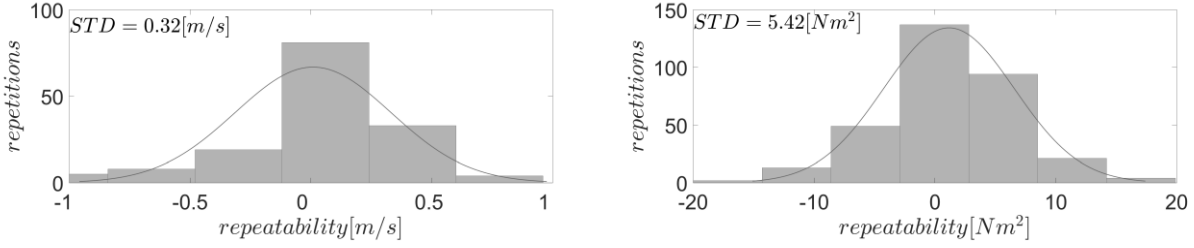


Figure 13: Distribution of the repeatability of circumferential velocity component (a) and surface pressure (b) for $S = 0.69$.

The standard deviation (STD) of the corresponding distributions was chosen as a representative measure to evaluate the repeatability, which is shown in table 3 for velocity components and surface pressures. Absolute values are given instead of percentages as these uncertainties are assumed to apply for all corresponding time averaged results. Similar to the statistical uncertainty, also table 3 shows that the repeatability of surface pressure measurements is dependent on the swirl ratio. For $S = 0.14$ and $S = 0.30$ the repeatability is not only dependent on the swirl ratio but also on the radial distance. For that reason, a repeatability dependent on r is introduced because a uniform value would highly underestimate the repeatability of measurement positions close to the vortex centre, and highly overestimate the repeatability for positions further away from the vortex centre (Table 3). No significant dependence on the radial distance was found for the repeatability of surface pressure measurements for $S = 0.69$ and all velocity measurements. Consequently, a uniform measurement repeatability independent from r is used for those cases (Table 3).

Table 3: Repeatability of time-averaged velocity components and surface pressures.

	$S=0.14$	$S=0.30$	$S=0.69$
	<i>Velocity</i>		
$u_{\theta,uncertainty} [m/s]$	± 0.51	± 0.44	± 0.32
$u_r,uncertainty [m/s]$	± 0.46	± 0.58	± 0.22
$W_{uncertainty} [m/s]$	± 0.17	± 0.16	± 0.08
	<i>P_{uncertainty} [Nm⁻²]</i>		
$-0.05m < r < 0.05m$	± 53.58	± 21.74	± 5.42
$r = \pm 0.1m$	± 12.98	± 17.34	± 5.42
$r > 0.1m \ \& \ r < -0.1m$	± 1.86	± 3.82	± 5.42

The low repeatability close to the vortex centre for $S = 0.14$ and $S = 0.30$ is not too surprising as the statistical uncertainty found at those positions (Table 2) is limiting the repeatability. Therefore, large uncertainties for $S = 0.14$ and $S = 0.30$ close to the vortex core can partly be explained by the large statistical uncertainty at these positions. Furthermore, these findings suggest that especially

708 positions close to the centre are sensitive to small variations in the boundary conditions such as
709 guide vane angle positioning.

710

711 3.2.3 Device uncertainty

712

713 In this study, pressure transducers (HCLA12X5DB) with a typical uncertainty of $\pm 5 \text{ Nm}^{-2}$ are
714 used. The Cobra Probe is accurate within $\pm 0.5 \text{ m/s}$ for the velocity vector up to a turbulence
715 intensity of $\sim 30\%$. Therefore, positions with a turbulence intensity greater than 30% are excluded
716 from the analysis. Since the device uncertainty for the Cobra Probe is given for the velocity vector
717 (\vec{V}); the uncertainty of each velocity component needs to be calculated for each measurement
718 position based on the obtained average of the corresponding velocity component. Equation 24
719 shows as an example the calculation of the device uncertainty for the circumferential velocity
720 component at a certain position.

721

$$722 \quad u_{\theta, \text{uncertainty}} = 0.5 \left\langle \frac{|u_{\theta}|}{\bar{v}} \right\rangle \quad (24)$$

723

724 3.2.4 Experimental uncertainty

725

726 The associated experimental uncertainty of velocity and surface pressure measurements, which is
727 shown as envelope in later figures is based on a combination of the repetition uncertainty (shown
728 in table 3) and the device uncertainty.

729

730

731 4. Comparison

732

733 In this section, flow field and surface pressure data for three different swirl ratios ($S = 0.14$, $S =$
734 0.30 , and $S = 0.69$) are analysed and the capability of the vortex models to replicate the
735 experimental results is evaluated.

736

737 4.1. The flow structure

738

739 In order to address the complex nature of the analysed flow fields, the 3-D velocity fields obtained
740 for $S = 0.14$, $S = 0.30$ and $S = 0.69$ are shown in figure 14. Also shown are results of selected
741 analytical vortex models, which for the sake of brevity are not repeated for all swirl ratios. The
742 ‘viscosity’ parameter for the Burgers-Rott and Sullivan model is assumed to be $\nu = 0.05 \text{ m}^2 \text{ s}^{-1}$.
743 However, for the analysis presented in figure 14b, the actual ‘viscosity’ value is not that crucial
744 because the focus of this analysis lies rather on the flow structure than on the actual velocity
745 magnitude, which is affected by the ‘viscosity’.

746

747 Velocity components are normalised by the maximum circumferential velocity ($u_{\theta,max}$) of the
 748 corresponding swirl ratio. Radial/vertical distances are normalised by the corresponding core
 749 radius (R). All of the simplified models presented in section 2 assume a height independent core
 750 radius and all vortex models besides the Baker vortex model assume a height independent $u_{\theta,max}$.
 751 For this reason, velocity components are normalised by the height average of the maximum
 752 circumferential velocity ($u_{\theta,max,average}$) of the corresponding swirl ratio (Eq. 25) and
 753 radial/vertical distances are normalised by the corresponding height averaged core radius ($R_{average}$,
 754 Eq. 26). The use of averaged quantities is considered to give the best indication of a representative
 755 core radius and maximum circumferential velocity for all heights.

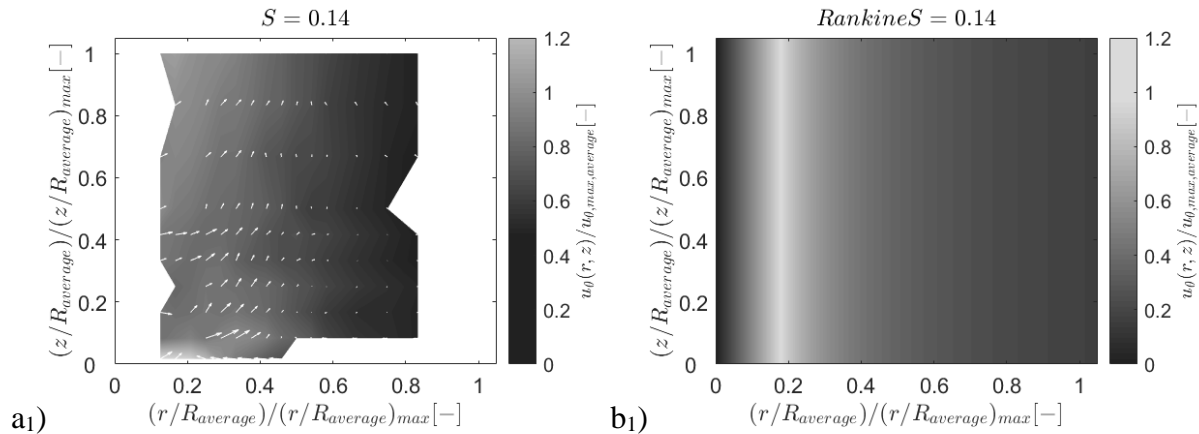
$$756 \quad u_{\theta,max,average} = \frac{1}{N} \sum_{i=1}^N u_{\theta,max}(z)_i \quad (25)$$

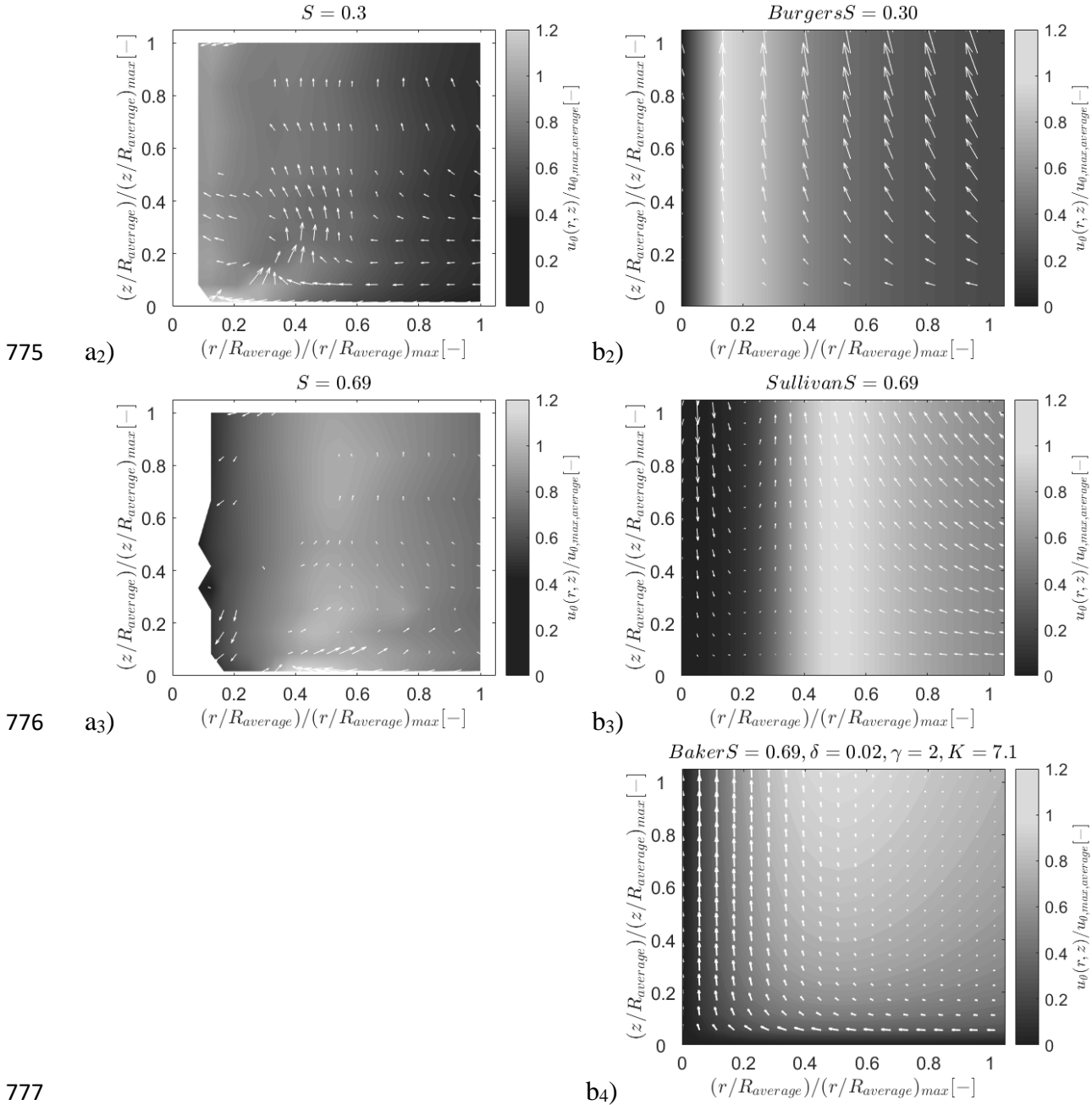
$$757 \quad R_{average} = \frac{1}{N} \sum_{i=1}^N R(z)_i \quad (26)$$

760 where N is the number of measurement heights.

761
 762 A further degree of normalisation is undertaken to aid visual comparisons, i.e., each height (radial
 763 distance) is normalised by the corresponding maximum of $z/R_{average}$ ($r/R_{average}$) for each swirl ratio
 764 – the actual maxima used are given in the figure captions. Experimentally obtained $R_{average}$ and
 765 $u_{\theta,max,average}$ (shown in figure 15) are used to calculate the flow field of the Rankine, Burgers-
 766 Rott, Sullivan and Baker vortex models for corresponding swirl ratios. Due to the restrictions when
 767 measuring with the Cobra probe, the velocity field close to the vortex centre for all swirl ratios and
 768 near the surface at larger radial distances for $S = 0.14$ could not be captured. Inside the vortex core
 769 ($r < R_{average}$), velocity vectors are only shown at positions where absolute values of time averaged
 770 radial and vertical velocities are larger than the corresponding measurement uncertainty. Hence,
 771 only positions for which a clear directionality can be defined are presented in figure 14.

772
 773





775

a₂)

b₂)

776

a₃)

b₃)

777

b₄)

Figure 14: Experimental results of the 3-D velocity field are shown in a₁ ($S = 0.14$), a₂ ($S = 0.30$) and a₃ ($S = 0.69$). Corresponding results of Rankine, Burgers-Rott, Sullivan and Baker vortex models are shown in b₁, b₂, b₃ and b₄, respectively. The normalised circumferential velocity component is shown as contour and radial and vertical velocity components are shown as 2-D vector field. To normalise heights and distances, maximum values of experimentally obtained $(z/R_{average})_{max}$ and $(r/R_{average})_{max}$ are used. For $S = 0.14$, $S = 0.30$ and $S = 0.69$ $(r/R_{average})_{max} = (z/R_{average})_{max} = 5.58, 7.27$ and 1.97 , respectively.

784

In general, experimentally obtained flow characteristics reveal much more complex flow patterns compared to the vortex models (Figure 14). The measured circumferential velocity component for all swirl ratios increases towards the core radius and reaches the overall maximum close to the surface. Furthermore, a strong decrease in circumferential velocity can be observed with height in

789 the lower heights for all swirl ratios (Figure 14a). The circumferential velocity components of
790 Rankine, Burgers-Rott and Sullivan vortex models (Figure 14b₁, Figure 14b₂, and Figure 14b₃) are
791 not a function of height and consequently, cannot represent the height dependence found in the
792 simulator (Figure 14a). The circumferential velocity of the Baker vortex model is height dependent
793 and increases with increasing height from the ground (Figure 14b₄), which is also not obtained
794 experimentally. It can be noticed in figure 14a that for all swirl ratios, a strong inflow can be
795 detected close to the surface, up to the position where the overall maximum circumferential
796 velocity occurs. At this position, the radial velocity drops drastically and the vertical velocity
797 increases significantly.

798

799 Contrary to what may be expected, Figure 14a₁, shows a radial outflow from the vortex centre.
800 However, this is supported by the work of Mishra et al., (2008) and Haan et al., (2008), where
801 tentative evidence of a radial outflow close to the vortex centre can be inferred. Various possible
802 reasons for this behaviour (including vortex core unsteadiness with respect to height) could be
803 inferred, however, firm conclusions as to why this may be the case cannot, at present, be drawn.

804

805 With increasing swirl ratio ($S = 0.69$), a downdraft is detected close to the centre of the simulator
806 (Figure 14a₃). This flow structure is expected for a two-celled vortex (see Sullivan vortex model,
807 Figure 14c₃). However, the downdraft is directed slightly towards the simulator's centre which
808 was also observed by Haan et al., (2008) for a high swirl ratio. The lack of detailed, fine scale,
809 experimental data at the centre of the vortex make further conclusions difficult.

810

811 Due to non-existing radial and vertical velocity components, this behaviour cannot be represented
812 with the Rankine vortex model (Figure 14b₁). Notwithstanding the more complex structure of
813 Burgers-Rott, Sullivan, and Baker vortex models, the measured flow pattern are far more complex
814 than the analytical models would suggest.

815

816 A more detailed analysis of the flow field can be found in figures 15 - 17 illustrating the
817 circumferential, radial and vertical velocity components for $z = 0.01\text{m}$, $z = 0.10\text{m}$, $z = 0.20\text{m}$ and
818 $z = 0.40\text{m}$, for $S = 0.14$ (a), $S = 0.30$ (b) and $S = 0.69$ (c).

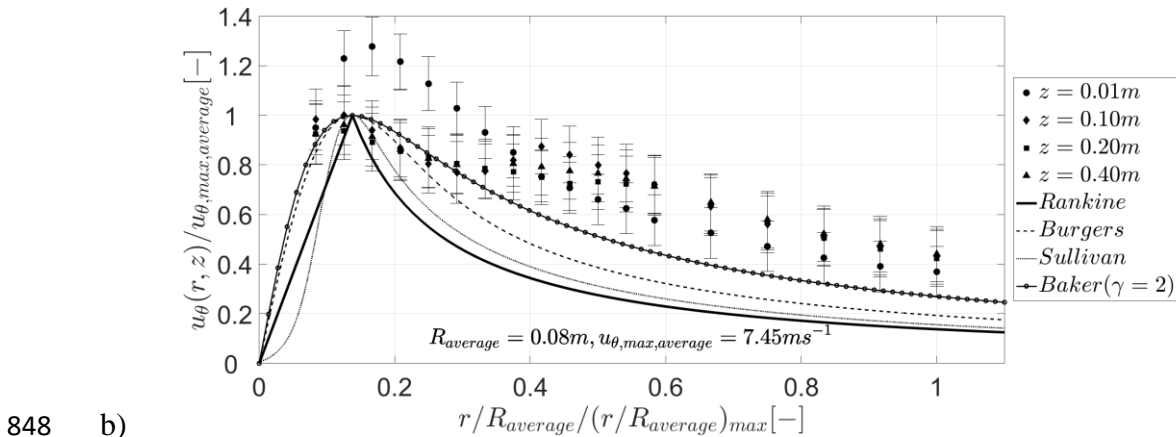
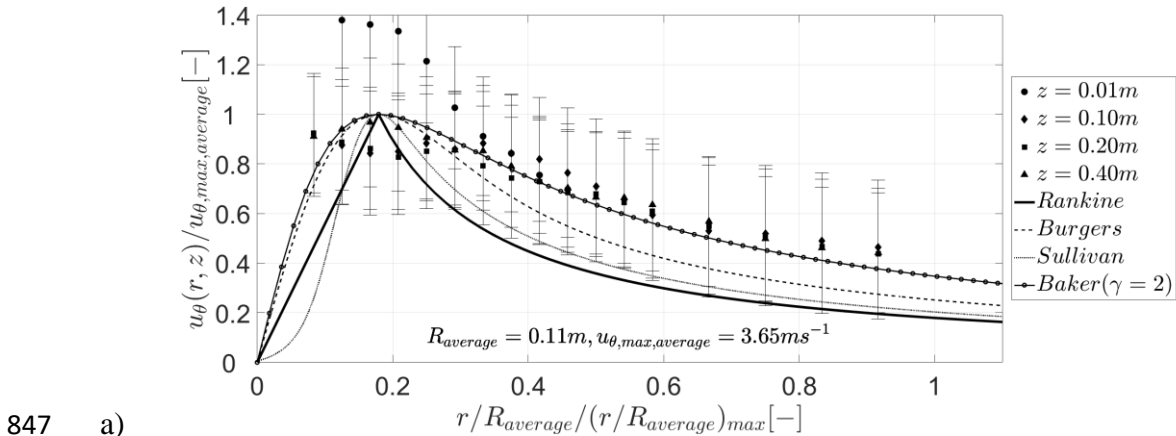
819

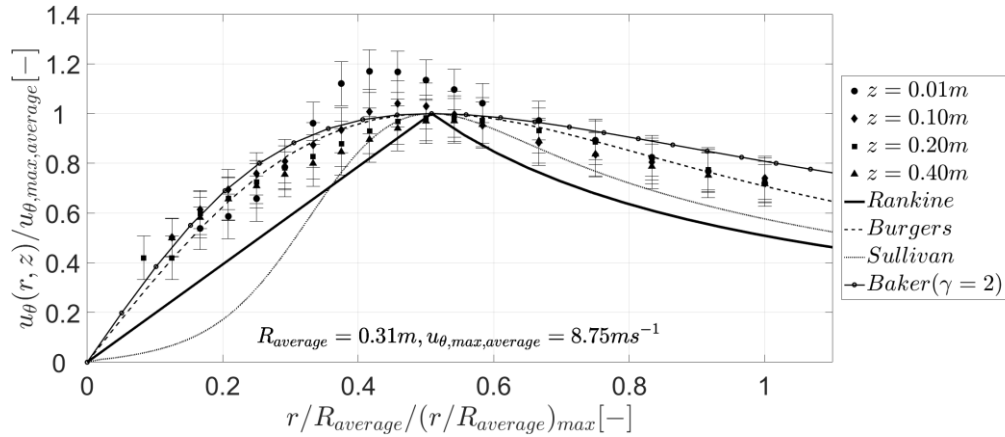
820 Experimentally obtained results for $\overline{u_\theta}$ (Figure 15) show that for all swirl ratios, the overall
821 maxima of $\overline{u_\theta}$ occurs at the lowest measurement height ($z=0.01\text{m}$, Figure 15). The distribution of
822 circumferential velocity components at greater heights is relatively uniform and differences lie
823 within the experimental uncertainty (Figure 15). The lowest height reveals an entirely different
824 flow structure compared to the rest of the generated vortex. This suggests a similar vertical profile
825 of circumferential velocity components as observed by Kosiba and Wurman (2013). Figure 15 also
826 outlines differences in the circumferential velocity profile for different swirl ratios. Figure 15
827 shows that $u_{\theta,max,average}$ increases with increasing swirl ratio from 3.65 – 8.75m/s. Also, the core
828 radius increases with increasing swirl ratio from 0.11 – 0.31m. Results obtained for $S = 0.30$ do

829 not follow this trend and reveal an averaged core radius of 0.08m, which is smaller than the one
 830 observed for $S = 0.14$. However, it is worth noting that the difference between $R_{average}$ for $S = 0.14$
 831 and $S = 0.30$ is $\sim 0.03m$. This difference is within the experimental uncertainty of determining
 832 $R_{average}$ and would therefore be masked when taking the uncertainty of determining $R_{average}$ into
 833 account, which is approximately $\pm 0.02m$.

834
 835 In general, the Rankine, Burgers-Rott and Sullivan vortex models appear to underestimate the
 836 trend of $\overline{u_\theta}$ for $S = 0.14$ (Figure 15a), although the differences are close to or within the
 837 experimental uncertainty in most cases. However, the Baker model appears to predict the trend
 838 reasonably well for this swirl ratio. All models appear to fail to capture the distribution of $\overline{u_\theta}$ at
 839 relatively large values of S (Figure 15b).

840
 841 Inside the vortex core, a comparison between experimentally obtained results and vortex models
 842 is difficult due to the lack of good experimental data for $S = 0.14$ and $S = 0.30$ (Figure 15a and
 843 Figure 15b). The circumferential velocity component of the Burgers-Rott and Baker vortex model
 844 match the experimental data obtained for $S = 0.69$ (Figure 15c). Results from the Rankine and
 845 Sullivan vortex model, again underestimate the magnitude of obtained circumferential velocities
 846 for the highest swirl ratio.





849 c)

850 Figure 15: Measured circumferential velocity component for $S = 0.14$ (a), $S = 0.30$ (b) and $S = 0.69$ (c).

851 Additionally, results of the circumferential velocity component of Rankine, Burgers-Rott, Sullivan, and Baker
 852 vortex models are shown. To normalise the radial distance, maximum values of experimentally obtained $(r/R_{\text{average}})_{\max}$
 853 $_{\max}$ are used. For $S = 0.14$, $S = 0.30$ and $S = 0.69$ $(r/R_{\text{average}})_{\max} = (z/R_{\text{average}})_{\max} = 5.58, 7.27$ and 1.97 , respectively.

854

855 Figure 16 illustrates the radial distribution of $\overline{u_r}$ for $z = 0.01\text{m}$, $z = 0.10\text{m}$, $z = 0.20\text{m}$ and $z = 0.40\text{m}$.
 856 Additionally a height average is calculated and shown for all velocities obtained for $z > 0.01\text{m}$. It
 857 needs to be mentioned here that the swirl ratio (S) defined in equation 23 is, unlike the definition
 858 adopted in the Baker vortex model (S_{Baker}), solely a function of the tornado generator
 859 characteristics. For that reason, values of S and S_{Baker} differ for the same flow field in the following
 860 figures. As illustrated in figure 16a and 16b, the Baker model fits the data obtained reasonably
 861 well close to the surface, whereas the Sullivan and Burgers-Rott vortex model are a better fit for
 862 experimental data obtained in greater heights. This is perhaps not too surprising given the
 863 assumptions embedded in the models. However, for the largest swirl ratio (Figure 16c), the Baker
 864 model only represents the trend in the lowest height for normalised radial distances greater than
 865 0.6, whereas the Sullivan model performs better for the lowest height and for lower normalised
 866 radial distances.

867

868 For $S = 0.14$, the radial outflow inside the vortex core suggests the structure of a (limited height)
 869 two-celled vortex. Thus, in general, the height averaged structure of $\overline{u_r}$ appears to be reasonably
 870 represented by the Sullivan vortex model (Figure 16a). Even though one feature of a two-celled
 871 vortex is present for $S = 0.14$, the vertical downdraft suggested by the Sullivan vortex model at
 872 the vortex centre was not capture (Figure 17a). For the vertical velocity component obtained with
 873 $S = 0.14$, none of the vortex models is capable of replicating the maximum updraft just outside the
 874 vortex core (Figure 17a). However, for larger radial distances, results of Burgers-Rott and Sullivan
 875 vortex model can be used to reproduce height averaged vertical velocities (Figure 17a). Certainly,
 876 it needs to be mentioned here that for this case, the vortex models fail to replicate the complex
 877 behaviour observed experimentally. Although there are only few positions available for a
 878 comparison in the lowest height, radial and vertical velocity component of the Baker vortex model
 879 are in good agreement with the experimental data (Figure 16a and Figure 17a).

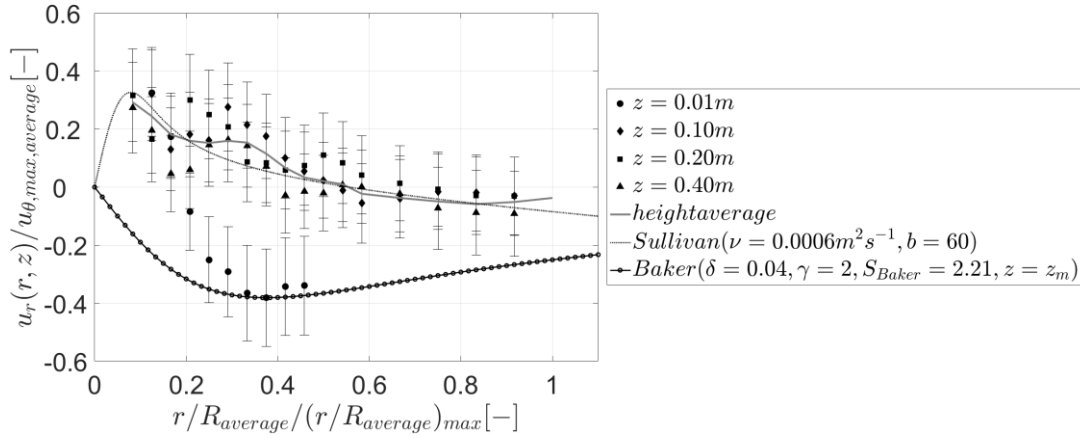
880
881 For $S = 0.30$, radial velocities are mainly directed towards the vortex centre (radial inflow). The
882 magnitude of radial velocities in greater heights $z > 0.01\text{m}$ is relatively low compared to radial
883 velocities obtained in the lowest height ($z=0.01\text{m}$). The best fit for this ‘single-celled’ flow
884 behaviour is achieved with the Burgers-Rott vortex model with a relatively low ‘viscosity’
885 parameter ($\nu=0.0015\text{m}^2\text{s}^{-1}$) to minimise the increase of radial inflow from the vortex centre to
886 larger radial distances. Similar to the vertical velocity component found for $S = 0.14$, also here the
887 maximum updraft outside the vortex core (Figure 17b) cannot be replicated by any of the
888 introduced vortex models. However, for larger radial distances the Burgers-Rott vortex model can
889 be used to model the height averaged behaviour (Figure 17b). Nevertheless, the complex vertical
890 flow structure cannot be captured. Radial and vertical velocity component of the Baker vortex
891 model on the other hand are in good agreement with the experimental data for $z=0.01\text{m}$ (Figure
892 16b and Figure 17b).

893
894 For $S = 0.69$, radial velocities (Figure 16c) are found to be directed inwards, towards the vortex
895 centre (radial inflow), even though a downdraft close to the vortex centre is found (Figure 17c). In
896 terms of vortex models, this means that the radial velocity shows a ‘single-celled’ behaviour with
897 radial inflow (Burgers-Rott and Baker), whereas the vertical velocity illustrates a ‘two-celled’
898 structure with downdraft (Sullivan). None of the presented vortex models is capable of
899 representing both of the observed flow patterns. For $S = 0.69$, the flow field obtained is far more
900 complex than the assumed flow structure of a single or a double-celled vortex. In this case, the
901 flow reveals a ‘multi-celled’ structure with a weak radial inflow and updraft outside the core vortex
902 and relatively strong radial inflow and downdraft inside the core vortex (Figure 16c and Figure
903 17c).

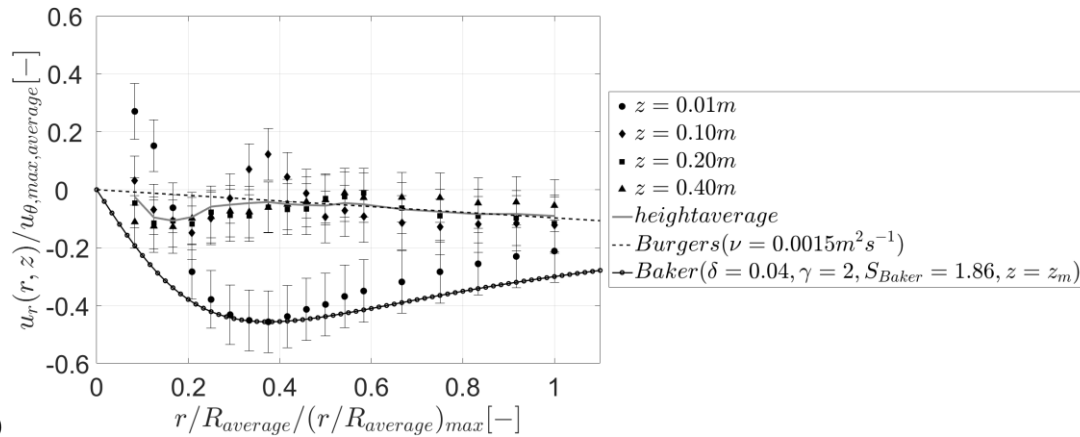
904
905 For radial and vertical velocity components of $S = 0.69$, an attempt is shown to use Sullivan and
906 Baker vortex model to replicate some of the flow patterns observed in the lowest height ($z =$
907 0.01m). For the radial velocity component in the lowest height (Figure 16c), the Baker vortex
908 model captures the radial inflow outside the vortex core but overestimates velocities inside the
909 vortex core. The decrease in radial inflow around the core radius can be replicated with the Sullivan
910 vortex model, however, close to the vortex centre, the Sullivan vortex model is not able to replicate
911 the flow field and also for larger radial distances the Sullivan vortex model fails due to its
912 increasing radial velocity component with increasing radial distance (Figure 16c).

913
914 For the vertical velocity (Figure 17c), the Sullivan vortex model can represent parts of the observed
915 results. By means of the shape parameter, b , the downdraft region around the vortex centre of the
916 Sullivan vortex model can be extended in the radial direction so that for $b = 12$, results of the
917 Sullivan model show some similarity with the increase in radial inflow from the vortex core
918 towards the vortex centre (Figure 17c). However, increasing, b , also results in an increasing
919 downdraft at the core radius, which for $S = 0.69$ is highly overestimated. The Baker vortex model

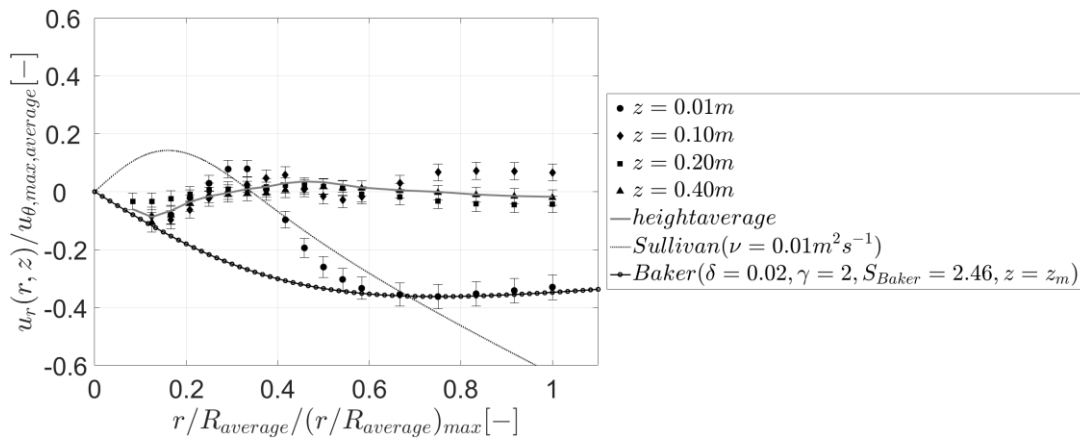
920 is not able to replicate the downdraft close to the vortex centre and the relatively strong vertical
 921 updraft around the core radius.



922 a)

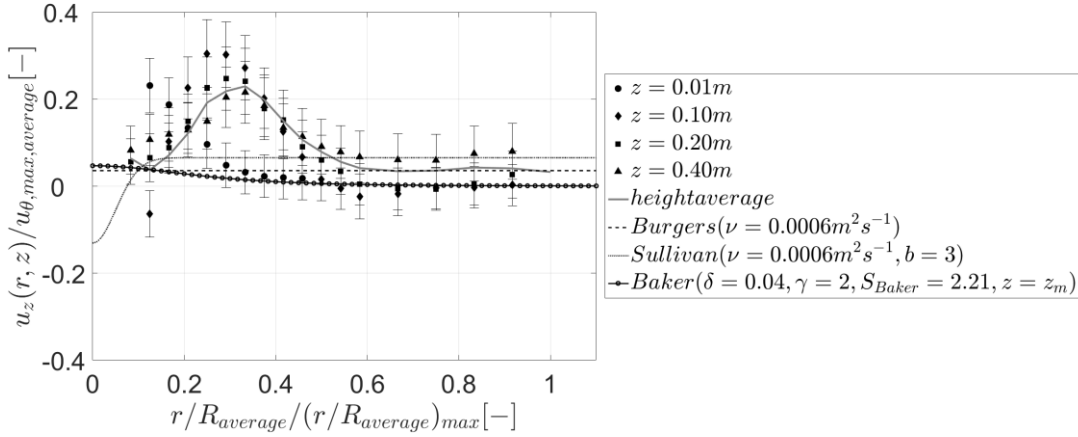


923 b)

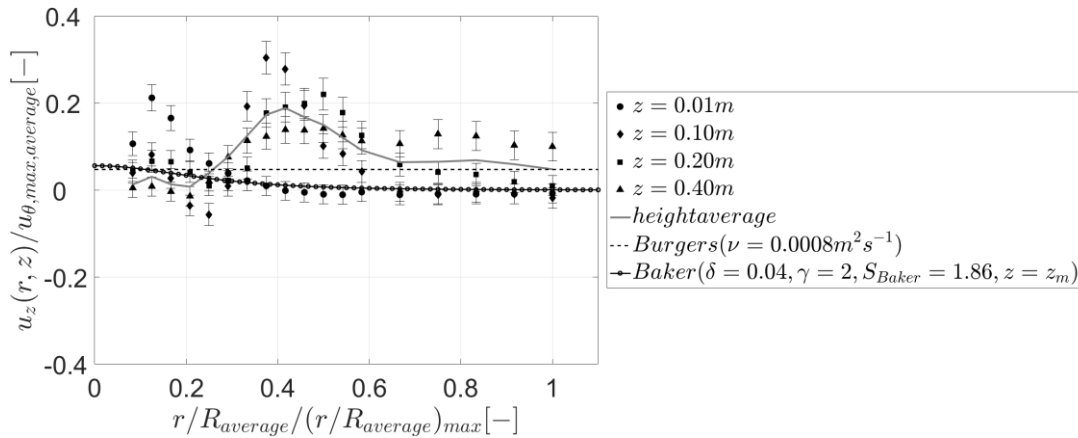


924 c)

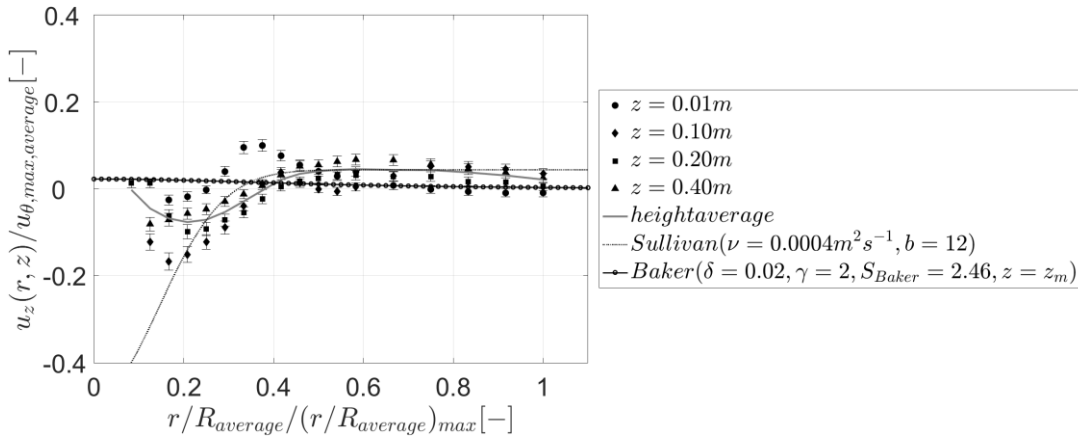
925 Figure 16: Measured radial velocity component for $S = 0.14$ (a), $S = 0.30$ (b) and $S = 0.69$ (c). Additionally, results
 926 of the radial velocity component of Burgers-Rott (b) and Sullivan vortex model (a and c) are shown. Results of the
 927 Baker vortex model are shown for the lowest height ($z = 0.01$). To normalise the radial distance, maximum values of
 928 experimentally obtained $(r/R_{average})_{max}$ are used. For $S = 0.14$, $S = 0.30$ and $S = 0.69$ $(r/R_{average})_{max} = (z/R_{average})_{max} =$
 929 $5.58, 7.27$ and 1.97 , respectively.



930 a)



931 b)



932 c)

933 Figure 17: Measured vertical velocity component for $S = 0.14$ (a), $S = 0.30$ (b) and $S = 0.69$ (c). Additionally, results
 934 of the vertical velocity component of Burgers-Rott (a and b) and Sullivan vortex model (a and b) are shown. Results
 935 of the Baker vortex model are shown for the lowest height ($z = 0.01$). To normalise the radial distance, maximum
 936 values of experimentally obtained $(r/R_{average})_{max}$ are used. For $S = 0.14$, $S = 0.30$ and $S = 0.69$ $(r/R_{average})_{max} =$
 937 $(z/R_{average})_{max} = 5.58, 7.27$ and 1.97 , respectively.

938

939 4.2 Surface pressure distribution

940

941 Figure 18 and figure 19 show the surface pressure distribution for $S = 0.14$, $S = 0.30$, and $S = 0.69$.
942 Additionally, the surface pressure distributions of the Rankine, Burgers-Rott and Sullivan vortex
943 models are illustrated in figure 18 and results of the Baker vortex model are shown in figure 19.
944

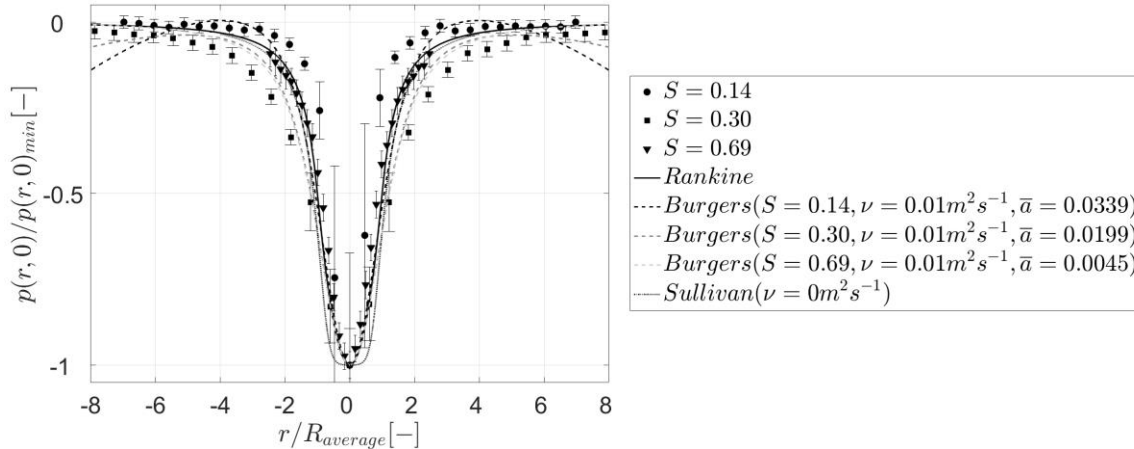
945 Looking at figures 18 and 19, it could be concluded that differences in the measured surface
946 pressure distribution depend significantly on the swirl ratio. As expected, the smallest vortex
947 shows the fastest increase in surface pressure from the vortex centre towards larger radial distances
948 ($S = 0.14$). The slope of the surface pressure change is determined by the measured surface
949 pressure distribution but also affected by the normalisation of radial distances. This means, for the
950 same surface pressure distribution, a larger core radius leads to a surface pressure distribution,
951 which is increasing faster from the vortex centre towards larger radial distances. For that reason,
952 the surface pressure distribution of $S = 0.30$ and $S = 0.60$ reveal an unexpected behaviour. The
953 surface pressure distribution obtained with $S = 0.69$ displays a faster increase of surface pressure
954 with increasing radial distance compared to the pressure distribution measured with $S = 0.30$. This
955 can be explained by the relatively small averaged core radius for $S = 0.30$ ($R_{average, (S=0.30)} = 0.08\text{m}$)
956 compared to $R_{average}$ of $S = 0.14$ ($R_{average, (S=0.14)} = 0.11\text{m}$) and $S = 0.69$ ($R_{average, (S=0.69)} = 0.31\text{m}$).
957

958 To avoid an unphysical decrease in surface pressure with increasing radial distance, the ‘viscosity’
959 parameter of the Sullivan vortex model is chosen to be zero and for the Burgers-Rott vortex model,
960 v and \bar{a} are adjusted for each swirl ratio to guaranty a physically reasonable surface pressure
961 increase with increasing radial distance (Figure 18).
962

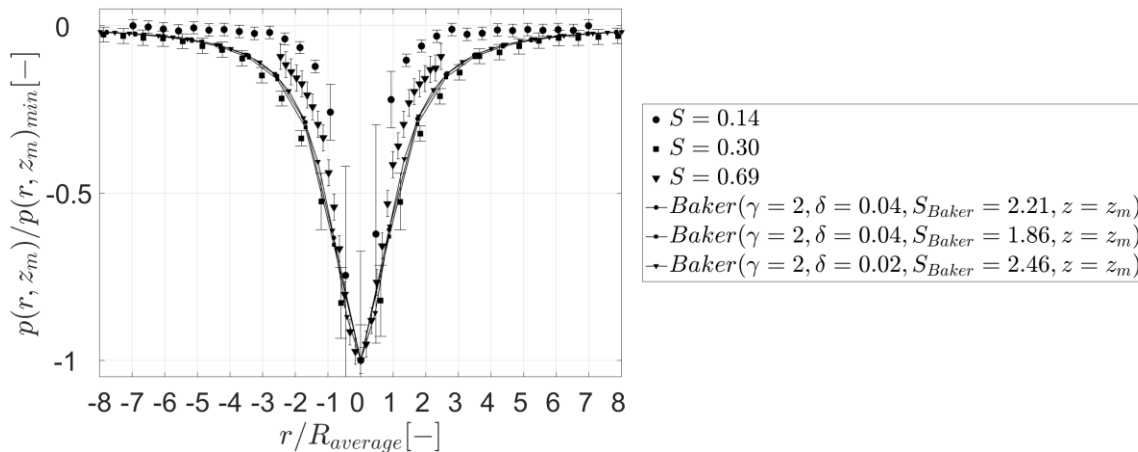
963 The largest differences in the surface pressure distribution of the vortex models is found close to
964 the vortex centre, whereas largest differences in the experimental surface pressure data for the
965 three different vortices are found outside the vortex core (Figure 18). The surface pressure
966 distributions of the Rankine and Burgers-Rott vortex models are in good agreement with the
967 experimental data for $S = 0.69$ (Figure 18), albeit the Burgers-Rott vortex model fails for larger
968 radial distances due to a physically unrealistic decrease in surface pressure, which is explained in
969 section 2.8. Also, the Sullivan vortex model shows good agreement for this swirl ratio ($S = 0.69$)
970 but only for larger radial distances. Close to the vortex centre, the Sullivan vortex model
971 underestimates experimental results due to its two-celled structure (Figure 18). Even though a
972 downdraft is present close to the vortex centre for $S = 0.69$, the effect of it is not visible in the
973 experimentally obtained surface pressure distribution.
974

975 All three solutions of the Baker vortex model match the experimental surface pressure for $S = 0.30$
976 the best (Figure 19). The swirl ratio of the Baker vortex model (S_{Baker}) is responsible for differences
977 in the surface pressure distribution. However, differences in S_{Baker} for the three analysed vortices
978 are too small to cause significant differences in the surface pressure distribution (Figure 19). To
979 allow a more flexible surface pressure model, the shape parameter, γ , in the Baker model needs to
980 be treated as a variable when deriving the static pressure distribution (Eq. 19). This would allow

981 different core radii (R_{Baker}) and also different shapes of the surface pressure distribution to be
 982 generated. However, in this work, the surface pressure equation for the Baker vortex model (Eq.
 983 19) assumes a shape parameter of $\gamma=2$. Consequently, the calculated surface pressure distributions
 984 shown in figure 19 assume a constant core radius (R_{Baker}) for all three vortices. For this reason, the
 985 surface pressure distributions of the Baker vortex model does not allow an accurate representation
 986 of the experimentally obtained surface pressure profiles.



987
 988 Figure 18: Measured surface pressure distribution for $S=0.14$ (a), $S=0.30$ (b), and $S=0.69$ (c) and surface pressure of
 989 Rankine, Burgers-Rott, and Sullivan vortex model.
 990



991
 992 Figure 19: Measured surface pressure distribution for $S=0.14$ (a), $S=0.30$ (b), and $S=0.69$ (c) and surface pressure of
 993 the Baker vortex model.
 994

996 5. Conclusion

997
 998 Based on this analysis, the following main conclusions can be drawn:
 999

- 1000 • Despite the simplicity of all the models examined, it has been shown that this area of
1001 research is highly complex, largely due to the interpretation of the different parameters
1002 involved.
- 1003 • The Burgers-Rott and Sullivan vortex models are able to replicate some parts of the flow
1004 field. However, parameters, which need to be chosen to make the model results fit the
1005 experimental data (v , \bar{a} and b) differ for surface pressure and different velocity
1006 components of the same vortex.
- 1007 • The Baker vortex model seemed to be the best model to replicate the radial inflow close to
1008 the ground. However, it fails for larger heights over the range tested.
- 1009 • Measured flow pattern are far less structured and organised than the pattern suggested by
1010 any of the vortex models. Consequently, none of the presented models can be used to
1011 represent the three dimensional vortex structures of experimentally generated tornado-like
1012 vortices.
- 1013 • Rankine, Burgers-Rott and Sullivan vortex model are able to replicate the surface pressure
1014 distribution of one of the analysed vortices ($S = 0.69$) but due to their limitations, these
1015 models are not adequate enough to replicate a variety of differently shaped pressure
1016 distributions.
- 1017 • The Baker vortex model with a shape parameter of $\gamma = 2$ allows the representation of the
1018 surface pressure distribution obtained for ($S = 0.30$). A more general expression for the
1019 surface pressure is required to represent experimental data for all three vortices.

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