

# Neural adaptive control for uncertain nonlinear system with input saturation

Gao, Shigen; Dong, Hairong; Ning, Bin; Chen, Lei

DOI:

[10.1016/j.neucom.2015.02.012](https://doi.org/10.1016/j.neucom.2015.02.012)

License:

Other (please specify with Rights Statement)

*Document Version*

Peer reviewed version

*Citation for published version (Harvard):*

Gao, S, Dong, H, Ning, B & Chen, L 2015, 'Neural adaptive control for uncertain nonlinear system with input saturation: state transformation based output feedback', *Neurocomputing*.  
<https://doi.org/10.1016/j.neucom.2015.02.012>

[Link to publication on Research at Birmingham portal](#)

## **Publisher Rights Statement:**

NOTICE: this is the author's version of a work that was accepted for publication in Neurocomputing. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Neurocomputing, DOI: 10.1016/j.neucom.2015.02.012.

Eligibility for repository checked March 2015

## **General rights**

Unless a licence is specified above, all rights (including copyright and moral rights) in this document are retained by the authors and/or the copyright holders. The express permission of the copyright holder must be obtained for any use of this material other than for purposes permitted by law.

- Users may freely distribute the URL that is used to identify this publication.
- Users may download and/or print one copy of the publication from the University of Birmingham research portal for the purpose of private study or non-commercial research.
- User may use extracts from the document in line with the concept of 'fair dealing' under the Copyright, Designs and Patents Act 1988 (?)
- Users may not further distribute the material nor use it for the purposes of commercial gain.

Where a licence is displayed above, please note the terms and conditions of the licence govern your use of this document.

When citing, please reference the published version.

## **Take down policy**

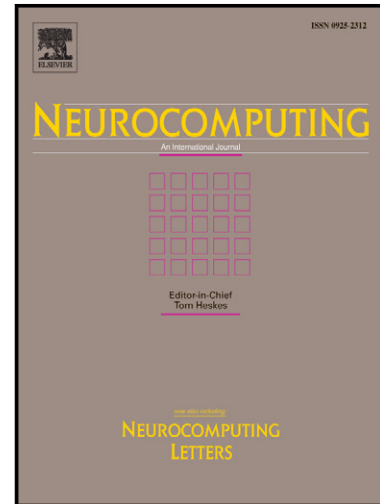
While the University of Birmingham exercises care and attention in making items available there are rare occasions when an item has been uploaded in error or has been deemed to be commercially or otherwise sensitive.

If you believe that this is the case for this document, please contact [UBIRA@lists.bham.ac.uk](mailto:UBIRA@lists.bham.ac.uk) providing details and we will remove access to the work immediately and investigate.

# Author's Accepted Manuscript

Neural adaptive control for uncertain non-linear system with input saturation: state transformation based output feedback

Shigen Gao, Hairong Dong, Bin Ning, Lei Chen



[www.elsevier.com/locate/neucom](http://www.elsevier.com/locate/neucom)

PII: S0925-2312(15)00161-7  
DOI: <http://dx.doi.org/10.1016/j.neucom.2015.02.012>  
Reference: NEUCOM15139

To appear in: *Neurocomputing*

Received date: 19 April 2014  
Revised date: 12 January 2015  
Accepted date: 8 February 2015

Cite this article as: Shigen Gao, Hairong Dong, Bin Ning, Lei Chen, Neural adaptive control for uncertain nonlinear system with input saturation: state transformation based output feedback, *Neurocomputing*, <http://dx.doi.org/10.1016/j.neucom.2015.02.012>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting galley proof before it is published in its final citable form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



# Neural adaptive control for uncertain nonlinear system with input saturation: state transformation based output feedback<sup>1</sup>

Shigen Gao<sup>2a</sup>, Hairong Dong<sup>3a</sup>, Bin Ning<sup>a</sup>, Lei Chen<sup>b</sup>

<sup>a</sup>State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China

<sup>b</sup>School of Electronic, Electrical and Computer Engineering, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK

---

## Abstract

This paper presents neural adaptive control methods for a class of nonlinear systems in the presence of actuator saturation. Backstepping technique is widely used for the control of nonlinear systems. By introducing alternative state variables and implementing state transformation, the system can be reformulated as output feedback of a canonical system, which ensures that the controllers can be developed without backstepping methodology. To reduce the influence caused by actuator saturation, an effective auxiliary system is constructed to prevent the stability of closed loop system from being destroyed. Radial basis function (RBF) neural networks (NNs) are used in the online learning of the unknown dynamics. High-order sliding mode (HOSM) observer is used in the output feedback case of the achieved canonical system. Ultimate and transient tracking errors can be adjusted arbitrarily small by choosing proper design parameters in an explicit way with input saturation in effect. Simulation results are presented to verify the effectiveness of proposed schemes.

© 2015 Published by Elsevier Ltd.

**Keywords:** Neural adaptive control, Actuator saturation, Sliding-mode observer, Nonlinear system

---

## 1. Introduction

Recent decades have witnessed great advance and development of adaptive control methods for nonlinear systems in theoretical studies and practical applications [1] [2] [3] [4] [5] [6] [7], and multifarious controllers have been developed using advanced techniques (e.g., inversion control [8], sliding mode control [9], backstepping [10][11][12], and so on). Among various control methods, adaptive backstepping control has been acknowledged as a powerful methodology and widely used in nonlinear control field [13]. To eliminate the difficulty and challenge caused by unknown nonlinear dynamics, approximation-based control methods have been used [14][15], where either neural networks or fuzzy logic systems act as the function approximators. The merits of above-mentioned approximation-based control are that the assumption of *linear in unknown parameters* can be removed, and the adaptive laws of

---

<sup>1</sup>This work is supported jointly by the National High Technology Research and Development Program (“863” Program) of China (No. 2012AA041701), Fundamental Research Funds for Central Universities (No. 2013JBZ007), National Natural Science Foundation of China (Nos. 61233001, 61322307 and 61304196), and Beijing Jiaotong University Research Program (No. RCS2012ZZ003). Part of this work was completed while Shigen Gao was Visiting Research Student in University of Birmingham, and the financial support of China Scholarship Council is gratefully appreciated.

<sup>2</sup>Co-corresponding authors. Email: gaoshigen@bjtu.edu.cn

<sup>3</sup>Co-corresponding authors. Email: hrdong@bjtu.edu.cn

adjustable weights of neural networks or fuzzy logic systems can be obtained on the basis of Lyapunov theorem, which guarantees the stability of closed-loop system.

Input saturation is encountered commonly in control systems design and exists in practical applications due to the fact that it is usually impossible to implement unlimited control signals. This phenomenon can cause performance degradation and even instability of the closed-loop system for practical systems if it is not explicitly considered during design of the controller [16]. Control input saturation in particular has a significant effect on adaptive control in that adaptation laws will act unexpectedly with saturation, which further tends to be aggressive in seeking the desired tracking performance. Therefore, the study of the problem of controller design subject to input saturation is of great significance, both from practical and theoretical points of view.

There are several interesting works attempting different adaptive control design methods for nonlinear systems in the presence of actuator saturation. In [17], a concept of augmented error signal (AES) is introduced, the proposed AES is generated by the auxiliary input, i.e.,  $\Delta u = v - u$ , with  $v$  and  $u$  being designed control signal and actuator output, respectively. The AES can absorb the excess control signal, therefore prevent the stability of closed-loop system from being destroyed. The AES method has been demonstrated to be effective to deal with input saturation in adaptive flight control [18] [19], pseudo-control hedging [20], and so on. In [16], an on-line approximation based adaptive control is presented for preventing the presence of input saturation from destroying the learning capabilities and memory of an on-line approximator based on AES method. As is indicated in [16], backstepping method is an effective tool to deal with adaptive control of high-order nonlinear systems with input saturation based on AES method using a regressive design procedure. Therefore, in [21], backstepping-based adaptive control is developed for unknown nonlinear chaotic systems, where fuzzy neural networks are used to on-line approximate the unknown dynamics, an auxiliary system with the same order of the controller plant is used to compensate the effect of the input saturation based on core idea of AES method. However, the backstepping method in [21] suffers from the curse of dimensionality problem [22] [23] caused by the repeated differentiations of certain nonlinear functions [24], which can be solved using a dynamic surface control (DSC) technique [25]. In [26], robust adaptive neural networks control for a general class of uncertain multiple-input-multiple-output nonlinear systems with input nonlinearities, including saturation and deadzone, variable structure control in combination with backstepping and Lyapunov synthesis is proposed for adaptive neural networks control, the problem of “curse of dimensionality” are solved using the DSC technique. The proposed control method guarantees the stability of the closed-loop adaptive system and the tracking errors converging to small residual sets. However, from a point of view of practical application, the methods developed in [26] [27] [28] are computationally expensive because the approximators (neural networks or fuzzy systems) are used in every step to online approximate the unknown dynamics, that is to say, there are at least  $\sum_{i=1}^n p_i$  adaptation laws in the control of a  $n$  order system, where  $p_i$  is the numbers of neural nodes or fuzzy rules.

This paper concerns the control design for a class of unknown single-input-single-output (SISO) nonlinear system in the presence of saturated actuator without using backstepping methodology. By selecting proper new state variables, the SISO nonlinear system is represented in a Brunovsky form while the control target is retained by controlling the transformed canonical system. This transformation allows a simpler control design by abandoning the backstepping design. This idea is partially inspired by [29] where adaptive neural control is designed for strict-feedback systems without backstepping. Following, neural adaptive control framework is presented to prevent the adaptation capabilities from being destroyed in the presence of input saturation. First, a neural adaptive controller is proposed based on state feedback. Auxiliary systems are constructed to attenuate the effects of input saturation inspired by AES method [17], but it's a quite nontrivial and different design procedure comparing the ones in [16] and [17]. Indeed, one cannot use the method for proving the results in [16][17][21] to prove our results in this paper, as will be seen in the development throughout this paper. After that, a neural adaptive controller is proposed using output feedback, because it is always very difficult to measure all the system states. In this case, a HOSM observer is introduced [30][31]. The prominent feature of the HOSM observer lies in that it guarantees finite-time observer error convergence and therefore is an admirable tool for feedback control design with separation principle [32] trivially satisfied. RBF-NNs are used in the online learning the unknown nonlinear dynamics by virtue of the RBF-NNs' approximation capability. The stability of the closed-loop system is obtained using Lyapunov theorem. The tracking error between the output and the reference signal converges to compact sets around zero. The bounded values of ultimate and transient compact sets can be adjusted to be small-enough by adjusting the parameters left to users in the proposed control schemes, which is a highly-desired property in theoretical analysis and practical engineering. The features of the developed methods are briefly summarized as follows:

1. Prescribed bound sets of ultimate and transient tracking errors are obtained with adjustable design parameters in an explicit form, which can be tuned to arbitrarily small via selecting proper design parameters.
2. Gain selection technique is given with consideration of initial tracking and estimation errors based on invariant set theorem.
3. By choosing new state variables, the nonlinear systems considered is transformed into a canonical one, which can decrease the calculated amount since only one neural network is needed in the online learning of uncertain dynamics, while the methods in in [26] [33] [28] require  $n$  neural networks in the online learning of uncertain dynamics for  $n$ th system.
4. The problem of “explosion of complexity” is avoided by designing direct neural adaptive control, which exists in the methods in [21] caused by repeated differentiations of virtual controllers.
5. Both state and output feedback control schemes are given. In the output feedback case, HOSM observer is utilized to extract to state estimation from noisy output information, finite-time observation error convergence is ensured and separation principle is trivially satisfied.

The rest of the paper is organized as follows. Section 2 presents the problem formulation and some preliminaries. Two neural adaptive controllers using state feedback and output feedback are described in Section 3. Simulation and comparative results are given in Section 4 to demonstrate the effectiveness of proposed controllers. Section 5 presents the conclusions.

## 2. Problem Formulation and Preliminaries

Consider a class of nonlinear systems subject to actuator saturation in the following form

$$\begin{cases} \dot{z}_i = h_i(z_i) + z_{i+1}, & 1 \leq i \leq n-1 \\ \dot{z}_n = h_n(z_n) + v \\ v = \text{sat}(u) \\ y = z_1 \end{cases} \quad (1)$$

where  $z_i$ ,  $h_i(\cdot)$ ,  $i = 1, 2, \dots, n$ ,  $y$  are state variables, unknown smooth functions and measurable output, respectively,  $z = [z_1, z_2, \dots, z_n]^T \in R^n$ .  $u$  is the designed input,  $v$  is the output of saturated actuator, i.e.,  $v$  is the actual input to the system. The relationship of  $v$  and  $u$  is of the following form:

$$v = \text{sat}(u) = \begin{cases} u^+, & \text{if } u > u^+; \\ u, & \text{if } u^- \leq u \leq u^+; \\ u^-, & \text{if } u < u^-; \end{cases} \quad (2)$$

where  $u^+$  and  $u^-$  are the upper and lower bounded limitation caused by actuator saturation, the saturation limitation is asymmetric, i.e.,  $|u^+| \neq |u^-|$ .

The objective is to design a proper control law  $u$  such that:

1. The output  $y$  tracks a reference signal  $y_r$  with the tracking error  $y - y_r$  adjustable and bounded;
2. All the signals in the closed-loop system are remained uniformly bounded.

**REMARK 1.** When there is no actuator saturation, the system given by Eq. (1) can be viewed as a simplified version of semi-strict feedback nonlinear system [34], strict feedback nonlinear system [35] and pure feedback nonlinear system [36]. However, in the case that actuator is constrained by saturation, the methods developed in [34]-[36] are not effective any more. Further more, the methods used in above literatures are all based on backstepping technique. Backstepping method is recognized as an effective tool to design control for nonlinear systems, but it requires complicated design procedure especially for high-order system, because one has to finish  $n$  times design procedure for  $n$  order system. Moreover, if NNs are adopted to approximate unknown nonlinear functions in every step, then, the derivatives of the so-called “virtual control” are included in the NNs, therefore, the input vector of NNs will be twice dimension as usual. Based on these arguments, it can be seen that backstepping method is computationally expensive to be implemented in practical engineering. This motivates us to develop effective control method for system in the form of Eq. (1) without using backstepping method and by explicitly taking the actuator saturation into consideration.

To facilitate the control design, we choose the new state variables as  $x_i = z_1^{(i-1)}$ ,  $i = 1, 2, \dots, n$ , then

$$\begin{cases} \dot{x}_2 = \dot{z}_1 = \frac{dh_1(z_1)}{dz_1} + h_2(z_2) + z_3 \\ \dot{x}_3 = \dot{z}_1^{(3)} = \frac{d^2h_1(z_1)}{dz_1^2} + \frac{dh_2(z_2)}{dz_2} + h_3(z_3) + z_4 \\ \dots \\ \dot{x}_i = \dot{z}_1^{(i)} = \frac{d^{i-1}h_1(z_1)}{dz_1^{i-1}} + \frac{d^{i-2}h_2(z_2)}{dz_2^{i-2}} + \dots + h_i(z_i) + z_{i+1} \\ \dots \\ \dot{x}_n = \dot{z}_1^{(n)} = \frac{d^{n-1}h_1(z_1)}{dz_1^{n-1}} + \frac{d^{n-2}h_2(z_2)}{dz_2^{n-2}} + \dots + \frac{d^{n-i}h_i(z_i)}{dz_i^{n-i}} + \dots + h_n(z_n) + v \end{cases}$$

It's clear that Eq. (1) is reformulated as the following form by selecting new state variables  $x_i = z_1^{(i-1)}$ ,  $i = 1, 2, \dots, n$

$$\begin{cases} \dot{x}_i = x_{i+1}, 1 \leq i \leq n-1 \\ \dot{x}_n = f(x) + v \\ f(x) = \frac{d^{n-1}h_1(z_1)}{dz_1^{n-1}} + \frac{d^{n-2}h_2(z_2)}{dz_2^{n-2}} + \dots + \frac{d^{n-i}h_i(z_i)}{dz_i^{n-i}} + \dots + h_n(z_n) \\ v = \text{sat}(u) \\ y = x_1 \end{cases} \quad (3)$$

where  $x_i$ ,  $i = 1, 2, \dots, n$  are the new state variables,  $x = [x_1, x_2, \dots, x_n]^T$ ,  $f(\cdot)$  is an unknown nonlinear smooth function. It is now clear that by selecting  $x_i = z_1^{(i-1)}$  for  $i = 1, 2, \dots, n$ , the nonlinear system Eq. (1) is reformulated as a Brunovsky system with output  $x_1$ . Since the fact that  $y = z_1 = x_1$ , the control target of Eq. (1) is retained by controlling system Eq. (3). In view of the fact that only the output information of Eq. (1) is included in Eq. (3), therefore, all the designed control methods for Eq. (3) are essentially output feedback control for Eq. (1), and this is clarified here to avoid the confusion caused by the following ‘‘state feedback’’ and ‘‘output feedback’’ control design, which are terms for Eq. (3) in fact. The following assumption and lemmas are used throughout the remaining contents.

**ASSUMPTION 1.** *The system given by Eq. (3) is input-to-state stable.*

**REMARK 2.** *Assumption 1 is reasonable because there does not exist a feasible control which can stabilize an unstable plant with saturated actuator [37], which is thus used in this work.*

**Lemma 1.** *Consider the dynamic system  $\dot{\rho}(t) = -a\rho(t) + b$ , where  $a$  and  $b$  are positive constants, then, for any given bounded initial condition  $\rho_0$ , one has  $\lim_{t \rightarrow \infty} \rho(t) \rightarrow b/a$ .*

*Proof.* The solution to the equation  $\dot{\rho}(t) = -a\rho(t) + b$  with bounded initial condition  $\rho_0$  can be obtained as

$$\begin{aligned} \rho(t) &= e^{-at} \rho_0 + b e^{-at} \int_{t_0}^t e^{as} ds \\ &= \frac{b}{a} + e^{-at} \rho_0 - \frac{b}{a} e^{-a(t-t_0)}, \end{aligned}$$

hence, there exists a moment  $T$  such that for any  $t > T$ ,  $\lim_{t \rightarrow \infty} \rho(t) \rightarrow b/a$  as  $e^{-at} \rho_0 - \frac{b}{a} e^{-a(t-t_0)}$  decays to zero. This completes the proof.  $\square$

**Lemma 2.** *Consider the dynamic system in the form of  $\dot{\psi}(t) = -c|\eta(t)|\psi(t) + d\eta(t)$ , where  $c$  and  $d$  are positive constants,  $\eta(t)$  is a bounded smooth function,  $|\eta(t)| \leq \varepsilon_\eta$ , then,  $\psi(t)$  is bounded by  $|\psi(t)| \leq d/c$  in finite-time.*

*Proof.* By selecting a Lyapunov candidate  $V = \frac{1}{2}\psi^2$ , one obtains the derivative of  $V$  as follows:

$$\begin{aligned}\dot{V} &= -c|\eta(t)|\psi^2(t) + d\eta(t)\psi(t) \\ &\leq -|\eta(t)|\psi(t)(c|\psi(t)| - d)\end{aligned}\quad (4)$$

It's clear that  $\dot{V} \leq 0$  if  $|\psi(t)| > d/c$ . In view of Lyapunov theorem [32],  $\psi(t)$  is bounded by  $|\psi(t)| \leq d/c$  in finite-time.  $\square$

*Lemma 3.* RBF-NNs [38] are widely used to model unknown continuous function  $F(\cdot)$  in control engineering, which is in the form of

$$F(\cdot) = W^T S(Z),$$

where  $Z \in \Omega_Z \subset \mathbb{R}^q$  is the input to the neural network with  $q$  being the NN input dimension,  $W = [w_1, w_2, \dots, w_l]^T$  is the adjustable parameters vector,  $l$  is the number of neurons;  $S(Z) = [s_1(Z), s_2(Z), \dots, s_l(Z)]^T$ , with  $s_i(Z)$  being Gaussian functions, i.e.,  $s_i(Z) = \exp(-(Z - \mu_i)^T(Z - \mu_i)/\eta_i^2)$ ,  $i = 1, 2, \dots, l$ , with  $\mu_i$  and  $\eta_i$  representing the centers and widths of the Gaussian functions. As indicated in [38], RBF NNs can approximate  $F(\cdot)$  to arbitrary accuracy over a compact set  $\Omega_Z \subset \mathbb{R}^q$ :

$$F(\cdot) = W^{*T} S(Z) + \varepsilon(Z),$$

where  $W^*$  is an ‘‘optimal’’ bounded weight vector which minimizes  $\varepsilon(Z)$ :

$$W^* := \arg \min_{W \in \mathbb{R}^l} \left\{ \sup_{Z \in \Omega_Z} |F(\cdot) - W^T S(Z)| \right\},$$

with  $\|W^*\| \leq \varepsilon_N$ ,  $\varepsilon(Z)$  is the bounded approximation error,  $|\varepsilon(Z)| \leq \varepsilon_n$ .

*Lemma 4.* Consider a signal  $\ell(t)$  defined on  $[0, \infty]$  composed of a bounded noise and an unknown base signal  $\ell_0(t)$ , with the noise being Lebesgue-measurable and the  $n$ -th derivative of base signal  $\ell_0(t)$  having a known Lipschitz constant  $L > 0$ . In order to estimate  $\dot{\ell}_0(t)$ ,  $\ddot{\ell}_0(t)$ ,  $\dots$ ,  $\ell_0^{(n)}(t)$ , one can construct a HOSM observer in the following form

$$\begin{cases} \dot{\hat{x}}_1 = p_1, \\ p_1 = -\lambda_1 |\hat{x}_1 - \ell(t)|^{\frac{n}{n+1}} \text{sign}(\hat{x}_1 - \ell(t)) + \hat{x}_2, \\ \dot{\hat{x}}_i = p_i, \\ p_i = -\lambda_i |\hat{x}_i - p_{i-1}|^{\frac{n-i+1}{n-i+2}} \text{sign}(\hat{x}_i - p_{i-1}) + \hat{x}_{i+1}, \\ \quad i = 2, 3, \dots, n, \\ \dot{\hat{x}}_{n+1} = -\lambda_{n+1} \text{sign}(\hat{x}_{n+1} - p_n), \end{cases}\quad (5)$$

where  $\lambda_i$ ,  $i = 1, \dots, n$  are positive constants selected by the designers. Based on this HOSM observer, one has the following two conclusions:

1. There exists a time moment  $T$ , such that for any  $t > T$  (i.e., finite time), one can achieve the following equalities in the absence of input noise by choosing proper  $\lambda_i$ ,  $i = 1, \dots, n$ :

$$\hat{x}_i(t) = \ell_0^{(i-1)}(t), \quad i = 1, 2, \dots, n+1.$$

Furthermore, the solutions of Eq. (5) are finite-time stable based on [31][39].

2. Assume a bounded noise exists, satisfying  $|\ell(t) - \ell_0(t)| \leq \varepsilon$ , then the following inequalities hold in finite time:

$$\begin{aligned}|\hat{x}_i(t) - \ell_0^{(i-1)}(t)| &\leq \varsigma_i \varepsilon^{\frac{n-i+2}{n+2}}, \quad i = 1, 2, \dots, n+1; \\ |p_i - \ell_0^{(i)}(t)| &\leq \mu_i \varepsilon^{\frac{n-i+1}{n+2}}, \quad i = 1, 2, \dots, n,\end{aligned}$$

where  $\varsigma_i$  and  $\mu_i$  are positive constants depending exclusively on the parameters of Eq. (5).

REMARK 3. It is known that standard sliding modes are capable of providing finite time convergence, precise keeping of the constraint and robustness in the case of internal and external disturbances [40][41]. However, this methodology also presents disadvantages, i.e., the relative degree of the constraint must be 1, and the chattering phenomenon is inevitable. Yet HOSM preserves the merits of standard sliding modes and overcomes above disadvantages [30]. At the same time, Lemma 4 means that Eq. (5) are kept in two-sliding mode, i.e.,  $\hat{x}$  can achieve precise estimation of  $x$  in finite time. C2 implies that the observer error will converge to a small bounded region associated with the magnitude of noise in finite time. Without loss of generality, it is assumed that noise exists when measures the system output  $y$ , thus, there is a positive constant  $\nu$  and a time moment  $T$ , such that for any  $t > T$ ,  $\|\hat{x} - x\| \leq \nu$ .

REMARK 4. It's noticeable that Lemma 4 guarantees the trivial realizability of separation principle [32], i.e., the controller and observer can be developed separately. This property allows the control scheme can be developed with a similar thinking to state feedback control. As is pointed out in [30], the constraints to implement Eq. (5) in practice are the requirement of the boundedness of some high order derivatives of the input, and the guarantee of non-happening of finite time escape. It's known to all that there's no system can operate in a infinite operation region in the presence of physical characteristics, therefore, above constraints can be trivially fulfilled by assuming large enough bounds.

### 3. Control Design

In this section, the neural adaptive control is proposed based on the available states first, following on, an output feedback control is investigated by using a HOSM observer to deal with the situation that only output is measurable in practical systems.

Define the tracking error vector as

$$e = x - \bar{y}_r - \chi, \quad (6)$$

where the bounded reference vector  $\bar{y}_r$  is defined as  $\bar{y}_r = [y_r, y_r^{(1)}, \dots, y_r^{(n-1)}]^T$ ,  $\chi = [\chi_1, \chi_2, \dots, \chi_n]^T$  is a auxiliary vector introduced to reduce the influence caused by actuator saturation and is generated by the following auxiliary systems:

$$\dot{\chi} = -B\chi + I\Delta u, \quad (7)$$

where  $B = \begin{bmatrix} b_1 & -1 & & & \\ & b_2 & -1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & -1 \\ & & & & b_n \end{bmatrix}$  is adjustable by user by tuning positive constants  $b_i$ ,  $i = 1, 2, \dots, n$ ,  $I = [0 \dots 0 \ 1]^T$ ,  $\Delta u = v - u$ ,  $v$  is defined in Eq. (2).

REMARK 5. It can be seen that the tracking error defined in Eq. (6) is quite different from the usual form, i.e.,  $e = x - \bar{y}_r$ , which is widely used in existing tracking control [42]. As a matter of fact, it is just the tracking error in the form of Eq. (6) that guarantees the adjustability of the bounded values of compact region in which the tracking error is ultimately confined in. Detail explanations are given below.

A filtered tracking error can be defined as follows:

$$e_s = [\Lambda^T \ 1]e, \quad (8)$$

where  $\Lambda = [\lambda_1, \lambda_2, \dots, \lambda_{n-1}]^T$  is properly chosen vector such that the polynomial  $s^{n-1} + \lambda_{n-1}s^{n-2} + \dots + \lambda_1$  is Hurwitz. In this way, the tracking error  $e$  is bounded with bounded  $e_s$ .



### 3.1. Neural Adaptive Control: State Feedback Case

Differentiating both sides of Eq. (8), one obtains the error dynamic equation:

$$\dot{e}_s = [\Lambda^T \ 1]\dot{e} = [0 \ \Lambda^T]e + f(x) - y_r^{(n)} + u + [\Lambda^T \ 1]B_1\chi, \quad (9)$$

where  $B_1 = [b_1, b_2, \dots, b_n]^T$ , with  $b_i, i = 1, 2, \dots, n$  being the adjustable positive constants used in the matrix  $B$  in Eq. (7).  $f(x)$  is an unknown function approximated by neural networks as  $f(x) = W_1^{*T}S(Z) + \varepsilon_1(Z)$ ,  $\varepsilon_1(Z)$  is a bounded approximation error  $|\varepsilon_1(Z)| \leq \varepsilon_{1n}$  with  $\varepsilon_{1n}$  being small constant.

Design the following neural adaptive control law:

$$\begin{cases} u = -\left(k_1 + \frac{1}{2}\right)e_s - \hat{W}_1^T S(Z) + u_0, \\ u_0 = y_r^{(n)} - [\Lambda^T \ 1]B_1\chi - [0 \ \Lambda^T]e, \end{cases} \quad (10)$$

where  $k_1 > 0$  is a design parameter,  $\hat{W}_1$  is the estimated value of “optimal” weight vector  $W_1^*$ , then, the following theorem holds.

**Theorem 1.** Consider the system described by Eq. (3), in view of any positive constant  $p$ , for initial conditions satisfying  $e_s^2(0) + \tilde{W}_1^T(0)\Gamma_1^{-1}\tilde{W}_1(0) \leq 2p$ , neural adaptive controller Eq. (10) and the neural adaptation laws  $\dot{\hat{W}}_1 = \Gamma_1(S(Z)e_s - \sigma_1\hat{W}_1)$  guarantee the following conclusions, where  $\Gamma_1 = \Gamma_1^T > 0$  is adaptive gain matrix,  $\sigma_1 > 0$  is a small constant introduced as  $\sigma$ -modification [43] to prevent the estimated value  $\hat{W}_1$  from drifting to be very large:

1. All the signals in the closed-loop system are uniformly ultimately bounded;
2. The ultimate tracking error between system output  $y$  and reference signal  $y_r$  is adjustable by the following inequalities:

$$|[\Lambda^T \ 1](x - \bar{y}_r - \chi)| \leq \sqrt{\frac{2\tau_2}{\tau_1}}, \quad \|\chi\| \leq \frac{|\Delta u|}{\prod_{k=1}^n b_k}$$

3. The transient tracking error between system output  $y$  and reference signal  $y_r$  is adjustable by the following inequalities:

$$|[\Lambda^T \ 1](x - \bar{y}_r - \chi)| \leq \sqrt{e_s^2(0) + \tilde{W}_1^T(0)\Gamma_1^{-1}\tilde{W}_1(0) + \frac{2\tau_2}{\tau_1}}, \quad \|\chi\| \leq \frac{|\Delta u|}{\prod_{k=1}^n b_k}$$

$e_s(0)$  and  $\tilde{W}_1(0)$  are the initial values of  $e_s(t)$  and  $\tilde{W}_1(t)$ , respectively, one refers to the proof of this theorem for the definitions of  $\tau_1$  and  $\tau_2$ .

*Proof.* Integrating the control law Eq. (10) and open-loop error dynamic equation Eq. (9), one obtains the following closed-loop error dynamic equation:

$$\dot{e}_s = -\left(k_1 + \frac{1}{2}\right)e_s + \tilde{W}_1^T S(Z) + \varepsilon_1(Z), \quad (11)$$

choosing a Lyapunov candidate

$$V_1 = \frac{1}{2}e_s^2 + \frac{1}{2}\tilde{W}_1^T\Gamma_1^{-1}\tilde{W}_1, \quad (12)$$

the derivative of  $V_1$  along Eq. (11) can be calculated as:

$$\begin{aligned} \dot{V}_1 &= e_s\dot{e}_s + \tilde{W}_1^T\Gamma_1^{-1}\dot{\tilde{W}}_1 \\ &= -\left(k_1 + \frac{1}{2}\right)e_s^2 + e_s\tilde{W}_1^T S(Z) + e_s\varepsilon_1(Z) + \tilde{W}_1^T\Gamma_1^{-1}\dot{\tilde{W}}_1 \end{aligned} \quad (13)$$

By virtue of adaptation law and Young's Inequality [44], Eq. (13) becomes

$$\begin{aligned} \dot{V}_1 &\leq -\left(k_1 + \frac{1}{2}\right)e_s^2 + |e_s|\varepsilon_{1n} + \sigma_1\tilde{W}_1^T\tilde{W}_1 \\ &\leq -k_1e_s^2 + \frac{1}{2}\varepsilon_{1n}^2 + \frac{1}{2}\sigma_1\|\tilde{W}_1\|^2 - \frac{1}{2}\sigma_1\tilde{W}_1^T\tilde{W}_1 \\ &\leq -\tau_1V_1 + \tau_2 \end{aligned} \quad (14)$$

where  $\tau_1 := \min\{k_1, \frac{\sigma_1}{2}\} / \max\{\frac{1}{2}, \frac{\|\Gamma_1^{-1}\|}{2}\}$ ,  $\tau_2 := \frac{1}{2}\varepsilon_{1n}^2 + \frac{1}{2}\sigma_1\|W_1^*\|^2$ , are all constants. Select proper design parameters such that  $\tau_1 > \tau_2/p$ ; as a result,  $\dot{V}_1 \leq 0$  on  $V_1 = p$ , which implies that  $V_1 \leq p$  is an invariant set and guarantees  $V_1(t) \leq p$  for  $V_1(0) \leq p$  and  $t \geq 0$ . In view of Lemma 1 and the definition of  $V_1$ , it is clear that  $V_1$  will be ultimately confined in the closed region  $0 \leq V_1 \leq \frac{\tau_2}{\tau_1}$ , which further guarantees the boundedness of  $e_s$  and  $e$ .  $x$  is also bounded because the input is bounded based on Assumption 1. Subsequently, the signals  $\chi$ ,  $\hat{W}_1$  and  $u$  are guaranteed bounded based on Eq. (6), Eq. (12) and Eq. (10), respectively. From Eq. (14), it can be drawn a conclusion that  $V_1(t)$  is transiently bounded by  $V_1(0) + \tau_2/\tau_1$  from the following inequality:

$$V_1(t) \leq \left(V_1(0) - \frac{\tau_2}{\tau_1}\right) \exp(-\tau_1 t) + \frac{\tau_2}{\tau_1}$$

In view of the definition of  $V_1$ , one has that  $e_s$  is ultimately confined in the closed region  $|e_s| \leq \sqrt{\frac{2\tau_2}{\tau_1}}$ , that is,  $|\Lambda^T 1|e| \leq \sqrt{\frac{2\tau_2}{\tau_1}}$ , and is transiently confined in the closed region  $|\Lambda^T 1|e| \leq \sqrt{2V_1(0) + \frac{2\tau_2}{\tau_1}}$  with  $V_1(0) = (1/2)(e_s^2(0) + \tilde{W}_1^T(0)\Gamma_1^{-1}\tilde{W}_1(0))$ . Actually, we can set  $\chi(0) = 0$  and choose proper initial states such that  $e_s(0) = 0$ , therefore,  $V_1(0)$  is a decreasing function of  $\Gamma_1$ , that is to say, the effect caused by initial neural estimation errors can be attenuated by choosing proper matrix  $\Gamma_1$ . Applying Lemma 1 again, one obtains  $\|\chi\| \leq \frac{\|\Lambda u\|}{\|B\|} = \frac{\|\Lambda u\|}{\prod_{k=1}^n b_k}$ .

The proof is completed.  $\square$

**REMARK 6.** The control scheme given by Theorem 1 is based on the assumption that the state vector  $x$  is available, which is a rigorous condition for many practical plants. At the same time, one cannot simply obtain  $x_i$  for  $i = 2, \dots, n$  by differentiating  $x_{i-1}$  because the measurement noise of output  $x_1$  will be potentially amplified for  $i - 1$  times when calculating  $\dot{x}_i$  using discretization by choosing a small-enough sampling time. This motivates us to develop a output feedback control scheme, which is the topic of the following subsection.

### 3.2. Neural Adaptive Control: Output Feedback Case

In this subsection, output feedback neural adaptive control is developed where  $x$  is not available any more, instead, only measured output  $y$  is available.

To proceed the control design now, define the error between observed states vector and reference vector as

$$\hat{e} = \hat{x} - \bar{y}_r - \chi, \quad (15)$$

and its corresponding filtered error as

$$\hat{e}_s = [\Lambda^T 1]\hat{e}, \quad (16)$$

where  $\hat{x}$  is obtained by virtue of observer given by Eq. (3). Then, one obtains the observer error  $\tilde{x}$  and  $\tilde{e}_s$  as

$$\tilde{x} = e - \hat{e} = x - \hat{x}, \quad \tilde{e}_s = e_s - \hat{e}_s = [\Lambda^T 1]\tilde{x}. \quad (17)$$

In view of Eq. (3) and Eq. (8), the error dynamic equation is obtained as

$$\dot{e}_s = [\Lambda^T 1]\dot{e} = [0 \ \Lambda^T]e + f(x) - y_r^{(n)} + u + [\Lambda^T 1]B_1\chi, \quad (18)$$

where  $B_1 = [b_1, b_2, \dots, b_n]^T$ , with  $b_i$ ,  $i = 1, 2, \dots, n$  being the adjustable positive constants used in the matrix  $B$  in Eq. (7). Based on Eq. (17), Eq. (18) becomes

$$\dot{e}_s = [0 \ \Lambda^T]\hat{e} + f(x) - y_r^{(n)} + u + [\Lambda^T 1]B_1\chi + [0 \ \Lambda^T]\tilde{x}, \quad (19)$$

where  $f(x)$  is an unknown function approximated by neural networks as  $f(x) = W_2^{*T}S(Z) + \varepsilon_2(Z)$ , and  $\varepsilon_2(Z)$  is bounded by small constant  $\varepsilon_{2n}$ .

**REMARK 7.** Although  $\varepsilon_2(Z)$  is defined as the approximation error of neural networks, it can be seen as one or mixture of external disturbance, modeling uncertainties and approximation error.

For the system described by Eq. (3) with observer given by Eq. (3), the following neural adaptive control is designed:

$$\begin{cases} u = -\left(k_2 + \frac{1}{2}\right)\hat{e}_s - \hat{W}_2^T S(Z) + u_0, \\ u_0 = y_r^{(n)} - [\Lambda^T \ 1]B_1\chi - [0 \ \Lambda^T]\hat{e}, \end{cases} \quad (20)$$

where  $k_2 > 0$  is a positive constant chosen by designer,  $\hat{W}_2$  is the estimation of the optimal weight vector  $W_2^*$ ,  $S(Z)$  is the basis functions vector of neural networks with the input  $Z = [x, \hat{e}]$ ,  $S(Z)$  is bounded by  $\|S(Z)\| \leq \varepsilon_b$ , other notations hold the same definition as above. The neural networks weights are updated by:

$$\dot{\hat{W}}_2 = \Gamma_2(\hat{e}_s S^T(Z) - \sigma_2|\hat{e}_s|\hat{W}_2). \quad (21)$$

The estimated value  $\hat{W}_2$  in Eq. (21) is bounded by  $\|\hat{W}_2\| \leq \varepsilon_b/\sigma_1$ . This proposition can be proved by virtue of Lemma 2. Therefore, the estimated error  $\hat{W}_2$  is bounded by  $\|\hat{W}_2\| \leq \varepsilon_N + \varepsilon_b/\sigma_2$ .

The following theorem holds.

**Theorem 2.** Consider the system described by Eq. (3) with observer given in Eq. (3), in view of any positive constant  $q$ , for initial conditions satisfying  $e_s^2(0) \leq 2q$ , neural adaptive controller Eq. (20) and adaptation laws Eq. (21) guarantee the following conclusions:

1. All the signals in the closed-loop system are uniformly ultimately bounded;
2. The ultimate tracking error between system output  $y$  and reference signal  $y_r$  is adjustable by the following inequalities:

$$|[\Lambda^T \ 1](x - \bar{y}_r - \chi)| \leq \sqrt{\frac{\theta}{k_2}}, \quad \|\chi\| \leq \frac{|\Delta u|}{\prod_{k=1}^n b_k}.$$

3. The transient tracking error between system output  $y$  and reference signal  $y_r$  is adjustable by the following inequalities:

$$|[\Lambda^T \ 1](x - \bar{y}_r - \chi)| \leq \sqrt{e_s^2(0) + \frac{\theta}{k_2}}, \quad \|\chi\| \leq \frac{|\Delta u|}{\prod_{k=1}^n b_k}$$

one refers to the proof of this theorem for the definitions of  $\theta$ .

*Proof.* By integrating Eq. (19) and Eq. (20), the closed-loop error system is obtained as:

$$\dot{e}_s = -\left(k_2 + \frac{1}{2}\right)\hat{e}_s + \tilde{W}_2^T S(Z) + \varepsilon_2(Z) + [0 \ \Lambda^T]\tilde{x}. \quad (22)$$

Consider the Lyapunov candidate  $V_2 = \frac{1}{2}e_s^2$ , its derivative along Eq. (22) can be calculated as:

$$\begin{aligned} \dot{V}_2 &= -\left(k_2 + \frac{1}{2}\right)e_s\hat{e}_s + e_s \left\{ \tilde{W}_2^T S(Z) + \varepsilon_2(Z) + [0 \ \Lambda^T]\tilde{x} \right\} \\ &= -\left(k_2 + \frac{1}{2}\right)e_s^2 + e_s \left\{ \left(k_2 + \frac{1}{2}\right)[\Lambda^T \ 1]\tilde{x} + \tilde{W}_2^T S(Z) + \varepsilon_2(Z) + [0 \ \Lambda^T]\tilde{x} \right\} \\ &\leq -\left(k_2 + \frac{1}{2}\right)e_s^2 + |e_s| \left[ \left(k_2 + \frac{1}{2}\right)v_1 + \varepsilon_N + \frac{\varepsilon_b}{\sigma_2} + \varepsilon_{2n} + v_2 \right] \end{aligned} \quad (23)$$

where  $v_1 := \max\{\lambda_1, \dots, \lambda_{i-1}, 1\}v \geq |[\Lambda^T \ 1]\tilde{x}|$ ,  $v_2 := \max\{\lambda_1, \dots, \lambda_{i-1}\}v \geq |[0 \ \Lambda^T]\tilde{x}|$ . In view of Young's Inequality [44], one has

$$\dot{V}_2 \leq -2k_2 V_2 + \theta \quad (24)$$

where  $\theta := \frac{1}{2} \left[ \left(k_2 + \frac{1}{2}\right)v_1 + \varepsilon_N + \frac{\varepsilon_b}{\sigma_2} + \varepsilon_n + v_2 \right]^2$  is an unknown constant. By selecting proper design parameters satisfying  $k_2 \geq \theta/(2q)$ , we guarantee that  $V_2 \leq q$  is an invariant set. The remain of the proof is similar to the proof of Theorem 1, the conclusions in Theorem 2 can be easily obtained, which is not discussed in detail here.

The proof is completed.  $\square$

REMARK 8. Since the approximation property of RBF NNs in Lemma 3 is only guaranteed in a compact set, and the initial value of tracking error should be located in the domain of attraction to satisfy Assumption 1, the stability results in this work, therefore, is semi-global.

REMARK 9. In Theorem 1 and Theorem 2, initial conditions are all imposed. Since  $p$  and  $q$  can be selected large enough and practical states in applications are always finite, the initial conditions are not restrictive in this sense. We can use the technique in [22] to choose initial conditions properly, which is not discussed in detail here.

#### 4. Numerical examples

In this section, simulation results are presented to illustrate the validity of Theorem 1 and Theorem 2. First, the results are applied to a nonlinear system in the form of Eq. (3), then, the controllers are directly applied to a nonlinear system in the form of Eq. (1) without adjusting any design parameters, which demonstrates the effectiveness of the developed control methods.

##### 4.1. Example 1

The nonlinear system considered is described by the following equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 0.2(x_1^2 + x_2^2) + v \\ y = x_1 \end{cases}$$

the upper and lower bounded values of actuator are  $u^+ = 1$ ,  $u^- = -1.5$ , respectively. The reference signal is  $y_r = \sin(t)$ .

The parameters used in simulations are given below. Adjustable parameters are chosen as:  $k_1 = k_2 = 15$ ,  $\lambda_1 = 2$ ,  $b_1 = 1$ ,  $b_2 = 2$ ,  $\sigma_1 = \sigma_2 = 0.001$ ,  $\Gamma_1 = \Gamma_2 = \text{diag}\{1\}$ . Initial values are chosen as:  $x_1(0) = 0.5$ ,  $x_2(0) = 0.5$ ,  $\chi_1(0) = 0$ ,  $\chi_2(0) = 0$ ,  $\hat{W}_1 = \hat{W}_2 = 0$ . The neural networks contain 41 nodes, with centers evenly spaced in  $[-20, 20]$  and widths are all 2. The NN input vector  $Z = [x_1, x_2]^T$ . The observer used in Theorem 2 is constructed as:

$$\begin{cases} \dot{\hat{x}}_1 = p_1 \\ p_1 = -12|\hat{x}_1 - x_1|^{\frac{2}{3}} \text{sign}(\hat{x}_1 - x_1) + \hat{x}_2 \\ \dot{\hat{x}}_2 = p_2 \\ p_2 = -15|\hat{x}_2 - p_1|^{\frac{1}{2}} \text{sign}(\hat{x}_2 - p_1) + \hat{x}_3 \\ \dot{\hat{x}}_3 = -20\text{sign}(\hat{x}_3 - p_2) \end{cases}$$

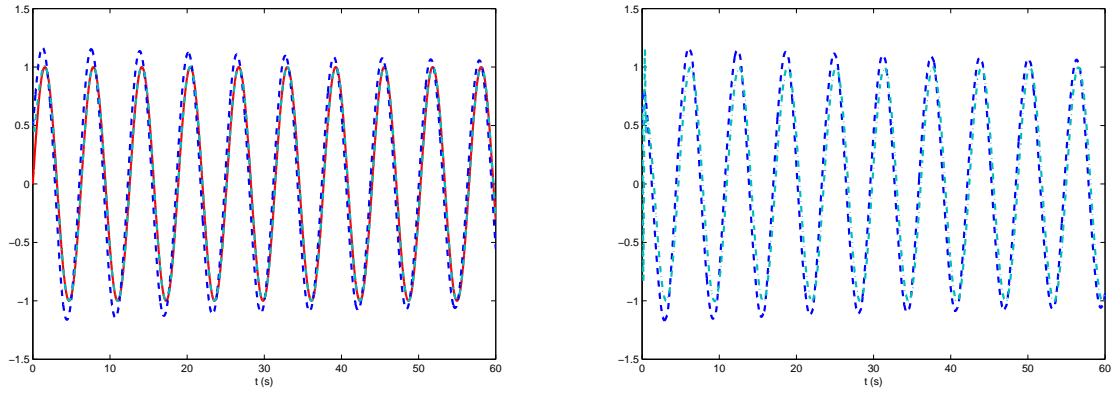
Note that the parameters used in the simulations of Theorem 1 and Theorem 2 are exactly the same for justice and equity, except for some new introduced parameters in HOSM observer.

Figure 1 illustrates the simulation results of example 1. It can be seen from Figure 1 (a) that the tracking performance of Theorem 1 and Theorem 2 is satisfactory. Figure 1 (b) presents the trajectories of  $x_2$  in two controllers. The trajectories of control inputs are shown in Figure 1 (c), it is observed the designed control laws  $u$  do exceed the output capacity of actuator. Even in this situation, the developed methods guarantee satisfactory systematic performance by virtue of properly designed auxiliary system, trajectories of auxiliary signals  $\chi_1$  and  $\chi_2$  are given in Figure 1 (d). Figure 1 (e) shows the trajectories of norms of neural networks weights. Estimation performance can be observed in Figure 1 (f).

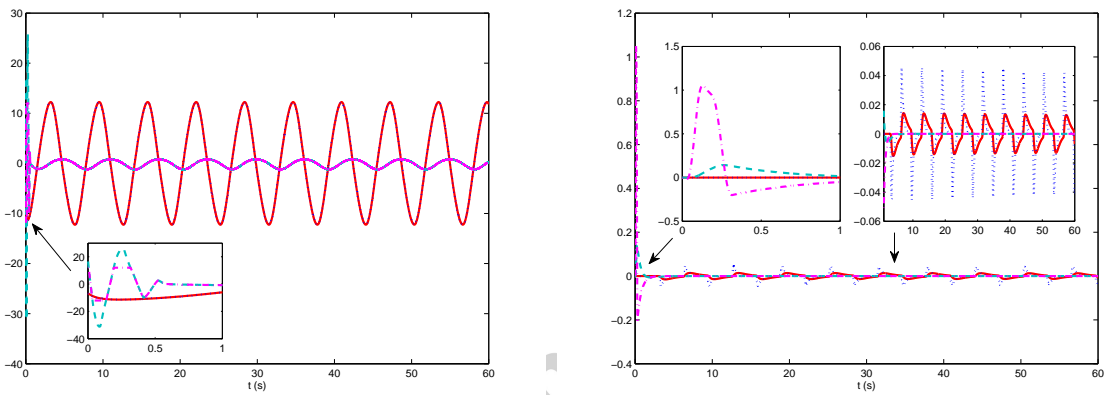
##### 4.2. Example 2

Consider the following nonlinear system:

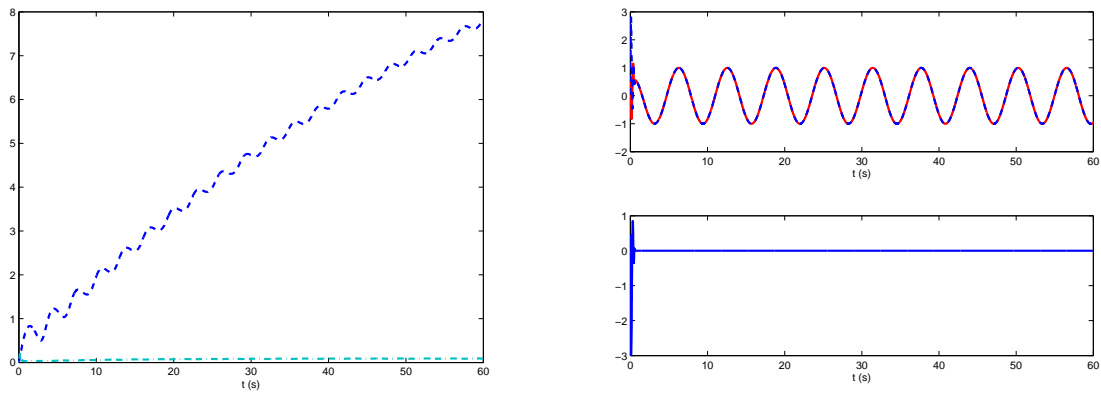
$$\begin{cases} \dot{x}_1 = -0.5 \sin(0.5x_1) + x_2 \\ \dot{x}_2 = \cos(0.5x_1x_2) + 0.2 \sin(0.5x_1x_2) + v \\ y = x_1 \end{cases}$$



(a) Tracking performance. Solid line: reference  $y$ , dash line: output  $y$  using Theorem 1, dot-dash line: output  $y$  using Theorem 2. (b) Trajectories of  $x_2$ . Dash line: Theorem 1, dot-dash line: Theorem 2.



(c) Control signals. Solid line:  $u$  using Theorem 1, dot line:  $v$  using Theorem 1, dash line:  $u$  using Theorem 2, dot-dash line:  $v$  using Theorem 2. (d) Trajectories of  $\chi_1$  and  $\chi_2$ . Solid line:  $\chi_1$  using Theorem 1, dot line:  $\chi_2$  using Theorem 1, dash line:  $\chi_1$  using Theorem 2, dot-dash line:  $\chi_2$  using Theorem 2.



(e) Norm of neural networks weights. Dash line: Theorem 1, dot-dash line: Theorem 2. (f) (i). Solid line:  $x_2$  using Theorem 2, dash line:  $\hat{x}_2$  using Theorem 2; (ii). Estimation error.

Figure 1. Simulation results of example 1.

The upper and lower bounded values of actuator are set to be  $u^+ = 2$ ,  $u^- = -2$ , respectively. To verify the generality of the developed controllers, all the parameters used here are the same as the ones in example 1. It's notable that the controllers used in the example 1 are directly applied to this example.

Figure 2 demonstrates the simulation results of example 2. It can be seen from Figure 2 (a) that the tracking performance of Theorem 1 and Theorem 2 is satisfactory. Figure 2 (b) presents the trajectories of  $x_2$  in two controllers. The trajectories of control inputs are given in Figure 2 (c), it is observed the designed control laws  $u$  exceed the output capacity of actuator. Trajectories of auxiliary signals  $\chi_1$  and  $\chi_2$  are given in Figure 2 (d). Figure 2 (e) shows the trajectories of norms of neural networks weights. Estimation performance can be observed in Figure 2 (f).

Adaptive (observer) control methods are used to provide comparative results by setting  $\chi = 0$ , Figure 3 (a) presents the tracking error  $e_s$  in example 1 under saturation value 11, it is observed that the proposed controllers guarantee better performance than adaptive method. While Figure 3 (b) demonstrates an apparent comparative results in example 2 using Theorem 2 and adaptive observer method.

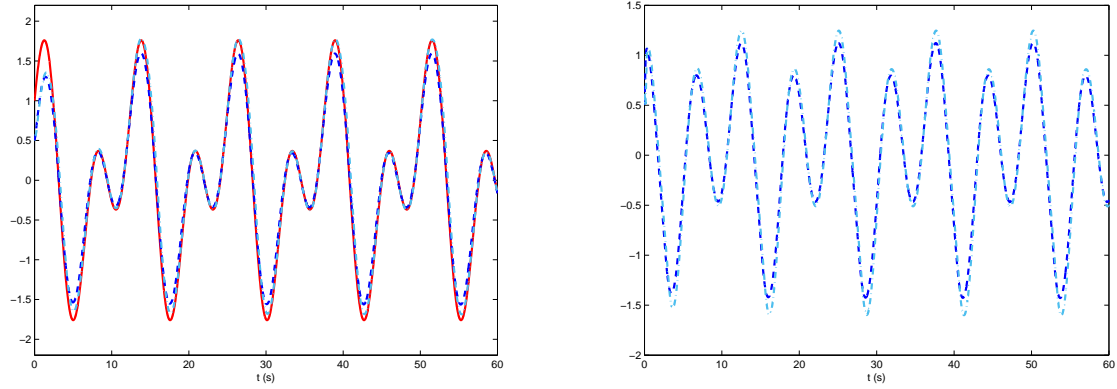
Until here, the effectiveness of Theorem 1 and Theorem 2 is well demonstrated.

## 5. Concluding Remarks

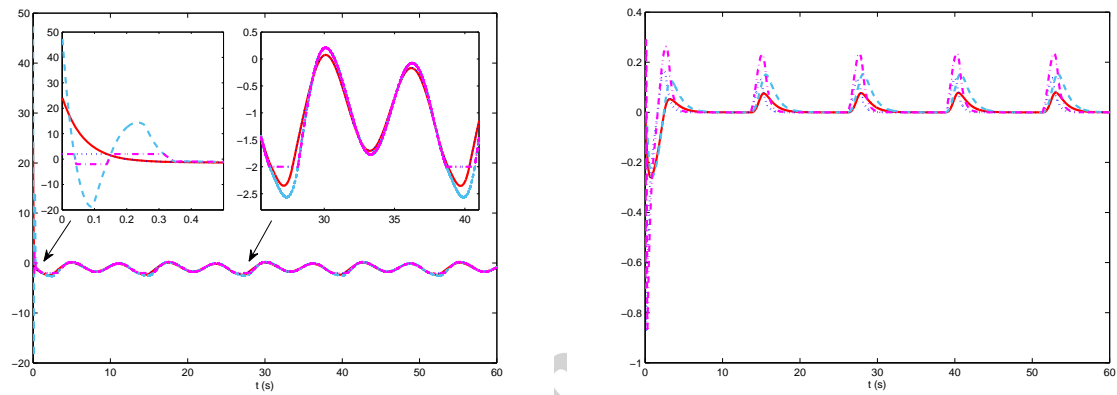
This paper proposes two neural adaptive control methods for a class of nonlinear system by state feedback and output feedback. The input to the controlled plant is constrained by actuator saturation, which is generally encountered in practical engineering, because it's impossible to implement unlimited control signal caused by the limited output capacity of physical actuators. The neural networks don't require an off-line training phase. Although asymptotical stability is not achieved, the overall system is proved to be uniformly ultimately bounded and the tracking error can be adjusted to closed regions which is dependent to parameters left to users. This is of high significance in practice. The theoretical analysis and simulation results prove the effectiveness of the proposed schemes. The methodological design in this paper can be extended to the time-delay systems if proper Lyapunov-Krasovskii function is developed and utilized, which is open problem and will be studied and reported in our future works.

## References

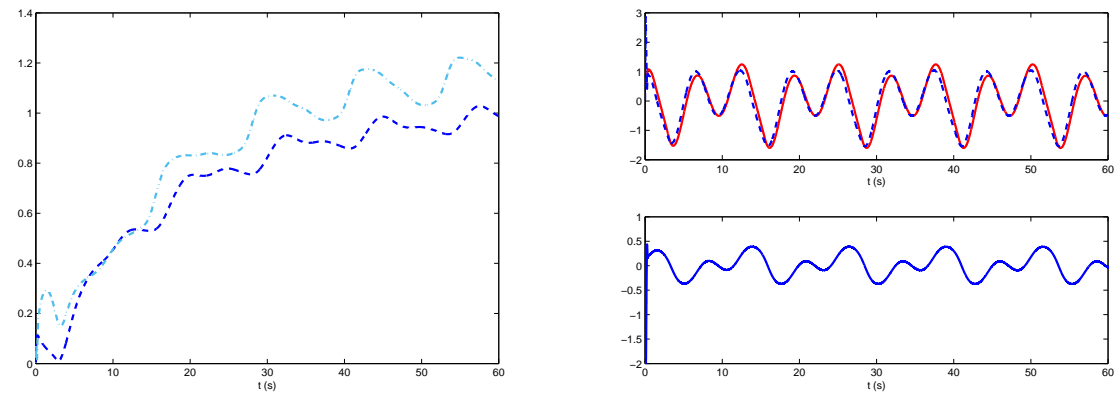
- [1] Y. Li, S. Tong, Y. Liu, T. Li, Adaptive fuzzy robust output feedback control of nonlinear systems with unknown dead zones based on a small-gain approach, *Fuzzy Systems, IEEE Transactions on* 22 (1) (2014) 164–176.
- [2] S. Tong, Y. Li, P. Shi, Observer-based adaptive fuzzy backstepping output feedback control of uncertain mimo pure-feedback nonlinear systems, *IEEE Transactions on Fuzzy Systems* 20 (4) (2012) 771–785.
- [3] F. Chen, F. Lu, B. Jiang, G. Tao, Adaptive compensation control of the quadrotor helicopter using quantum information technology and disturbance observer, *Journal of the Franklin Institute* 351 (1) (2014) 442–455.
- [4] F. Chen, S. Zhang, B. Jiang, G. Tao, Multiple model-based fault detection and diagnosis for helicopter with actuator faults via quantum information technique, *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering* 228 (3) (2014) 182–190.
- [5] F. Chen, B. Jiang, G. Tao, An intelligent self-repairing control for nonlinear mimo systems via adaptive sliding mode control technology, *Journal of the Franklin Institute* 351 (1) (2014) 399–411.
- [6] F. Chen, B. Jiang, G. Tao, Fault self-repairing flight control of a small helicopter via fuzzy feedforward and quantum control techniques, *Cognitive Computation* 4 (4) (2012) 543–548.
- [7] S. Gao, H. Dong, B. Ning, Y. Chen, X. Sun, Adaptive fault-tolerant automatic train operation using rbf neural networks, *Neural Computing and Applications* 26 (1) (2015) 141–149.
- [8] L. Hunt, G. Meyer, Stable inversion for nonlinear systems, *Automatica* 33 (8) (1997) 1549–1554.
- [9] Q. Khan, A. I. Bhatti, A. Ferrara, Dynamic sliding mode control design based on an integral manifold for nonlinear uncertain systems, *Journal of Nonlinear Dynamics* 2014.
- [10] J. Wu, W. Chen, D. Zhao, J. Li, Globally stable direct adaptive backstepping nn control for uncertain nonlinear strict-feedback systems, *Neurocomputing* 122 (2013) 134–147.
- [11] S. Tong, S. Sui, Y. Li, Fuzzy adaptive output feedback control of mimo nonlinear systems with partial tracking errors constrained, *IEEE Transactions on Fuzzy Systems* 00 (2015) in press.
- [12] Y. Li, S. Tong, T. Li, Observer-based adaptive fuzzy tracking control of mimo stochastic nonlinear systems with unknown control direction and unknown dead-zones, *IEEE Transactions on Fuzzy Systems* 00 (2015) in press.
- [13] S. Tong, Y. Li, Adaptive fuzzy output feedback control of mimo nonlinear systems with unknown dead-zone inputs, *IEEE Transactions on Fuzzy Systems* 21 (1) (2013) 134–146.
- [14] B. Yao, M. Tomizuka, Adaptive robust control of mimo nonlinear systems in semi-strict feedback forms, *Automatica* 37 (9) (2001) 1305–1321.
- [15] S.-C. Tong, Y.-M. Li, G. Feng, T.-S. Li, Observer-based adaptive fuzzy backstepping dynamic surface control for a class of mimo nonlinear systems, *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics* 41 (4) (2011) 1124–1135.



(a) Tracking performance. Solid line: reference  $y$ , dash line: output  $y$  using Theorem 1, dot-dash line: output  $y$  using Theorem 2. (b) Trajectories of  $x_2$ . Dash line: Theorem 1, dot-dash line: Theorem 2.

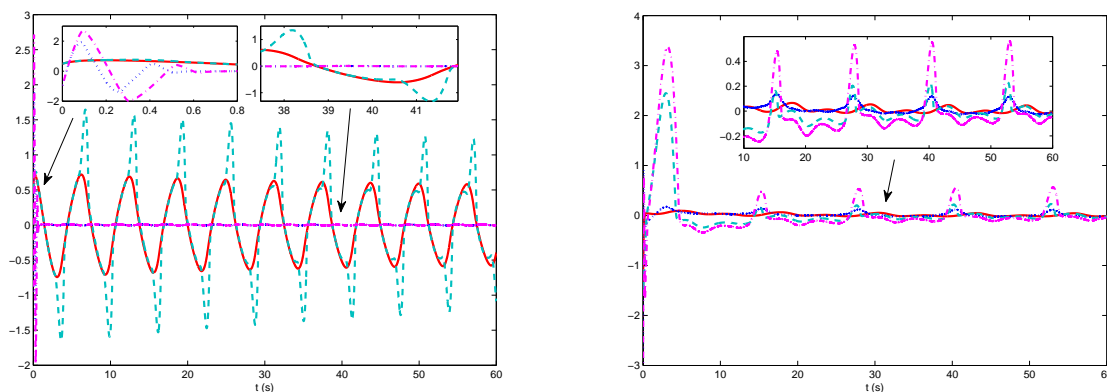


(c) Control signals. Solid line:  $u$  using Theorem 1, dot line:  $v$  using Theorem 1, dash line:  $u$  using Theorem 2, dot-dash line:  $v$  using Theorem 2. (d) Trajectories of  $x_1$  and  $x_2$ . Solid line:  $x_1$  using Theorem 1, dot line:  $x_2$  using Theorem 1, dash line:  $x_1$  using Theorem 2, dot-dash line:  $x_2$  using Theorem 2.



(e) Norm of neural networks weights. Dash line: Theorem 1, dot-dash line: Theorem 2. (f) (i). Solid line:  $x_2$  using Theorem 2, dash line:  $\hat{x}_2$  using Theorem 2; (ii). Estimation error.

Figure 2. Simulation results of example 2.



(a) Trajectories of  $e_s$  of example 1. Solid line: Theorem 1, dash line: (b) Trajectories of  $e_s$  of example 2. Solid line: Theorem 1, dash line: adaptive control, dot line: Theorem 2, dot-dash line: adaptive observer control.

Figure 3. Comparative results.

- [16] M. Polycarpou, J. Farrell, M. Sharma, On-line approximation control of uncertain nonlinear systems: issues with control input saturation, in: American Control Conference, 2003. Proceedings of the 2003, Vol. 1, IEEE, 2003, pp. 543–548.
- [17] R. Monopoli, Adaptive control for systems with hard saturation, in: Decision and Control including the 14th Symposium on Adaptive Processes, 1975 IEEE Conference on, Vol. 14, IEEE, 1975, pp. 841–843.
- [18] J. Farrell, M. Sharma, M. Polycarpou, Backstepping-based flight control with adaptive function approximation, *Journal of Guidance, Control, and Dynamics* 28 (6) (2005) 1089–1102.
- [19] E. N. Johnson, A. J. Calise, Neural network adaptive control of systems with input saturation, in: American Control Conference, 2001. Proceedings of the 2001, Vol. 5, IEEE, 2001, pp. 3527–3532.
- [20] E. N. Johnson, A. J. Calise, Pseudo-control hedging: a new method for adaptive control, in: Advances in Navigation Guidance and Control Technology Workshop, 2000, pp. 1–2.
- [21] D. Lin, X. Wang, Y. Yao, Fuzzy neural adaptive tracking control of unknown chaotic systems with input saturation, *Nonlinear Dynamics* 67 (4) (2012) 2889–2897.
- [22] D. Swaroop, J. C. Gerdes, P. Yip, J. Hedrick, Dynamic surface control of nonlinear systems, in: American Control Conference, 1997. Proceedings of the 1997, Vol. 5, IEEE, 1997, pp. 3028–3034.
- [23] P. P. Yip, J. K. Hedrick, Adaptive dynamic surface control: a simplified algorithm for adaptive backstepping control of nonlinear systems, *International Journal of Control* 71 (5) (1998) 959–979.
- [24] D. Wang, J. Huang, Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems in strict-feedback form, *Neural Networks, IEEE Transactions on* 16 (1) (2005) 195–202.
- [25] D. Swaroop, J. Hedrick, P. Yip, J. Gerdes, Dynamic surface control for a class of nonlinear systems, *Automatic Control, IEEE Transactions on* 45 (10) (2000) 1893–1899.
- [26] M. Chen, S. S. Ge, B. Voon Ee How, Robust adaptive neural network control for a class of uncertain mimo nonlinear systems with input nonlinearities, *Neural Networks, IEEE Transactions on* 21 (5) (2010) 796–812.
- [27] Y. Li, S. Tong, T. Li, Composite adaptive fuzzy output feedback control design for uncertain nonlinear strict-feedback systems with input saturation., *IEEE transactions on cybernetics* 00 (2015) in press.
- [28] T.-P. Zhang, Q. Zhu, Y.-Q. Yang, Adaptive neural control of non-affine pure-feedback non-linear systems with input nonlinearity and perturbed uncertainties, *International Journal of systems science* 43 (4) (2012) 691–706.
- [29] J.-H. Park, S.-H. Kim, C.-J. Moon, Adaptive neural control for strict-feedback nonlinear systems without backstepping, *Neural Networks, IEEE Transactions on* 20 (7) (2009) 1204–1209.
- [30] A. Levant, Robust exact differentiation via sliding mode technique, *Automatica* 34 (3) (1998) 379–384.
- [31] A. Levant, Higher-order sliding modes, differentiation and output-feedback control, *International Journal of control* 76 (9-10) (2003) 924–941.
- [32] H. K. Khalil, J. Grizzle, *Nonlinear systems*, Vol. 3, Prentice hall Upper Saddle River, 2002.
- [33] Y. Li, S. Tong, T. Li, Direct adaptive fuzzy backstepping control of uncertain nonlinear systems in the presence of input saturation, *Neural Computing and Applications* 23 (5) (2013) 1207–1216.
- [34] B. Yao, M. Tomizuka, Adaptive robust control of siso nonlinear systems in a semi-strict feedback form, *Automatica* 33 (5) (1997) 893–900.
- [35] D. Wang, J. Huang, Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems in strict-feedback form, *Neural Networks, IEEE Transactions on* 16 (1) (2005) 195–202.
- [36] S. S. Ge, C. Wang, Adaptive nn control of uncertain nonlinear pure-feedback systems, *Automatica* 38 (4) (2002) 671–682.
- [37] C. Wen, J. Zhou, Z. Liu, H. Su, Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance, *Automatic Control, IEEE Transactions on* 56 (7) (2011) 1672–1678.
- [38] R. M. Sanner, J.-J. Slotine, Gaussian networks for direct adaptive control, *Neural Networks, IEEE Transactions on* 3 (6) (1992) 837–863.



- [39] L. Rosier, Homogeneous lyapunov function for homogeneous continuous vector field, *Systems & Control Letters* 19 (6) (1992) 467–473.
- [40] V. I. Utkin, Variable structure systems with sliding modes, *IEEE Transactions on Automatic Control* 22 (2) (1977) 212–222.
- [41] W. Perruquetti, J.-P. Barbot, *Sliding mode control in engineering*, CRC Press, 2002.
- [42] S. Tong, T. Wang, J. T. Tang, Fuzzy adaptive output tracking control of nonlinear systems, *Fuzzy Sets and Systems* 111 (2) (2000) 169–182.
- [43] P. A. Ioannou, J. Sun, *Robust adaptive control*, Courier Dover Publications, 2012.
- [44] S. S. Ge, F. Hong, T. H. Lee, Robust adaptive control of nonlinear systems with unknown time delays, *Automatica* 41 (7) (2005) 1181–1190.

Accepted manuscript