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Field Test of Train Trajectory Optimisation on a Metro Line

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Abstract

Train trajectory optimisation plays a key role in improving energy saving performance and it is currently receiving increasing attention in railway research because of rising energy prices and environmental concerns. There have been many studies looking for optimal train trajectories with various different approaches. However, very few of the results have been evaluated and tested in practice.

This paper presents a field test of an optimal train trajectory on a metro line to evaluate the performance and the practicability of the trajectory with respect to operational energy computation. A train trajectory optimisation algorithm has been developed specifically for this purpose, and a field test of the obtained trajectory has been carried out on a metro line. In the field test the driver controls the train in accordance with the information given by a driving advisory system, which contains the results of the train trajectory optimisation.

The field test results show that, by implementing the optimal train trajectory, the actual energy consumption of the train can be significantly reduced, thereby improving the operational performance. Moreover, the field test results are very similar to the simulation results, proving that the developed train kinematics model is effective and accurate.

1 Introduction

Recent decades have seen the development of a significant number of metro systems worldwide, due to their convenience and efficiency in modern cities. However, metro systems use a considerable amount of energy in day-to-day operations, with the whole life cost of the energy used to operate a train potentially costing as much as the train itself. Due to increasing environmental concerns, metro operators are facing growing pressure to save energy. As a main foundation of metro operation, train trajectory plays a key role in metro energy consumption. An optimal train trajectory is able to provide a means of minimising energy consumption during train operation.

Research on the optimal railway operation performance began in the middle of 20^{th} century and since then various methods have been developed for the problem. Due to the complexity of the solution domain, metaheuristic methods such as genetic algorithms (GA) are often considered to driving speed curve optimisation. Bocharnikov introduced a method to calculate the most appropriate maximum and minimum coasting speeds to minimise train operation energy consumption using a mixed searching method including a fuzzy logic and a genetic algorithm [1, 2]. Umiliacchi introduced a combined macro and microscopic level approach in a train trajectory optimisation algorithm to consider the trade-off between train running time and energy consumption in a delay situation [3]. Chang presented a novel approach to obtain the best coasting control method using a genetic algorithm [4, 5]. Ye discussed a simulation model to calculate the optimal train speed as a function of time on a single-track railway line [6]. The authors have previously developed a multiple train simulator, and implemented one numerical algorithm and two exhaustive searching methods to optimise multiple train trajectories simultaneously. The comparison between the algorithms showed that the numerical algorithm is able to produce more accurate results, but with a higher computational time, when compared with the exhaustive methods [7]. However, metaheuristics methods use iteration methods or heuristic information to guide the search procedure converging. Therefore, in order to reduce the computational time, a number of researchers developed mathematical models and solutions to model the train network and optimise the train operation from different theoretical points of view [8, 9]. Howlett utilised a Pontryagin principle and proposed a method to analyse train operation into different sections in order to produce an optimal train trajectory in a relatively short time [10, 11]. Miyatake developed a mathematical formulation to find an energy-efficient train operation and compared three different methods to solve it [12].

All of the previous works have discussed train trajectory optimisation based on computer modelling. However, very few of them have been evaluated and tested in practice by field tests. There are significant differences between simulation and practice due to system delay, driver response delay, environmental disturbance and other uncertainties. It is therefore necessary to evaluate and test the optimal train trajectory on real trains in order to facilitate the understanding of the feasibility and robustness of the algorithm. It is also important to assess the practicability of implementing optimal train trajectories in the real world.

In this paper, a train kinematics model is introduced, followed by a description of the proposed train trajectory optimisation method. The method aims to minimise train energy consumption by calculating the most appropriate train movement mode on different route sections. This paper then presents a field test of the optimal train trajectory on a metro line.

The test aims to evaluate the developed optimal train trajectory by using a driving advisory system.

2 Model formulation

2.1 Nomenclature

Parameters	**Expansion**
A	Curve resistance constant number
a	Train resistance constant
a_{acc}	Train acceleration rate and braking rate, m/s
a_{brk}	Train braking rate, m/s
b	Train resistance constant
c	Train resistance constant
C_e	Unit energy cost per kWh, pound
D_{max}	Maximum delay time, seconds
D_{sg}	Delay time for a single train, seconds
E_{it}	Inter-station energy consumption, kWh
E_{sg}	Single train energy consumption, kWh
F	Train traction force or braking force, N
$f[v(t)]$	Train maximum tractive effort at the current vehicle speed v(t), N
$F_{br}(v)$	Train braking effort at the current vehicle speed v
F_{grad}	Force due to the gradient, N
$F_{tr}(v)$	Train traction effort at the current vehicle speed v
g	Gravitational acceleration, m/s
IT	Inter-station journey time, seconds
IT_r	Maximum variation between scheduled journey time and optimal journey time, seconds
IT_{sh}	Scheduled inter-station journey time, seconds
MC	Movement mode code for each inter-station journey (detailed in Figure 1)
M_{eff}	Train effective mass, kg
M_{ls}	Rolling stock mass, kg
M_{opt}	Train traction energy composition that needs to be optimised for a single journey
M_p	Passenger mass, kg
RAD	radius of the curve, m
R_{cu}	Train curve resistance, N

R_{mo}	Train resistance to motion, N
s	train position, m
S_{acc}	Train acceleration distance, m
S_{brk}	Train braking distance, m
S_{cur}	Train cruising distance, m
si	Number of sections
sn	Number of stations
s_t	Train position at the terminal station, m
t	Train time, seconds
T_{acc}	Train acceleration time, seconds
T_{brk}	Train braking time, seconds
T_{cur}	Train cruising time, seconds
TM	Train movement mode sequence
T_{sg}	Train journey time for a single train from Origin to Destination, seconds
T_{sh}	Scheduled single journey time, seconds
u_b	Train control signals for braking effort
u_f	Train control signals for traction effort
v	Train speed, m/s
$v_{limit}(s)$	Line speed limit at the current position s
V_{max}	Train cruising speed, m/s
x	The first sections in each inter-station journey that need to be considered in the optimisation
$x+j$	The last sections in each inter-station journey that need to be considered in the optimisation
α	Gradient angle
λ_w	Rotary allowance

2.2 Vehicle Kinematics Modelling

In this study, Lomonossoff's Equations are used in the kinematics modelling as the general equations of vehicle motion, which is based on Newton's second law of motion. The equations are as follows, and are subject to the constraints imposed on the train movement by the route and driving style [13-15].

$$\begin{cases} M_{eff}\dfrac{d^2s}{dt^2} = F(v) - R_{mo}(v) - R_{cu}(v) - F_{grad} \\ R_{mo}(v) = a + b|v| + cv^2 \\ R_{cu}(v) = \dfrac{A}{RAD}M_{eff}g \\ F_{grad} = M_{ls}gsin(\alpha) \end{cases} \tag{1}$$

The resistance to motion, the constants a, b, c being empirical and related to the track and aerodynamic resistance known as the Davis equation [16]; The curve resistance constant number, which may vary in different countries. The number is set at 600 in this study (English and Chinese standard). The effective mass (M_{eff}) can be calculated as follows.

$$M_{eff} = M_{ls}(1 + \lambda_w) + M_p \tag{2}$$

Time is a dependent variable in this vehicle kinematics model. Based on Equation (1), the state equation of the train motion can be further described as follows:

$$\begin{cases} \dot{s} = v \\ M_{eff}\dot{v} = u_f \cdot F_{tr}(v) - u_b \cdot F_{br}(v) - R_{mo}(v) - R_{cu}(v) - F_{grad}(s) \end{cases} \tag{3}$$

Some constraints are shown in following:

$$\begin{cases} v \le v_{\text{limit}}(s) \\ u_f \in [0,1] \\ u_b \in [0,1] \end{cases} \tag{4}$$

The traction or braking effort will be equal to zero when the corresponding control signal is set at 0.

The boundary condition, initial condition and final conditions are imposed as follows:

$$\begin{cases} v(0) = 0, s(0) = 0 \\ v(T) = 0, s(T) = s_t \end{cases} \tag{5}$$

Table 1. Control signals for different movement modes.

Movement Mode	u_f	u_b	Equations (6)
Motoring	1	0	$M_{eff}\,\dot{v} = F_{tr}(v) - R_{mo}(v) - R_{cu}(v) - F_{grad}(s)$
Cruising	1	0	$M_{eff}\,\dot{v} = F_{tr}(v) - R_{mo}(v) - R_{cu}(v) - F_{grad}(s) = 0$
Coasting	0	0	$M_{eff}\,\dot{v} = -R_{mo}(v) - R_{cu}(v) - F_{grad}(s)$
Braking	0	1	$M_{eff}\,\dot{v} = -F_{br}[v(t)] - R_{mo}(v) - R_{cu}(v) - F_{grad}(s)$

Four typical movement modes form a train motion are considered [17], as shown in Figure 1 and Table 1. In the motoring mode, the forward traction control signal is set at 1. Therefore the traction power is applied to achieve the required train speed. In the cruising mode, the traction power is used to overcome the resistances (motion resistance and curve resistance) and the force due to the gradient, so that the train can keep running at a constant speed. In the coasting mode, the forward traction control signal is set at 0. Therefore the traction power is switched off and the train motion is affected by the resistances and the force due to the gradient. Travelling in coasting mode as long as possible on an inter-station section is considered to be the most energy-effective method [18, 19]. In the braking mode, the forward control and backward braking control signals are set at 0 and 1 respectively. The train applies necessary braking effort to reduce the speed.

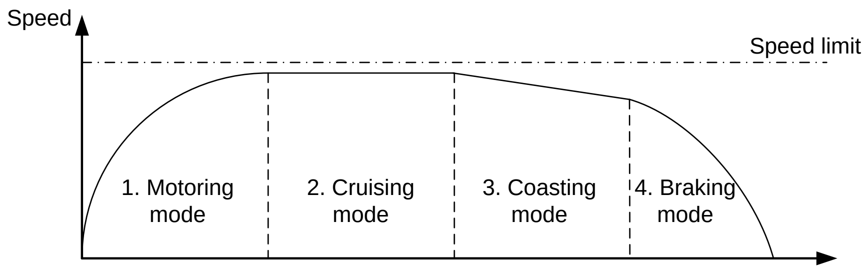

Figure 1. Four train movement modes.

3 Train Trajectory Optimization Algorithm

In this optimisation study, the route is divided into a number of sections with respect to gradient changes, line speed limit changes and section length, as shown by the vertical dot dash lines in Figure 2. Applying different movement modes (*TM*) in each section will result in different train trajectories (running profile).

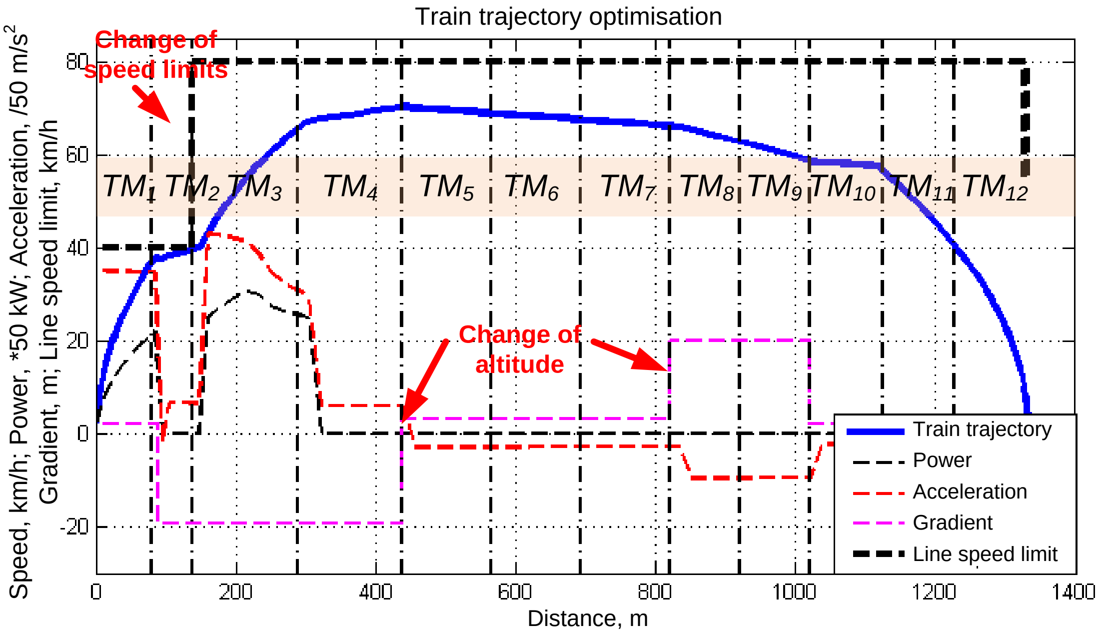

Figure 2. Train trajectory optimization for an inter-station section.

In this study, the aim of the train trajectory optimisation is to search the most appropriate train movement mode sequence (TM) to minimise train energy consumption (E_{sg}) within a given delay allowance (D_{sg}). f represents for the simulation process to calculate IT and E_{it}. The fitness function is shown in following:

$$\begin{cases} \min \quad M_{opt} = E_{sg}C_e\,, \; if \;\; D_{sg} \le D_{max} \\ \qquad [IT, E_{it}] = f(TM) \\ \qquad TM = [TM_1, TM_2, \dots, TM_{si}] \end{cases} \tag{7}$$

In order to minimise the impact of the timetable rescheduling, it is best to set D_{max} at a small number (1 second in this study). The single train energy consumption (E_{sg}), journey time (T_{sg}) and delay time (D_{sg}), which can be calculated using the following equations:

$$\begin{cases} T_{sg} = \sum_{i=1}^{sn-1} (IT_i), \quad if \; |IT_i - IT_{shi}| \in [0, IT_r] \\ E_{sg} = \sum_{i=1}^{sn-1} \int_0^{IT_i} f[v(t)]v(t)dt \\ D_{sg} = T_{sg} - T_{sh} \end{cases} \tag{8}$$

The maximum variation between scheduled journey time and optimal journey time (IT_r) is set at 5 seconds in this study.

As shown in Equation (7), each movement mode sequence is assumed to be a candidate solution. Depending on the assumed search boundary and the number of sections, the solution domain can be huge. Due to the complexity of the problem, it is important to find an appropriate algorithm to search for the optimum properly and efficiently.

As an exact algorithm, the Brute Force method is often used in computer science. It provides a more straightforward approach than metaheuristics (such as Genetic Algorithm), and, importantly, it guarantees to find the optimum solution by enumerating all possible solutions in the solution domain to prove optimality [20, 21]. However, the algorithm becomes impractical in some complex problems as the computational time increases rapidly when the complexity increases. To overcome this weakness, an enhanced Brute Force searching method has been developed in this study. The algorithm is able to address the complexity problem by constraining the solution domain [22] with the following steps:

Step 1: First, the method calculates an estimated movement mode sequence (TM_{est}) by using the simulator (g) based on the scheduled inter-station journey time (IT_{sh}).

$$[TM_{est}] = g(IT_{sh}) \tag{9}$$

$$\begin{cases} T_{acc} + T_{cur} + T_{brk} = IT_{sh} \\ S_{acc} + S_{cur} + S_{brk} = IS_{sh} \end{cases} \tag{10}$$

In this calculation, the coasting mode will not be implemented in order to simplify the process.

$$\begin{cases} T_{acc} = \dfrac{V_{max}}{a_{acc}} \\ S_{acc} = \dfrac{1}{2} a_{acc} T_{acc}^2 \\ S_{cur} = V_{max} T_{cur} \\ T_{brk} = \dfrac{V_{max}}{a_{brk}} \\ S_{brk} = V_{max} T_{brk} + \dfrac{1}{2} a_{brk} T_{brk}^2 \end{cases} \tag{11}$$

The train cruising speed (V_{max}) can be calculated using Equation (10) and Equation (11).

Step 2: The estimated movement mode sequence will be used to reduce the solution domain. The acceleration mode sections at the beginning of the journey (for example, TM_1 and TM_3 in Figure 2), and braking mode sections at the end of the journey (for example, TM_{11} and TM_{12} in Figure 2) will be retained. The algorithm will not re-calculate the movement modes for these sections in the following steps. The complexity of the Brute Force algorithm is $O(n^2)$ [23]. Therefore, reducing the number of sections (n) can significantly constrain the solution domain, thereby shortening the computational time.

Step 3: The algorithm then enumerates all possible solutions in the reduced solution domain. The following notation (journey time and energy consumption pairs) (*ALLSOL*) for each inter-station journey will be calculated using the following equations:

$$\begin{cases} ALLSOL([IT, E_{it}]pair) = \sum_{ST=1}^{sn-1} \sum_{MC_x=1}^{4} \dots \sum_{MC_{x+j}=1}^{4} f(TM), if \;\; D_{sg} \leq D_{max} \\ T_{sg} = \sum_{i=1}^{sn-1} (IT_i), \;\; if \; |IT_i - IT_{shi}| \in [0, IT_r] \\ D_{sg} = T_{sg} - T_{sh} \end{cases} \quad (12)$$

Step 4: Based on Equation (12), the solutions that do not meet the constraint conditions will be discarded. Furthermore, as shown in Figure 3, the results (*ALLSOL*) may contain solutions with the same journey time (T_{sg}) but different energy consumption (for example, a train runs at a constant median speed may achieves the same journey time as a train runs at a high speed at first and then runs at a low speed. But their energy consumptions will be different). In this study, only the solution with the lowest energy consumption will be retained as optimum for each journey time. Assume there are ζ solutions in *ALLSOL*, if:

$$E_\theta \geq E_{\theta-1} \; and \, T_{sg\theta} = T_{sg\theta-1}, \theta \in \zeta \quad (13)$$

Then the solution θ will be discarded because the solution θ-1 achieves lower energy consumption for the journey time $T_{sg\theta}$.

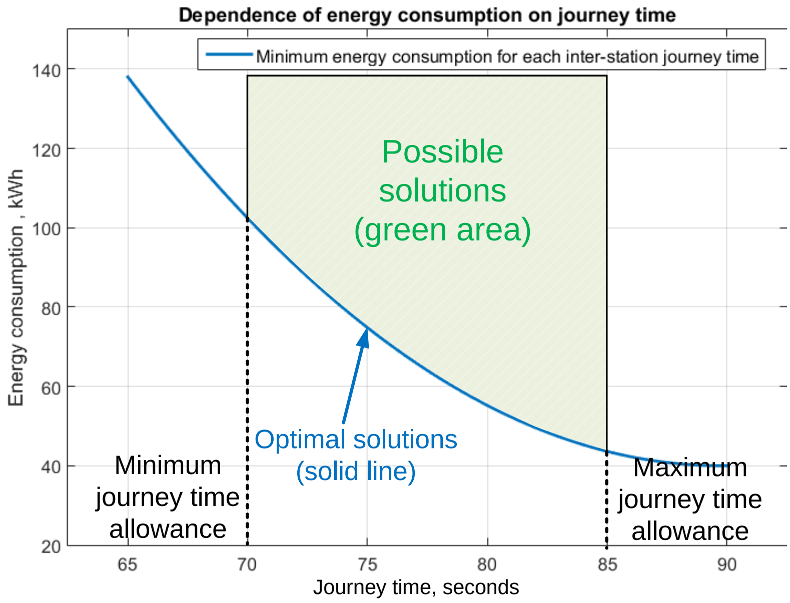

Figure 3. Dependence of energy consumption on journey time for each inter-station journey.

Step 5: After Step 4, only the optimal solutions remain and are ready to be converted into a driving advisory system for the field test.

4 Case Study

4.1 Route Introduction

In order to evaluate and identify the performance of the optimised train trajectory, a field test has been carried out on the China Beijing Yizhuang Line. It is a suburban metro line connecting Yizhuang Railway Station to Songjiazhuang Station (up direction). The line is 22.7 km long with 12 intermediate stations. The line speed limits and gradient profile are shown in Figure 4. The scheduled single journey time (one-way journey from the first station to the terminal) is 2087 seconds with 1632 seconds running time and 455 seconds dwell time for the up direction, as shown in Table 2.

Table 2. Scheduled timetable of Beijing Yizhuang Line.

	Station name	Scheduled journey time, -down direction-, seconds	Scheduled journey time, -up direction-, seconds	Distance between stations, m	Dwell time, seconds
1	Songjiazhuang				30
		193	190	2631	
2	Xiaocun				30
		104	106	1275	
3	Xiaohongmen				30
		155	156	2366	
4	Jiugong				30

		134	131	1982	
5	Yizhuangqiao				35
		85	86	993	
6	Yizhuang Park				30
		113	112	1538	
7	Wanyuanjie				30
		99	100	1280	
8	Rongjingdongjie				30
		103	103	1354	
9	Rongchangdongjie				30
		160	163	2338	
10	Tongjinanlu				30
		148	147	2265	
11	Jinghailu				30
		140	135	2086	
12	Ciqu South				35
		101	100	1286	
13	Ciqu				45
		105	103	1334	
14	Yizhaung Railway Station				40
Total		1640	1632	22728	455

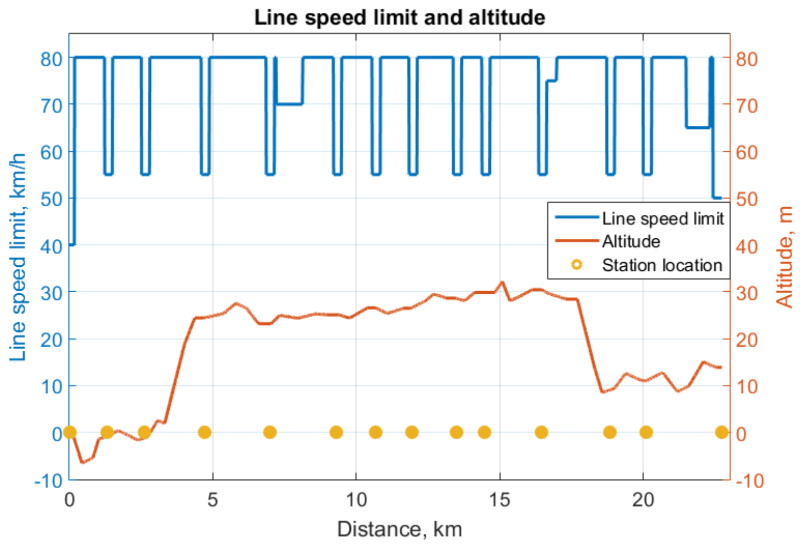

Figure 4. Beijing Yizhuang Line gradient, speed limits and station locations.

Table 3 and Figure 5 show the vehicle traction characteristics. The train uses a DC 750 V third-rail power supply and is equipped with a regenerative braking system. Each train is formed of 6 carriages and the total mass is 287 tonnes with a standard passenger load (AW2). The train can be controlled by an ATO system or by a manually driving system. The maximum service speed and average operation speed are 80 km/h and 40 km/h respectively.

Table 3. Train traction characteristics.

Parameters	Value/Equation
Overall train mass, tonnes	199 (3M3T)
Passenger mass, tonnes	88 (AW2)
Train formation	3M3T
Train length, m	138
Rotary allowance	0.08
Resistance, N/tonne	$3.48184+0.04025v+0.0006575v^2$ (V: km/h)
OHL power	DC 750V
Maximum traction power, kW	3144
Maximum braking power, kW	4237
Engine efficiency from electrical power to mechanical power	82%
Maximum operational speed, km/h	80
Tractive effort, kN	Figure 5 (maximum 289)
Braking effort, kN	238 (constant)
Train control system	Automatic Train Operation (ATO), manually

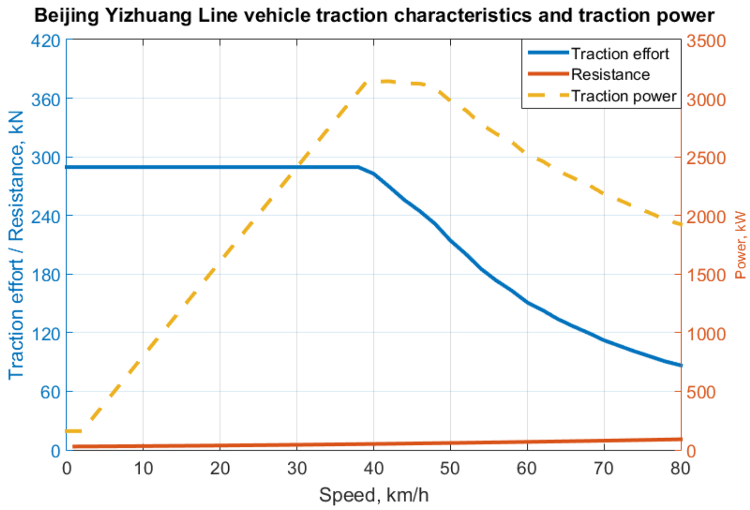

Figure 5. Train traction system characteristics.

4.2 Driver Advisory System Development

Due to the policy of the Beijing Yizhuang Metro Line operator, it is not possible to modify the existing ATO system in the field test due to safety concerns. Therefore, the field test is carried out by a human driver. A simple driver advisory system (DAS) has been developed using Microsoft PowerPoint. All of the proposed optimal train trajectories have been input into the DAS. The driver is expected to control the train in accordance with the instructions displayed by the DAS. The field test results will be compared with the existing ATO operation and existing manual driving operation.

As shown in Figure 6, the DAS contains a number of slides for each inter-station operation. Each slide shows the movement instructions for the current section (in red), and advanced instructions for the next section (in blue) with a countdown function. For example, in Figure 6, the train is running in Section 1 (S1). The DAS is instructing the driver to accelerate up to a speed of 38 km/h. Then, 15 seconds later, the driver should switch the train to the coasting mode, and a new slide will be displayed to the driver at that time.

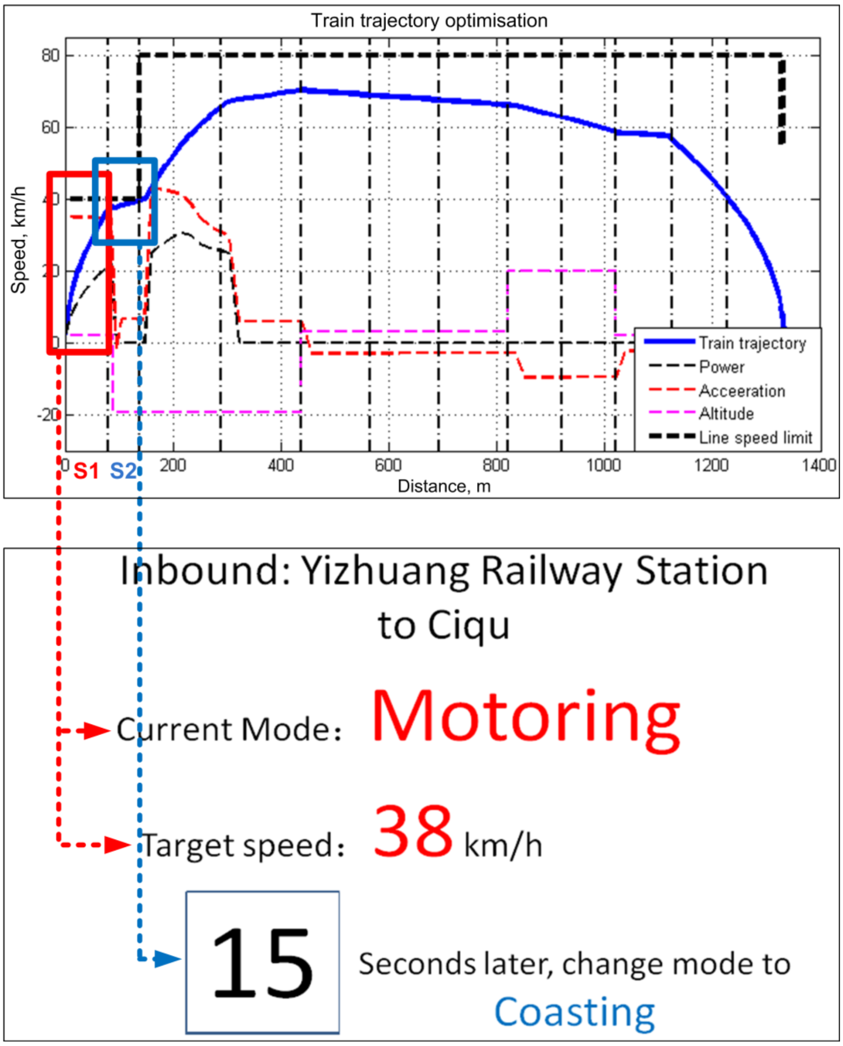

Figure 6. Converting the train trajectory into DAS instructions.

Figure 7 shows photos of the field test being carried out on the Beijing Yizhuang Line. A laptop is placed on the left-hand side of the cab desk, which displays instructions to the driver. The driver is watching the screen and controls the train in accordance with the instructions.

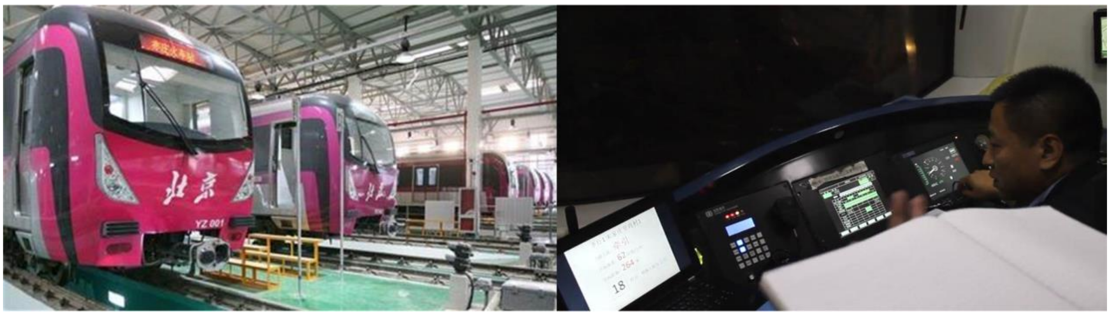

Figure 7. Field test on the Beijing Yizhuang Metro Line.

4.3 Comparison between Simulation and Practice

Figure 8 and Figure 9 show the train trajectory comparison between the existing operation (ATO), simulated optimal operation and actual optimal operation (manual driving) for the up direction and down direction, respectively. All of the actual operation data is obtained from the on-board Train Information Measurement System (TIMS). As shown in Figure 8(a) and Figure 9(a), in the existing operation, after the train reaches the maximum target speed (approximately 75 km/h), ATO tries to drive the train at a constant speed (cruising mode) until the train approaches the station stop. However, due to the limitations of the ATO speed tracking algorithm and the traction characteristic, the train movement is switched between motoring and braking modes frequently in order to maintain the given speed. The yellow lines (acceleration rate) in Figure 8(a) and Figure 9(a) are increasing and decreasing throughout the cruising period. Such a driving strategy will cause more energy to be consumed. Furthermore, the maximum target speeds for different inter-station stretches are not optimised but remain the same.

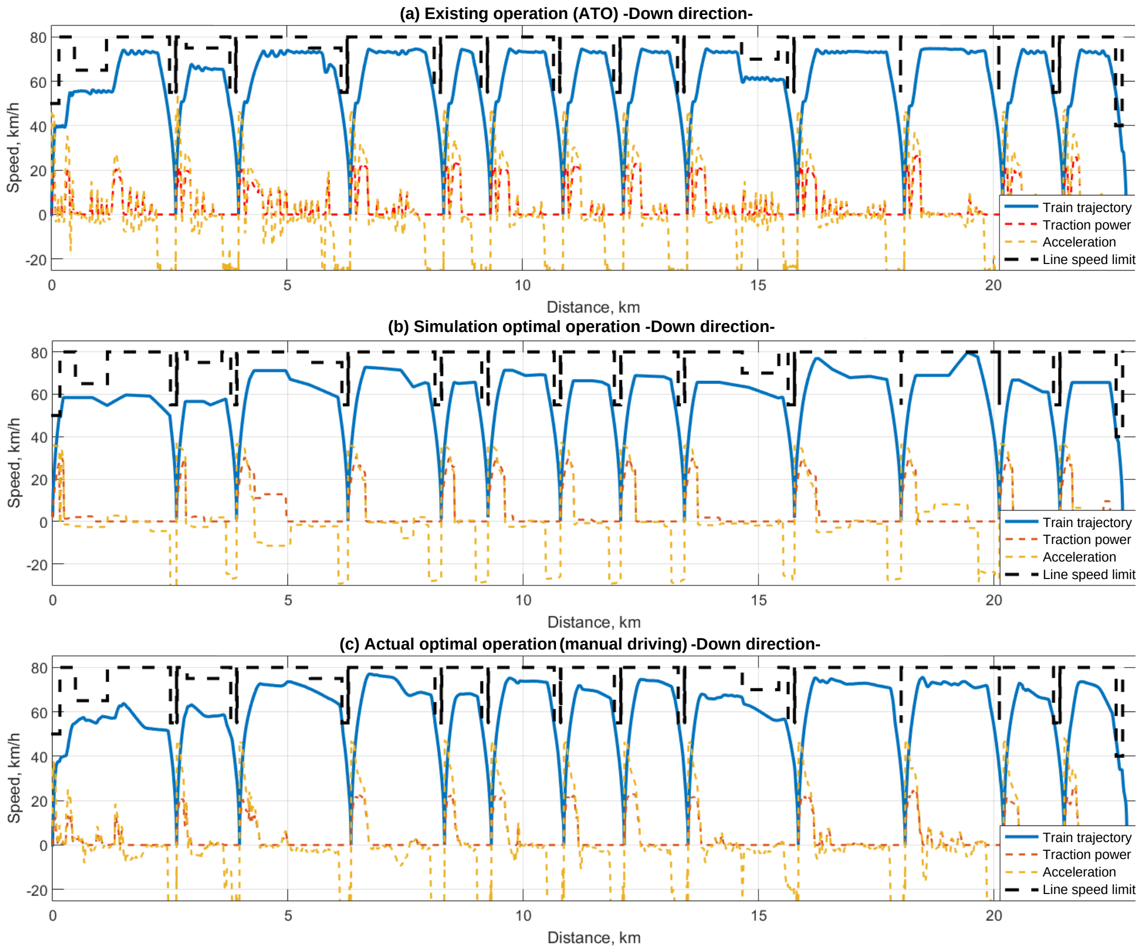

Figure 8. Comparison between existing operation (ATO) (a), simulated optimal operation (b) and actual optimal operation (manual driving) (c) -down direction-

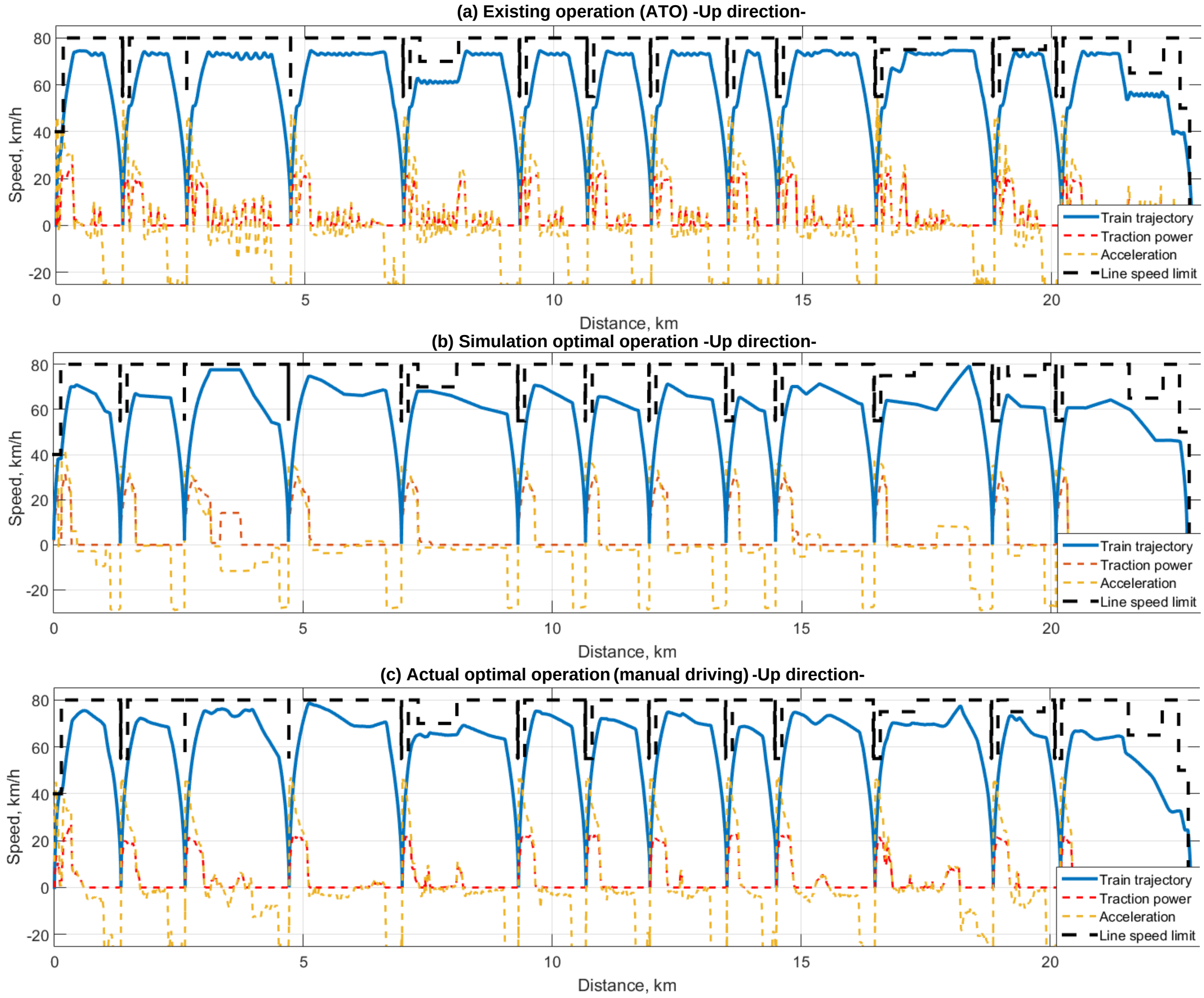

Figure 9. Comparison between existing operation (ATO) (a), simulated optimal operation (b) and actual optimal operation (manual driving) (c) -up direction-

Figure 8(b) and Figure 9(b) show the simulated optimal train trajectory obtained using the developed enhanced brute force algorithm. It can be observed that the train performs more efficiently. In order to reduce the energy consumption, the train control system coasts for as long as possible, rather than switching between motoring and braking modes frequently. Furthermore, the train takes full advantage of the gradient profile. For example, there is a steep downhill stretch from km18 to km 19 as shown in Figure 4; the train control system selects the coasting mode in these sections so that the train speed can be increased without using any traction power. Compared with the existing operation's one, the maximum train target speeds in different inter-station stretches are optimised based on the time requirements.

Figure 8(c) and Figure 9(c) show the actual optimal train trajectory from the field test. It can be seen that the actual optimal trajectory is similar to the simulated optimal trajectory (Figure 8(b) and Figure 9(b)). This shows that the developed vehicle kinematics model is accurate, and that the human driver is able to control the train following the instructions from the DAS in practice. The train acceleration rate (yellow lines) does not change frequently throughout the journey when compared with the existing operation.

4.4 Comparison between Different of Practical Operations

In the previous sections the differences between simulated optimal operation and actual optimal operation were discussed. In the following sections, different actual operations will be compared. All data (time, speed, energy usage, etc.) is obtained from train on-board Train Information Measurement System.

Figure 10 and Figure 11 show energy consumed in three different actual operations, which are: existing operation (ATO), existing operation (manual driving) and optimal operation (manual driving). It can be observed that the optimal operation (yellow line) achieves the lowest total energy usage, which is 13% and 19% lower than the existing operation (ATO) in the down-direction and up-direction respectively. In existing operations, a human being drives the train more energy efficiently than an ATO system, but worse than the optimum found in simulation.

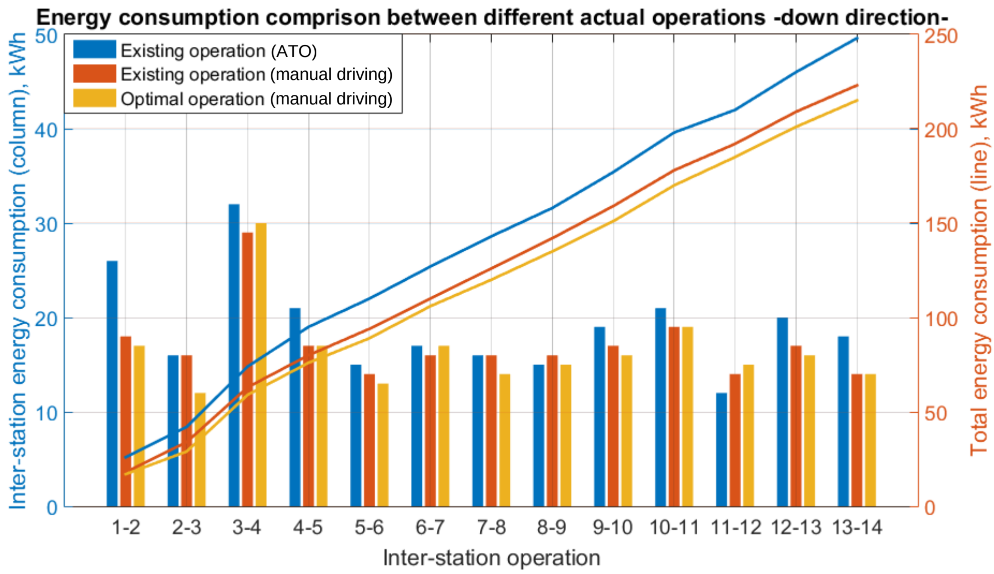

Figure 10. Energy comparison between existing operation (ATO), existing operation (manual driving) and optimal operation (manual driving) -Down direction-

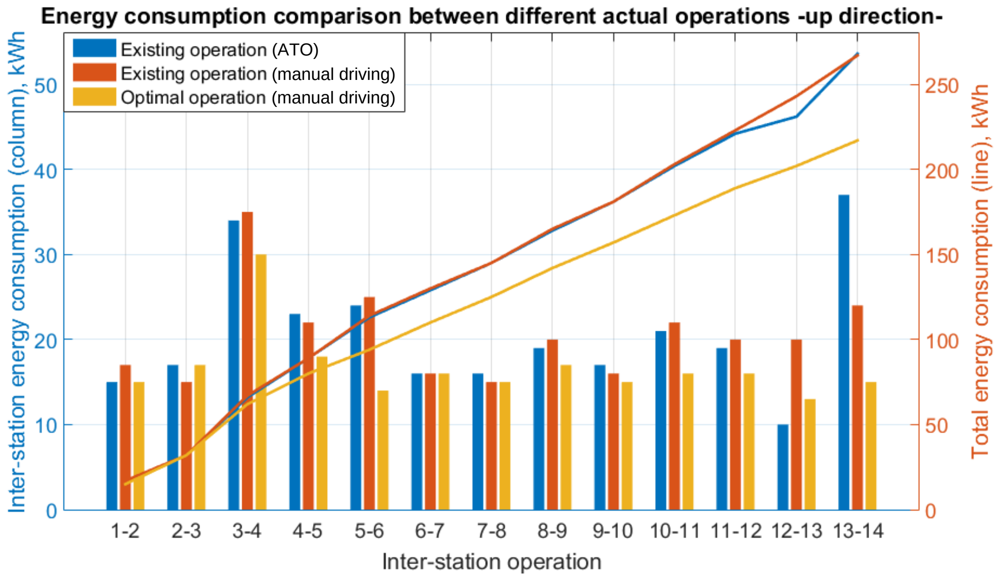

Figure 11. Energy comparison between existing operation (ATO), existing operation (manual driving) and optimal operation (manual driving) -Up direction-

Table 4 summarises a comparison of the journey time in three different actual operations. Compared with the scheduled timetable shown in Table 2, the differences in the total journey time between the scheduled operation and the optimal operation are very small (within 15 seconds). This result is in line with the policy of the metro operator, which requires that the difference should be less than 60 seconds. It can be observed that the existing operation (manual driving) and optimal operation achieves a higher energy usage when running in 12-13 but a much lower energy usage in 13-14 due to better journey time distribution application.

Table 4. The actual journey time comparison between existing operation (ATO), existing operation (manual driving) and optimal operation (manual driving).

	Station name	Actual journey time -down direction-, seconds			Actual journey time -up direction-, seconds		
		Existing operation (ATO)	Existing operation (manual driving)	Optimal operation (manual driving)	Existing operation (ATO)	Existing operation (manual driving)	Optimal operation (manual driving)
1	Songjiazhuang						
		191	202	199	196	199	200
2	Xiaocun						
		105	99	106	116	104	112
3	Xiaohongmen						
		155	152	154	158	156	159
4	Jiugong						
		131	126	135	132	135	132
5	Yizhuangqiao						
		82	80	85	82	84	81
6	Yizhuang Park						
		110	109	109	111	111	112
7	Wanyuanjie						
		96	98	100	96	99	97

8	Rongjingdongjie						
		100	99	100	101	104	100
9	Rongchangdongjie						
		163	162	163	160	163	169
10	Tongjinanlu						
		148	144	147	149	146	144
11	Jinghailu						
		136	136	140	139	143	140
12	Ciqu South						
		98	98	100	107	102	99
13	Ciqu						
		100	105	108	102	105	102
14	Yizhaung Railway Station						
Total		1615	1610	1646	1649	1651	1647

5 Conclusion

In this paper, a field test of the optimal train trajectory has been presented. The test aims to assess the practicability of the optimal train trajectory in day-to-day operation. A train kinematics model and an enhanced brute force searching method have been developed in order to obtain the optimal train trajectory. Furthermore, a driving advisory system has been produced to display the optimal train trajectory to the train driver in the field test.

The field test results show that the train trajectory of the actual optimal operation is similar to the simulated optimal operation one. *It is therefore considered* that the developed train kinematics model is accurate and meets the design requirements. The test also reveals that the human driver is able to follow the instructions from the DAS.

The energy consumption comparison between different actual operations shows that implementing an optimal trajectory could successfully reduce the energy consumption of the train by up to 51 kWh (19%) for each one-way operation. There are 242 services in each day on the Beijing Yizhuang Line. So the annual energy saving could be up to 4,504,830 kWh, that is, assuming a cost of 10 pence/kWh, £450 k per annum. Therefore, it can be concluded that implementing the optimal train trajectory is both practicable and convenient, and it could help the train operator to significantly reduce annual energy costs.

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