# UNIVERSITYOF <br> BIRMINGHAM <br> University of Birmingham Research at Birmingham 

# Self-adaptation via multi-objectivisation: an empirical study 

Qin, Xiaoyu; Lehre, Per Kristian

DOI:
10.1007/978-3-031-14714-2_22

License:
None: All rights reserved

## Document Version <br> Other version

Citation for published version (Harvard):
Qin, X \& Lehre, PK 2022, Self-adaptation via multi-objectivisation: an empirical study. in G Rudolph, AV
Kononova, H Aguirre, P Kerschke, G Ochoa \& T Tušar (eds), Parallel Problem Solving from Nature - PPSN XVII: 17th International Conference, PPSN 2022, Dortmund, Germany, September 10-14, 2022, Proceedings, Part I. 1 edn, Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), vol. 13398 LNCS, Springer, pp. 308-323, The seventeenth International Conference on Parallel Problem Solving from Nature, Dortmund, Germany, 10/09/22.
https://doi.org/10.1007/978-3-031-14714-2_22

Link to publication on Research at Birmingham portal

## General rights

Unless a licence is specified above, all rights (including copyright and moral rights) in this document are retained by the authors and/or the copyright holders. The express permission of the copyright holder must be obtained for any use of this material other than for purposes permitted by law.

- Users may freely distribute the URL that is used to identify this publication.
- Users may download and/or print one copy of the publication from the University of Birmingham research portal for the purpose of private study or non-commercial research.
$\bullet$ User may use extracts from the document in line with the concept of 'fair dealing' under the Copyright, Designs and Patents Act 1988 (?)
-Users may not further distribute the material nor use it for the purposes of commercial gain.
Where a licence is displayed above, please note the terms and conditions of the licence govern your use of this document.
When citing, please reference the published version.


## Take down policy

While the University of Birmingham exercises care and attention in making items available there are rare occasions when an item has been uploaded in error or has been deemed to be commercially or otherwise sensitive.
If you believe that this is the case for this document, please contact UBIRA@lists.bham.ac.uk providing details and we will remove access to the work immediately and investigate.

## Appendix 1 Omitted algorithms in this paper

```
Algorithm 3 Multi-objective sorting mechanism [33]
Require: Population sizes \(\lambda \in \mathbb{N}\). Population \(P_{t} \in \mathcal{Y}^{\lambda}\). Fitness function \(f\).
    1: Sort \(P_{t}\) into strict non-dominated fronts \(\mathcal{F}_{0}^{t}, \mathcal{F}_{1}^{t}, \ldots\) based on \(f_{1}(x, \chi):=f(x)\) and
    \(f_{2}(x, \chi):=\chi\).
    for \(\mathcal{F}=\mathcal{F}_{0}^{t}, \mathcal{F}_{1}^{t}, \ldots\) do
        Sort \(\mathcal{F}\) such that \(f_{1}(\mathcal{F}(1))>f_{1}(\mathcal{F}(2))>\ldots\)
    \(P_{t}:=\left(\mathcal{F}_{0}^{t}, \mathcal{F}_{1}^{t}, \ldots\right)\).
    return \(P_{t}\).
```

```
Algorithm 4 Strict non-dominated sorting [33]
Require: Population sizes \(\lambda \in \mathbb{N}\). Population \(P \in \mathcal{Z}^{\lambda}\), where \(\mathcal{Z}\) is a finite state
    space. Objective functions \(f_{1}, f_{2}, \ldots: \mathcal{Z} \rightarrow \mathbb{R}\) (assume to maximise all objective
    functions).
    for each individual \(P(i)\) do
        Set \(S_{i}:=\emptyset\) and \(n_{i}:=0\).
    for \(i=1, \ldots, \lambda\) do
        for \(j=1, \ldots, \lambda\) do
            if \(P(i) \prec P(j)\) based on \(f_{1}, f_{2}, \ldots\) then
                \(S_{i}:=S_{i} \cup\{P(i)\}\),
            else if \(P(j) \prec P(i)\) based on \(f_{1}, f_{2}, \ldots\) then
                \(n_{i}:=n_{i}+1\),
            else if \(f_{\ell}(P(i))=f_{\ell}(P(j))\) where \(\ell=1,2, \ldots\) then
                if \(P(i) \notin S_{j}\) then \(S_{i}:=S_{i} \cup\{P(i)\}\) else \(n_{i}:=n_{i}+1\).
        if \(n_{i}=0\) then \(\mathcal{F}_{0}=\mathcal{F}_{0} \cup\{P(i)\}\).
    Set \(k:=0\).
    while \(\mathcal{F}_{k} \neq \emptyset\) do
        \(Q:=\emptyset\).
        for each individual \(P(i) \in \mathcal{F}_{k}\) and \(P(j) \in S_{i}\) do
            Set \(n_{j}:=n_{j}-1\).
            if \(n_{j}=0\) then \(Q:=Q \cup\{P(j)\}\).
        Set \(k:=k+1, \mathcal{F}_{k}:=Q\).
    return \(\mathcal{F}_{0}, \mathcal{F}_{1}, \ldots\).
```

```
Algorithm 5 Multi-objective sorting mechanism (alternative)
Require: Population sizes \(\lambda \in \mathbb{N}\). Population \(P_{t} \in \mathcal{Y}^{\lambda}\). Fitness function \(f\).
    1: Sort \(P_{t}\) into \(P_{t}^{1}, P_{t}^{1}, \ldots\) where \(P_{t}^{1}\) containing all individuals with the highest fitness
    \(f, P_{t}^{2}\) containing all individuals with the 2nd highest fitness \(f, \ldots\).
    for \(i=1, \ldots, \lambda\) do
        Set \(\hat{\chi}:=-\infty\).
        for \(Q=P_{t}^{1}, P_{t}^{1}, \ldots\) do
            Find \(\left(x^{\prime}, \chi^{\prime}\right)\) which is the element with the highest \(\chi\) in \(Q\).
            if \(Q \neq \emptyset\) and \(\chi^{\prime}>\hat{\chi}\) then
                    \(P_{t}(i):=\left(x^{\prime}, \chi^{\prime}\right)\) and \(\hat{\chi}:=\chi^{\prime}\).
                    Pop ( \(x^{\prime}, \chi^{\prime}\) ) from \(Q\).
                    Break.
    return \(P_{t}\).
```

```
Algorithm \(6(\mu, \lambda)\) selection
Require: Population size \(\lambda \in \mathbb{N}\). Parameter \(\mu \in[\lambda]^{3}\).
    \(I_{t} \sim \operatorname{Unif}([\mu])\).
    return \(I_{t}\).
```

```
Algorithm 7 Fitness-first sorting mechanism [7]
Require: Population sizes \(\lambda \in \mathbb{N}\). Population \(P_{t} \in \mathcal{Y}^{\lambda}\). Fitness function \(f\).
    1: Sort \(P_{t}\) such that \(P_{t}(1) \succeq \cdots \succeq P_{t}(\lambda)\), according to
        \((x, \chi) \succeq\left(x^{\prime}, \chi^{\prime}\right) \Leftrightarrow f(x)>f\left(x^{\prime}\right) \vee\left(f(x)=f\left(x^{\prime}\right) \wedge \chi \geq \chi^{\prime}\right)\).
    return \(P_{t}\).
```

[^0]
## Appendix 2 Omitted statistical results of experiments

Table 2: Statistical results of experiments on random NK-LANDSCAPE problems. The $p$-values of each algorithm come from Wilcoxon rank-sum tests between the algorithm and MOSA-EA.

| $k$ | Stat. | RS | cGA | UMDA | RLS | SA-( $1, \lambda$ ) EA | $(1+1)$ EA | FastGA ( $1+(\lambda, \lambda)$ ) GA |  | $(\mu, \lambda)$ EA | 3-tour.EA | MOSA-EA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Median | 66.6591 | 72.9964 | 74.8631 | 71.3547 | 74.8418 | 76.6613 | 76.9230 | 79.2846 | 78.2089 | 79.2846 | 79.2846 |
|  | 'p-value ${ }^{\text {- }}$ | $2.1 \bar{e}-\overline{22}$ | ${ }^{2} .3$ è $\overline{0} 4$ | $\overline{0} \overline{0} 2 \overline{1} \overline{3}$ | $6.5 \overline{0}-\overline{8}$ | ${ }^{0} \cdot \overline{0} \overline{2} 2 \overline{6}$ | $0.2 \overline{6} 6 \overline{8}$ | $0.42 \overline{15}$ | 0.9299 | $\overline{0} .7985$ | 0.8805 |  |
|  | Median | 66.4442 | 69.5499 | 73.2968 | 68.3100 | 71.0248 | 75.5792 | 76.1340 | 77.1520 | 79.2680 | 78.7832 | 82.527 |
|  | p-vālue ${ }^{-1}$ | $2.6 \overline{6}-34^{-}$ | ${ }^{1} 1.5 \overline{5}^{-2} \overline{2}^{-}$ | $2.0 \overline{0}-15$ | 2. 6 e- $\overline{3} \overline{4}$ | $\overline{3} .5 \overline{\text { e }}$-3 ${ }^{\text {a }}$ | $5 . \overline{0}{ }^{-1} \overline{8}$ | $1.1 \overline{1}-1 \overline{1}^{-}$ | 2.2e-09 ${ }^{\text {a }}$ | $\overline{0} \overline{0} 0 \overline{3} 0^{\circ}$ | 0.0063 |  |
|  | Median | 66.2055 | 66.5517 | 70.9576 | 66.4446 | 67.8968 | ${ }^{73.7253}$ | 74.2253 | 74.6407 | 76.0777 | 76.9053 | 80.441 |
|  | $p$-value ${ }^{\text {- }}$ | $2.6 \overline{6}-34{ }^{-}$ | ${ }^{-2.6 \overline{6}} \overline{34}$ | $5.5 \overline{-1} 2{ }^{2}$ | $\overline{2} \overline{60} \overline{3} \overline{4}$ | $\overline{2} . \overline{\overline{6}}$ - $-3 \overline{4}$ | $\overline{2} . \overline{6}$ e-3 3 - | $\overline{1.8} \bar{e}^{-3 \overline{3}}$ | $5.2 \bar{e}-33$ | $1.3 \overline{\mathrm{e}}$-20 ${ }^{\circ}$ | 1.1e-17 |  |
|  | Median | 66.1233 | 64.4191 | 69.6786 | 64.9865 | 66.0533 | 72.8025 | 72.8783 | 73.0882 | 74.2580 | 75.3662 | 78.524 |
|  | $p$-value | $2.66 e^{-3}-$ | 2.6e-34 | 7.0е-31 | 2.6e-34 | 2. 6 e- -34 | $2.6{ }^{\text {e }}$-34 | $2.6 \mathrm{e}^{-34}$ | 2.6e-34 | $4.0 \mathrm{e}-33$ | 1.2e-31 |  |
|  | Median | 66.2207 | 63.1222 | 68.5683 | 64.3685 | 65.1886 | 70.8648 | 71.7564 | 71.9623 | 73.4398 | 74.8115 | 77.502 |
|  | - $p$-value | $2.6 \overline{6}-\overline{34}{ }^{-}$ | ${ }^{-2.6} \overline{6} \overline{3} 4$ | $2.6 \bar{e}-\overline{3} 4$ | $\overline{2} . \overline{6}$ e $\overline{3} \overline{4}$ | $\overline{2} . \overline{6} \mathrm{e}-3 \overline{4}$ | 2. $\overline{6}$ e-3 -3 | $2.6 \overline{\mathrm{e}}$-34 ${ }^{-}$ | $2.6 \overline{\mathrm{e}}$ - 34 | $2.6 \mathrm{e}^{-3}{ }^{-3}$ | 1.4e-33 |  |



Fig. 11: The $p$-values of Wilcoxon rank-sum tests between the algorithms and the MOSA-EA on 100 random $k$-SAT instances. The y-axis is log-scaled.


Fig. 12: The $p$-value of Wilcoxon rank-sum test between Open-WBO and the MOSA-EA on 100 random $k$-SAT instances. The y-axis is log-scaled.


[^0]:    ${ }^{3}$ For any $n \in \mathbb{N}$, we define $[n]:=\{1, \ldots, n\}$

