

Application of ISO4359 for discharge calculation in a narrow flume

Jesson, Michael; Sterling, Mark; Baker, David

DOI:

[10.1016/j.flowmeasinst.2016.06.015](https://doi.org/10.1016/j.flowmeasinst.2016.06.015)

License:

Creative Commons: Attribution-NonCommercial-NoDerivs (CC BY-NC-ND)

Document Version

Peer reviewed version

Citation for published version (Harvard):

Jesson, M, Sterling, M & Baker, D 2016, 'Application of ISO4359 for discharge calculation in a narrow flume', *Flow Measurement and Instrumentation*. <https://doi.org/10.1016/j.flowmeasinst.2016.06.015>

[Link to publication on Research at Birmingham portal](#)

Publisher Rights Statement:

Checked for eligibility: 01/07/2016

General rights

Unless a licence is specified above, all rights (including copyright and moral rights) in this document are retained by the authors and/or the copyright holders. The express permission of the copyright holder must be obtained for any use of this material other than for purposes permitted by law.

- Users may freely distribute the URL that is used to identify this publication.
- Users may download and/or print one copy of the publication from the University of Birmingham research portal for the purpose of private study or non-commercial research.
- User may use extracts from the document in line with the concept of 'fair dealing' under the Copyright, Designs and Patents Act 1988 (?)
- Users may not further distribute the material nor use it for the purposes of commercial gain.

Where a licence is displayed above, please note the terms and conditions of the licence govern your use of this document.

When citing, please reference the published version.

Take down policy

While the University of Birmingham exercises care and attention in making items available there are rare occasions when an item has been uploaded in error or has been deemed to be commercially or otherwise sensitive.

If you believe that this is the case for this document, please contact UBIRA@lists.bham.ac.uk providing details and we will remove access to the work immediately and investigate.

© 2016. This Accepted Manuscript version is made available under the CC-BY-NC-ND 4.0 license (see <http://creativecommons.org/licenses/by-nc-nd/4.0/>).

The formally published version may be accessed from:
<http://authors.elsevier.com/sd/article/S0955598616300656>

Application of ISO4359 for Discharge Calculation in a Narrow Flume

Michael Jesson¹, Mark Sterling² and David Baker³

¹Research Fellow, School of Civil Engineering, University of Birmingham, Birmingham, UK,
m.a.jesson@bham.ac.uk

²Beale Professor of Civil Engineering, School of Civil Engineering, University of Birmingham,
Birmingham, UK

³Visiting Student, School of Civil Engineering, University of Birmingham, Birmingham, UK,

Abstract

Flumes with either width contractions or raised beds to force modular flow conditions (i.e. a transition from sub- to super-critical flow) are a widely used instrument for the measurement of flow rate. The conservation of mass, written as the continuity equation, and the conservation of energy are combined with the existence of a critical depth point to derive a theoretical equation for the discharge as a function of the upstream water depth. This derivation requires a number of assumptions regarding the flow both upstream and in the throat of the contraction. The international standard covering the use of such flumes, ISO4359, places restrictions on the upstream position at which the water depth should be measured, which this work shows to be unnecessary, at least for the small (100mm throat width) flumes examined. The assumption that critical depth occurs at the end of the flume throat is also shown to be incorrect, but has negligible effect on discharge calculation using the ISO4359 method.

1 Introduction and Discussion of ISO4359

Many industries use open-channels, including partially-full pipes, to transfer fluids from one part of the industrial process to another. Often, the rate of flow along the channel, the discharge, must be known; for waste-water treatment in the UK, for example, the discharge must be measured to an accuracy of $\pm 8\%$ (Environment Agency, 2013). Flumes are a commonly used apparatus to open-channel discharge measurement, with the international standard ISO4359 (BSI, 2013) specifying how they should be used. A flume generally consists of a straight, prismatic channel into which is built a short lateral or vertical contraction (or both); the length of flume over which this contraction is applied is referred to as the throat. Under steady flow conditions, the fluid accelerates within the throat due to the reduction in cross-sectional area which, under the correct conditions, causes a transition between sub- and super-critical flow. It is these modular flow conditions, along with the conservation of mass and energy, which form the basis for the derivation of the discharge equation for flumes which follows. This equation allows the calculation of the discharge solely from the

measurement of the water depth upstream of the contraction. The discussion which follows assumes a rectangular flume of width b with a horizontal bed, though it is easily extended to trapezoidal flumes and vertical contractions (Chadwick et al., 2004 or BSI, 2013).

If the pressure distribution within the flow is hydrostatic and the channel bed is used as the reference datum, then the energy per unit weight, or head, H , is given by:

$$H = h + \frac{\alpha \tilde{u}^2}{2g} \quad (1)$$

where h is the flow depth, α is the kinetic energy correction coefficient (see below), \tilde{u} is a characteristic velocity which is taken as uniform over the cross-section, and g is gravitational acceleration. The mean streamwise velocity, $\bar{u} \equiv Q/A = Q/bh$ (for a rectangular flume; Q is the discharge, b is the channel width), is commonly used for \tilde{u} , but ISO4359 uses the “free-stream” velocity, u_0 . u_0 is the velocity in the main body of the flow, assuming that the velocity is uniform (u_0) throughout a cross-section apart from within a boundary layer adjacent to the walls and bed. However, since $u_0 > \bar{u}$, the discharge calculated using the cross-sectional area will exceed the true discharge: $Au_0 > A\bar{u}$. Considering a cross-section of unit width, the displacement thickness, δ^* , is defined as the adjustment required such that $b(h-\delta^*)u_0 = bh\bar{u}$. In reality, boundary layers will also form at the channel walls. For simplicity, it is assumed that the wall and bed boundary layer are identical, and so the adjusted dimensions (subscript e) are $b_e = b - 2\delta^*$ and $h_e = h - \delta^*$ and $Q = b_e h_e u_0$. Although ISO4359 uses \bar{u} in its initial definition of the energy equation, a switch to u_0 is implicit when the displacement thickness adjustment is made. The assumption of uniform velocity requires a correction coefficient, α , for the kinetic energy term in (1) (BSI, 2013), though it is noted that arguments have been made that this should be a momentum correction coefficient (see Liggett (1993), for example). α will depend on the characteristic velocity chosen, and a defining expression may be simply derived (see, for example, Chadwick et al. (2004)):

$$\alpha \equiv \frac{\int_0^h \frac{1}{2} \rho b u(z)^3 dz}{\frac{1}{2} \rho b h \tilde{u}^3} = \frac{1}{h} \frac{1}{\tilde{u}^3} \int_0^h u(z)^3 dz \quad (2)$$

The use of u_0 as the characteristic velocity is convenient as δ^* may be determined from empirically derived figures, which are provided in ISO4359.

When the adjusted dimensions b_e and h_e are used to calculate u_0 from Q , (1) becomes:

$$H = (h_e + \delta_*) + \frac{\alpha Q^2}{2gb_e^2 h_e^2} \quad (3)$$

Critical depth, h_{ce} , occurs when H is a minimum, which may be shown to occur when:

$$h_{ce} = \sqrt[3]{\frac{\alpha Q^2}{gb_e^2}} \quad (4)$$

Considering the critical depth point, where $h_e = h_{ce}$, and moving δ_* to the left-hand side of (3), gives:

$$H_e \equiv H - \delta_* = h_{ce} + \frac{\alpha Q^2}{2gb_e^2 h_{ce}^2} = h_{ce} + \frac{h_{ce}^3}{2h_{ce}^2} = \frac{3}{2}h_{ce} \quad (5)$$

where H_e is the effective head. Back substitution for h_{ce} from (5) into (4) gives the ISO4359 result:

$$Q = \frac{1}{\sqrt{\alpha}} \left(\frac{2}{3}\right)^{\frac{3}{2}} \sqrt{gb_e} (H_e)^{\frac{3}{2}} \quad (6)$$

If energy losses at the contraction are assumed to be negligible, the total head H may be measured upstream of the contraction rather than within the throat. ISO4359 specifies the dimensions of the contraction to ensure that this assumption is valid in any flume which meets the standard. Upstream, the streamwise velocity is relatively low and $H_e \approx h_e$. A velocity correction factor, C_v , is introduced, defined as:

$$C_v \equiv \left(\frac{H_e}{h_e}\right)^{\frac{3}{2}} \quad (7)$$

along with a displacement correction factor, C_d :

$$C_d \equiv \frac{b_e}{b} \left(\frac{h_e}{h} \right)^{\frac{3}{2}} \quad (8)$$

allowing (6) to be rewritten in terms of the true upstream water depth and contraction width:

$$Q = \frac{1}{\sqrt{\alpha}} \left(\frac{2}{3} \right)^{\frac{3}{2}} \sqrt{g} C_v C_d b h^{\frac{3}{2}} \quad (9)$$

Evidently it is necessary to evaluate C_v and C_d in order use (9) and so α and δ^* must be known. ISO4359 makes the assumption that the velocity distribution within the throat is sufficiently uniform for $\alpha = 1$ to be a reasonable approximation. Further, it provides figures relating the Reynolds number of the flow to δ^*/L , where L is the throat length. In using these figures it is necessary to assume that the critical depth is reached at the end of the throat, an assumption which Dabrowski and Polak (2012) refute. In an analysis of data from a variety of three flumes ($B = 0.6, 1.0$ and 1.5m), Dabrowski and Polak showed that the position of the critical depth point varied according to:

$$\frac{L_c}{L} = 1.715 \left(\frac{h}{B} \right)^3 - 3.920 \left(\frac{h}{B} \right)^2 + 3.772 \left(\frac{h}{B} \right) - 0.277 \quad (10)$$

where L_c is the distance from the start of the throat of the critical depth point and B is the pre-contraction width of the flume. For Reynolds number (Re) greater than 3×10^5 and “good surface finishes”, ISO4359 allows δ^*/L to be taken as a constant value of 0.003 (BSI, 2013). Yeung (2007) noted that Re will be low for narrow flumes ($b \leq 300\text{mm}$), with the flow in the laminar or transitional region where δ^*/L varies rapidly with Re , whereas for large flumes (high Re) δ^*/L is approximately constant. As such, Yeung considered that additional uncertainty may be associated with the calculation of δ^* for narrow flumes, and that variation of δ^*/L must be taken into account (i.e. the fixed value of 0.003 should not be used), though the Reynolds numbers used by Yeung (maximum $Re = 3 \times 10^5$) fall below the limit specified in ISO4359 anyway. Dabrowski and Polak also commented that the use of δ^* in (3) is incorrect, as it is a correction based on the continuity equation rather than the kinetic energy flux. They suggest that the kinetic energy thickness, δ^{***} , should therefore be used in its place. However, the authors of the current work consider this second suggestion to be unnecessary, and the use of δ^* to be correct based on the assumptions of ISO4359

– the kinetic energy thickness is implicit in α , while it is appropriate to use δ^* to modify the cross-sectional area used in the calculation of the representative velocity, u_0 .

Another modification to ISO4359 has been suggested by Dufresne and Vazquez (2013), who used numerical simulations to show that the ISO4359 restriction on the minimum throat length relative to the upstream head ($\frac{h}{L} < 0.5$) could be relaxed to $\frac{h}{L} < 1.0$.

The application of ISO4359 to calculate the discharge is an iterative process. The discharge must be estimated (ignoring the correction coefficients) in order to calculate the Reynolds number. The Reynolds number is then used to evaluate δ^*/L from the graphs provided, allowing the correction coefficients to be calculated, leading to an improved estimate of Q . This process may be repeated (as the Reynolds number has changed), though in the authors' experience the change is small and a single repetition is required.

As mentioned previously, ISO4359 allows calculation of Q from the upstream head, approximated as the upstream water depth. It has been assumed that drawdown of the water surface in the upstream region near the contraction distorts the water surface to such an extent that water depth measurements in this area will give a poor estimate of the head. Consequently, ISO4359 places restrictions on the upstream location at which the head must be measured, specifying that it lies between 3 and 4 times the maximum water depth upstream of the contraction.

The main aim of the research presented in this paper is to investigate this assumption, and to determine whether discharge may be calculated accurately using water depth measurements taken closer to the contraction than currently prescribed by the standard. Additional data, which provide more evidence to support the findings of Dabrowski and Polak (2012) (that critical depth occurs before the end of the contraction) are also presented. The following section describes the experimental work designed to examine this, after which the results are presented. The paper finishes with a discussion of the results and conclusions drawn.

2 Experimental Method

The experimental work was conducted in a horizontal, rectangular, glass-walled flume (illustrated in Figure 1) of pre-contraction width $B = 200 \pm 0.5$ mm in the University of Birmingham Civil Engineering laboratories. This flume is fed from a constant head tank in the laboratory roof, with the flow controlled by a manual valve. Flow meters, ultrasonic water depth gauges and a contraction, details of which follow, were kindly supplied by Hymetrics Ltd., a UKAS accredited manufacture of flumes. The contraction was a symmetrical, lateral contraction of nominally 0.05m on each side, resulting in

a throat width measured as $b = 99.5\text{mm}$. To within measurement accuracy of $\pm 0.5\text{mm}$ this falls on the lower boundary of set by ISO4359 of $b = 100\text{mm}$; additionally this lower limit only exists due to a lack of experimental data for smaller flumes (BSI, 2013).

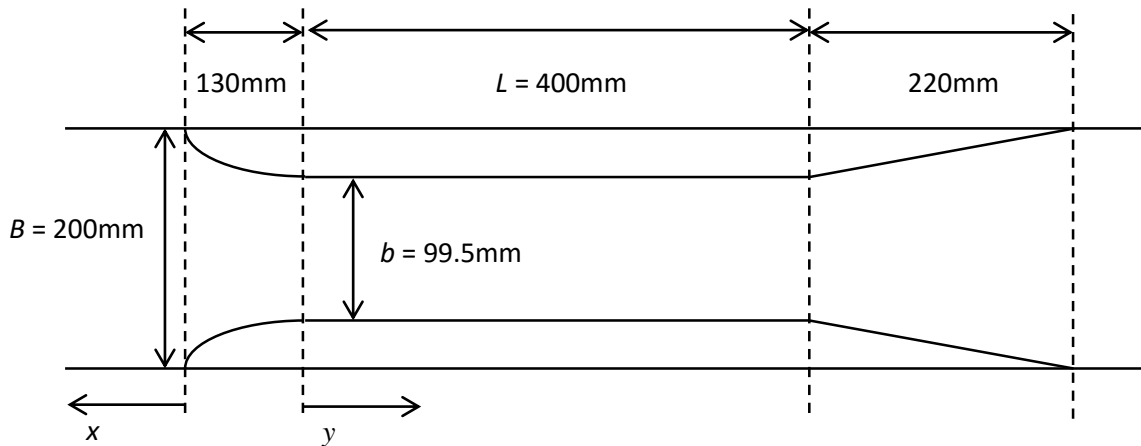


Figure 1 Plan view of the flume contraction

To permit comparison with calculated discharge values, the flow rate was measured using an electromagnetic flowmeter fitted to the supply pipe, with upstream water depth, h , measured using an ultrasonic depth gauge (details of both follow). The two instruments were connected to a PC running software supplied by Hymetrics Ltd. which simultaneously logged data from both sources at a rate of 1Hz . Measurements were made for discharges ranging from 2ls^{-1} ($0.002\text{m}^3\text{s}^{-1}$) to 20ls^{-1} ($0.020\text{m}^3\text{s}^{-1}$) in 2ls^{-1} increments at four distances upstream of the contraction: $x = 50$, $x = 100$, $x = 200\text{mm}$ and $x = 800\text{mm}$. Flow parameters are summarised in Table 1, from which it may be seen that the upstream Froude number is below the 0.5 limit recommended by ISO4359 (BSI, 2013), and the Reynolds number is in the laminar and transitional boundary layer regions. The first of the measurement locations is the closest the equipment could be positioned to the start of the contraction; the last meets the ISO4359 criterion that h should be measured between $3h_{max}$ and $4h_{max}$ upstream of the leading edge of the contraction, where h_{max} is the maximum water depth for which the flume will be used to measure discharge (BSI, 2013), which gives an allowed range of approximately $x = 750\text{mm}$ and $x = 1000\text{mm}$ in the current work.

Table 1 Flow parameters for the experimental discharges

Q (l/s)	h (mm)	Re ($\times 10^5$)	Fr (upstream)
2	53	2	0.27
4	81	2.5	0.27
6	108	2.9	0.27
8	131	3.2	0.27
10	151	3.5	0.27
12	172	3.7	0.27
14	0.188	3.9	0.27
16	0.207	4.1	0.27
18	0.222	4.2	0.27
20	0.236	4.4	0.28

For each location and discharge, 50 samples of instantaneous discharge and head were taken and mean values and 95% confidence intervals calculated, allowing surface fluctuation effects to be reduced.

The electromagnetic flowmeter was manufactured by ABB Limited (serial number G/06776/1/1) and calibrated by Hymetrics Ltd. to an accuracy of $\pm 0.12\%$. This calibration has previously been verified at the University of Birmingham using volumetric flow checks (Japy, 2012). Upstream h was measured using a Pulsar 3db ultrasonic meter (serial number 77150/2005). The Pulsar uses time-of-flight of the transmitted acoustic pulses to measure the distance to the water surface. In order to output water depth, the Pulsar must be calibrated using an object of known height as the reflector, allowing it to calculate its height above the channel bed and thus convert distance to depth. A 20mm high PVC plate was used for this purpose. It is known, however, that variation in air temperature (which changes the speed of sound through the air) can affect the measurements. To overcome this issue, the PVC plate was attached to an electronic height gauge (manufactured by Machine DRO, part number ME-HG-1000) and ten ultrasonic plate height (“water depth”) measurements were taken with the plate in its initial position on the channel bed, and the channel dry. The plate was then raised 20mm and ten more measurements made, and these steps repeated for a range of plate heights spanning the range of water depths to be investigated. The entire process was repeated at

each measurement location, x , immediately before the discharge and head measurements were made at that location. In this manner, the deviation of the ultrasonic measurements from the true height (taken from the electronic height gauge) with plate height was determined and shown to be linear (Figure 2). When applied to the ultrasonic data, the linear correction reduces the error to <1.5% for plate height 50mm (equivalent to the lowest water depths measured), and <0.5% for plate heights ≥ 80 mm. These linear correction models were applied to the experimental data.

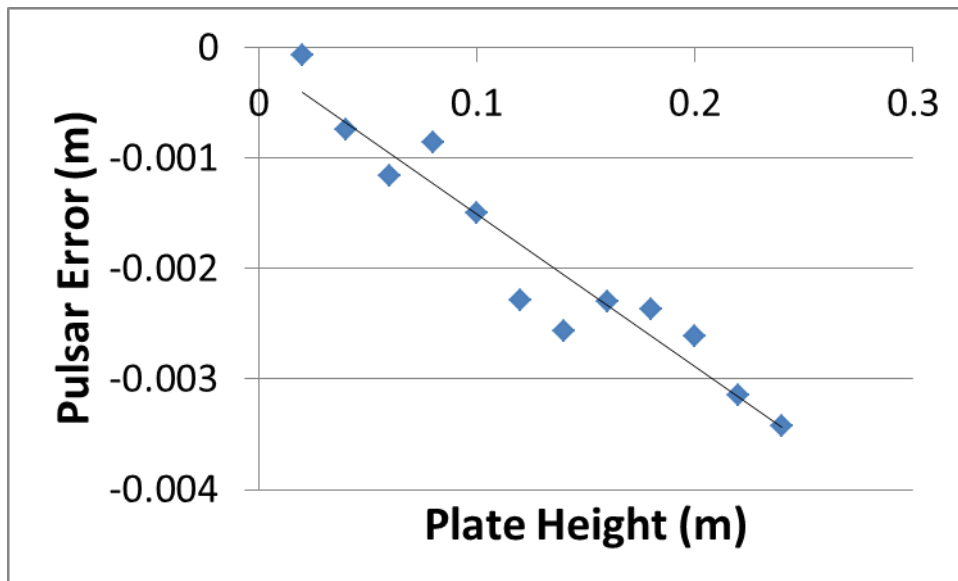


Figure 2 Example of pulsar measurement error variation with plate height

In addition to the head measurements, velocity measurements were made using a Nixon Instrumentation “Streamflo” propeller meter. These measurements were made at ten positions along the length of the contraction, at 50mm intervals along the channel centreline starting at $y = 0$ (see Figure 1), with a vertical profile of streamwise velocity measured at each location. These data allowed the position of the critical depth position to be estimated. Due to the small flow depths in the contraction for low discharges, these measurements were only made for discharges of 8l/s and greater. For two discharges (10.3l/s and 20.2l/s), the velocity was measured at points spanning a cross-section in the sub-critical flow region upstream of the contraction, at 20mm vertical and horizontal spacing. These measurements were used to validate the calibration of the propeller meter, by integration over the cross-section to obtain the mean velocity and hence discharge, and comparison with the flowmeter reading. Boundary layer effects were accounted for by fitting a log-law profile to the velocity data, and using this to extrapolate to the channel walls and bed. For the two flows, the integrated values were 106% and 107% of the flowmeter value respectively, and so a correction of $1/1.07$ was applied to the vertical profiles measured in the contraction. Water depth

was also measured at the locations of the vertical profiles using a pointer gauge with a Vernier scale, giving a second method of estimating the critical depth position.

3 Results and Analysis

The method of ISO4359 was applied to the channel using the measured h values. Due to the low Reynolds numbers, and noting the comments of Yeung (2007) mentioned earlier, δ^*/L was evaluated from the figures provided in ISO4359 (BSI, 2013) rather than using the fixed value of 0.003. Within the uncertainty based purely on the uncertainties in the experimental measurements (indicated by the error bars in Figure 3), the calculated ISO4359 discharges are consistent across all measurement locations, x . The calculated values do, however, indicate a consistent overestimation of the discharge for all but the lowest discharges ($Q < 5 \times 10^{-3} \text{m}^3/\text{s}$) which exceeds this experimental uncertainty. The percentage uncertainty due to errors in the calculation of C_v and C_d were evaluated using the formula provided by ISO4359 (BSI, 2013):

$$X_c \cong \pm 1 + 20(C_v - C_d) \quad (11)$$

and combined with the estimated uncertainties in the measurements of h and b . The percentage uncertainty in Q had a range of 2.8% to 3.8%, with variation with discharge but negligible variation with measured position, x , (Table 2). In the calculation of these uncertainties, X_c dominates and is an order of magnitude greater than uncertainty in the h measurements and flume dimension. For $h/L > 0.5$ X_c must be increased by 2% (BSI, 2013) and this is reflected in the table.

$Q \text{ (m}^3\text{s}^{-1}\text{)}$	Uncertainty (%)			
	$x=800\text{mm}$	$x=200\text{mm}$	$x=100\text{mm}$	$x=50\text{mm}$
0.020	2.9 (4.9)	2.9 (4.9)	2.8 (4.8)	2.8 (4.8)
0.018	2.8 (4.8)	2.8 (4.8)	2.8 (4.8)	2.8 (4.8)
0.016	2.9 (4.9)	2.8 (4.8)	2.9 (4.9)	2.9 (4.9)
0.014	2.9	2.9	2.9	2.9
0.012	2.9	2.9	2.9	2.9
0.010	2.9	2.9	3.0	3.0
0.008	3.0	3.0	3.0	3.0
0.006	3.1	3.1	3.1	3.1
0.004	3.3	3.3	3.3	3.3
0.002	3.8	3.8	3.8	3.8

Table 2: Discharge uncertainties calculated following ISO4359 (bracketed values include the 2% increase due to a high h/L ratio, as dictated by ISO4359 (BSI, 2013))

The ISO4359 uncertainties are shown for $x = 50\text{mm}$ (lower bound only for clarity) and $x = 100\text{mm}$ in Figure 3. These bounds are indicative of the largest deviations from the measured discharge which were seen. The deviation from the measured discharge is within the uncertainty defined by ISO4359 for all discharges, and is also within the 8% limit set by the Environment Agency (2013), even when the ISO4359 uncertainties are included (Figure 3). This shows that, at least for a flume such as that used in the current work, the ISO4359 requirement for the upstream distance at which the water depth is measured may be relaxed. Although the upstream water depth will vary slightly due to the gradually varied flow region upstream of the contraction, this has negligible effect on the discharge measurement. This may be due to associated changes to the C_v coefficient (7) used in (9).

The results do not indicate increasing scatter for $0.5 < \frac{h}{L} < 0.63$ (the maximum used), supporting the assertion of Dufresne and Vazquez (2013) that the maximum ratio of upstream head to throat length may be relaxed, and indicating that the additional 2% uncertainty specified by ISO4359 may be unnecessary.

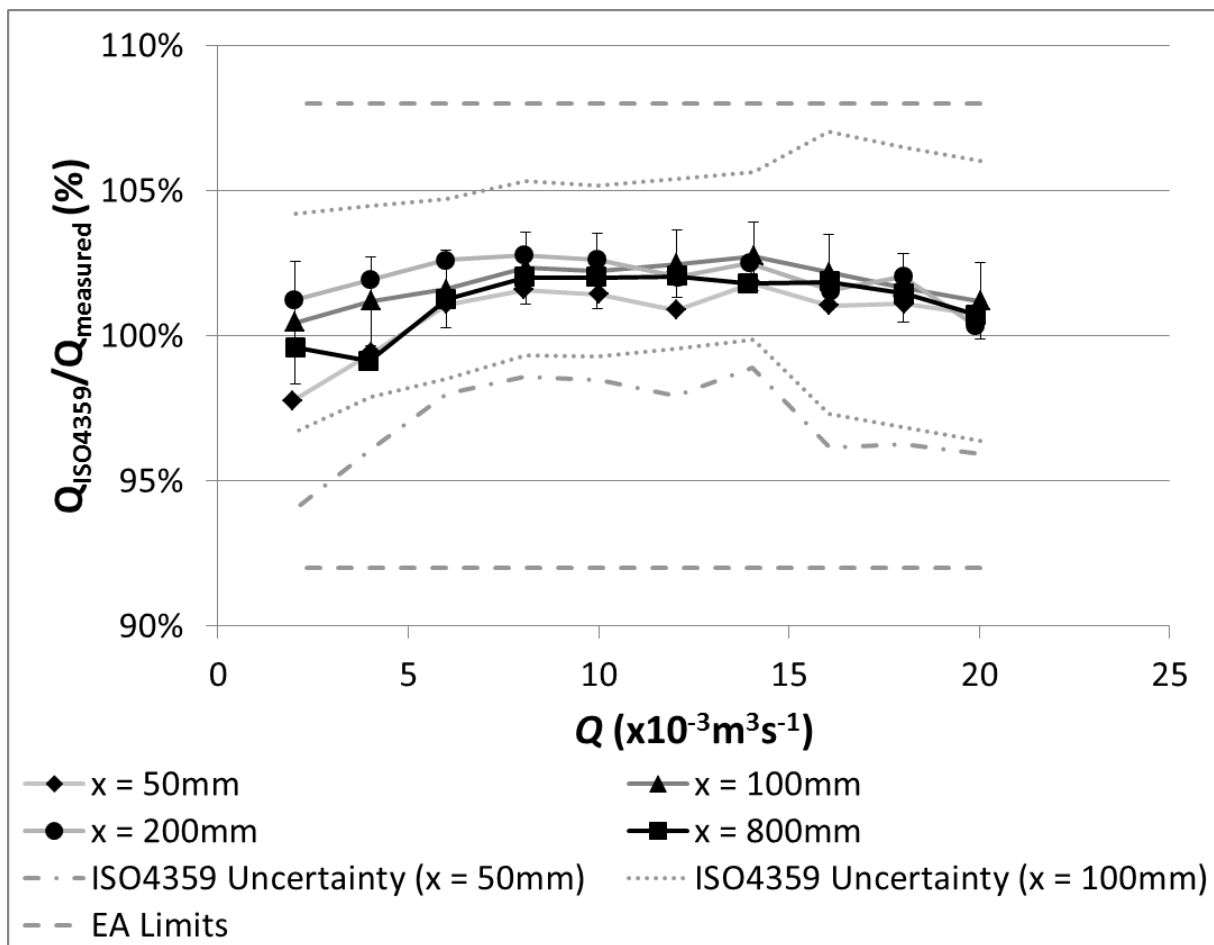


Figure 3 ISO4359 discharge values as a percentage of the flowmeter readings. For clarity, error bars are shown for $x = 100\text{mm}$ only; similar error ranges apply to the other measurements.

Dabrowski and Polak (2012) showed that the critical depth occurs within the throat rather than at the end of the throat, at position $L_c \ll L$. Their result is substantiated by the vertical profiles of streamwise velocity and depth measurements made in the contraction for this work. The critical depth locations calculated from the velocity measurements (indicated by the location where the mean velocity first exceeds the critical velocity) and depth measurements (by direct comparison to the theoretical h_c) are consistent (Figure 4), increasing with discharge but only being between $0.2L$ and $0.4L$. The formula for calculating L_c proposed by Dabrowski and Polak (10) does not match the results found here – due to the high ratio h/B for all but the lowest discharges, their formula gives $L_c \approx L$ or greater and is clearly unsuitable for the flume used in the current work.

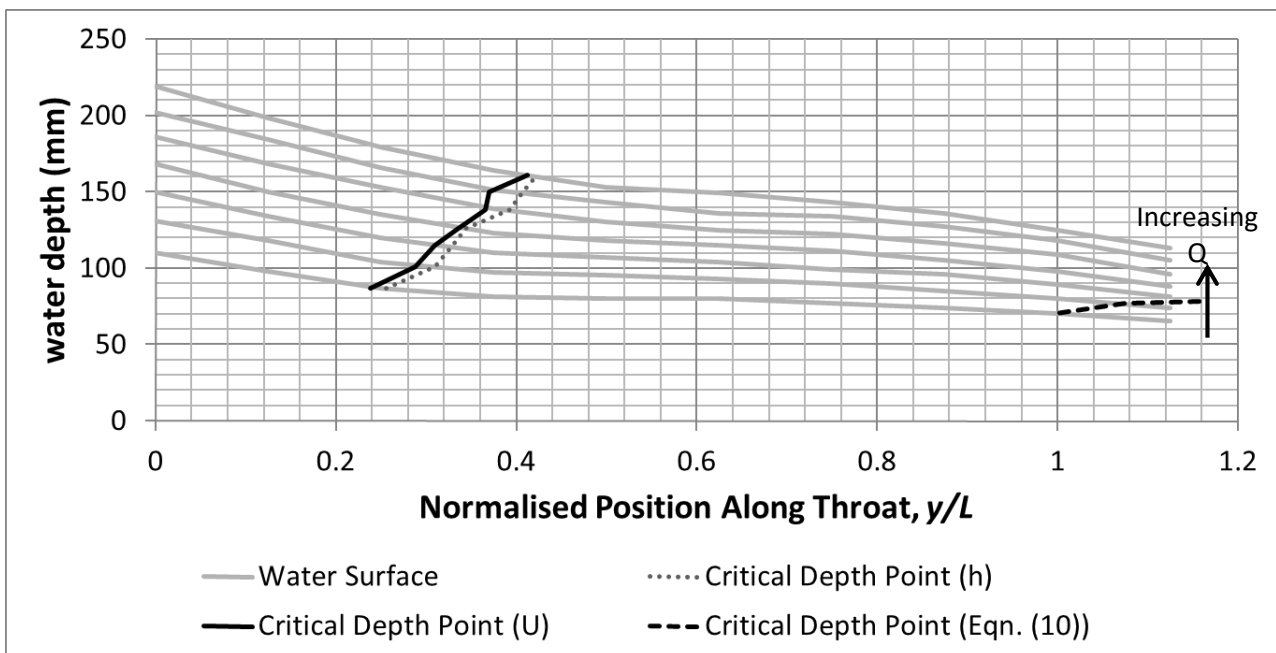


Figure 4 Water surface profiles along the contraction, and critical depth points ($Q = 8\text{ l/s}, 10\text{ l/s}, 12\text{ l/s}, 14\text{ l/s}, 16\text{ l/s}, 18\text{ l/s}, 20\text{ l/s}$)

Using the measured L_c values in place of L , Q was recalculated following the ISO4359 procedure for $x = 800\text{ mm}$. The difference in Q calculated using L and L_c was negligible, being a maximum of 0.3% (Figure 5). Dabrowski and Polak saw similar results when using L_c values calculated numerically, except for at low flow rates, at which significant improvements were seen (Dabrowski and Polak, 2012). It should be noted, however, that Dabrowski and Polak saw large errors in Q at these low flow rates ($\sim 13\%$ for the lowest flow rate, improved to $\sim 5\%$ when using L_c).

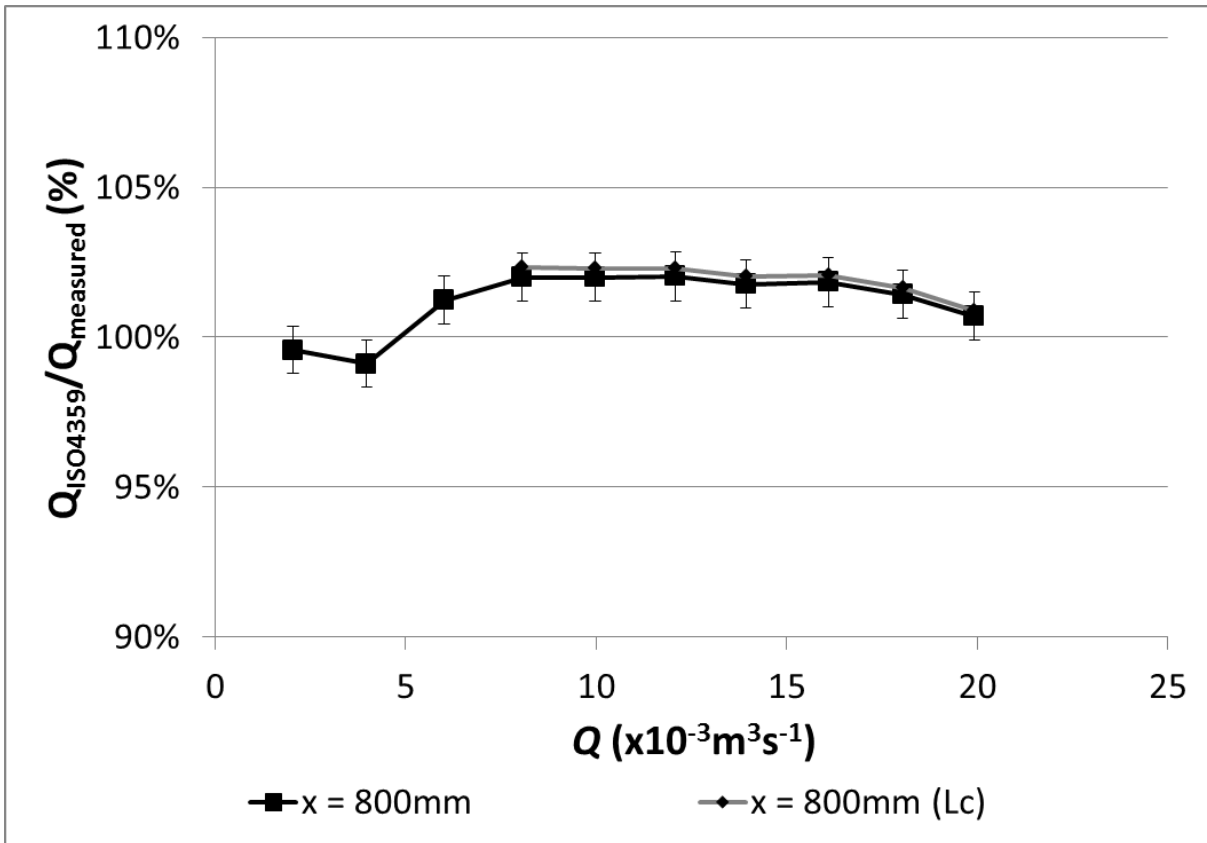


Figure 5 ISO4359 calculated discharge using L and L_c

4 Conclusions

The application of international standard ISO4359 to measure discharge in open-channels was investigated using an experimental flume with a flat-bed, double-sided contractions to create modular flow conditions. An ultrasonic depth meter was used to measure the upstream head at a number of positions relative to the start of the contraction, allowing a statistical analysis of the uncertainty in the measurements. Water surface profile measurements were made within the throat of the flume, along with vertical profiles of streamwise velocity, allowing the position at which critical depth occurred to be calculated. Through this work, certain assumptions of ISO4359 (i.e. that the upstream head must be measured at a position at least four times the maximum head upstream of the contraction, and that critical depth occurs at the downstream end of the throat) were examined and found to be inapplicable to a flume such as that used in the current work. The following conclusions are drawn from the experimental data:

- The upstream head may be measured as close as 50mm upstream of the start of the contraction without compromising the calculation of the discharge, Q .

- The critical depth occurs well before the end of the throat, at a position between 0.2 and 0.4 of the throat length.
- While use of the true critical depth position (rather than the throat length) in the Q calculation has a measurable effect, this is negligible when compared to the experimental/calculation uncertainty.
- The large improvement in the discharge calculation seen by Dabrowski and Polak (2012) for low discharges was not seen, though this may be in part due to the smaller error in the calculated discharge seen in the current work for those flow rates, in itself due to differences in the flow depth-to-flume width ratio.

Acknowledgements

The authors would like to thank Ashley Molloy for his assistance in setting up the equipment loaned by Hymetrics Ltd., and Mike Vanderstam for his maintenance of the University of Birmingham flumes. Special thanks must be given to Richard Jones, formerly of Hymetrics Ltd., for his support for this work and the associated MSc projects, which have been a source of inspiration for a number of students.

5 References

- BSI, 2013. BS ISO 4359:2013(E) Flow measurement structures - Rectangular, trapezoidal and U-shaped flumes.
- Chadwick, A., Morfett, J., Borthwick, M., 2004. Hydraulics in civil and environmental engineering. Spon Press.
- Dabrowski, W., Polak, U., 2012. Improvements in Flow Rate Measurements by Flumes. *J. Hydraul. Eng.* 138, 757–763. doi:10.1061/(asce)hy.1943-7900.0000567
- Dufresne, M., Vazquez, J., 2013. Head–discharge relationship of Venturi flumes: from long to short throats. *J. Hydraul. Res.* 51, 465–468. doi:10.1080/00221686.2013.781550
- Environment Agency, 2013. *Minimum requirements for the self-monitoring of effluent flow, Version 3.5*. Environment Agency.
- Japy, C., 2012. *An evaluation of the performances of ISO 4359 flumes*. University of Birmingham, Birmingham, UK.
- Liggett, J.A., 1993. Critical Depth, Velocity Profiles, and Averaging. *J. Irrig. Drain. Eng.* 119, 416–422. doi:10.1061/(ASCE)0733-9437(1993)119:2(416)
- Yeung, H., 2007. An examination of BS3680 4C (ISO/DIS 4369) on the measurement of liquid flow in open channels — flumes. *Flow Meas. Instrum.* 18, 175–182. doi:10.1016/j.flowmeasinst.2007.01.002