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Fatigue life modelling of railway prestressed concrete sleepers

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12 Abstract

13 The railway sleepers, which transfer wheel loads to the formation, are an important component of railway track systems. Prestressed concrete is the most commonly used type of railway sleeper around 14 world. Crushing is a common problem on concrete sleepers, which include excessive flexural, shear, 15 and bond stresses. The most causes of crushing in prestressed concrete sleepers are dynamic loads. 16 However, accumulated damage due to cyclic loads can also cause crushing. Much previous research 17 18 has investigated the impact load characteristics and the ultimate load capacity of prestressed concrete railway sleepers. There is a gap in the knowledge in fatigue failure for prestressed concrete sleepers. 19 This study presents new results of extensive numerical and analytical investigations aimed at predicting 20 fatigue lives under cyclic loads. A numerical study validated by 30 full-scale experimental tests is 21 executed to assess fatigue performance, while theoretical fatigue analysis methods based on S-N curve 22 and Miner linear cumulative damage are introduced for benchmarking. This paper presents a remaining 23 fatigue life assessment for prestressed concrete sleepers and contrasts with the theoretical results. 24 25 Parametric studies discuss the effect of support conditions, dynamic load distribution, and track stiffnesses on prestressed concrete sleepers. This paper highlights the rationales associated with the 26 development of fatigue limit state. The outcome of this paper will provide design flexibility and 27 improve railway sleeper maintenance and inspection criteria. 28

Keywords: prestressed concrete sleeper, fatigue, cyclic load, finite element method (FEM), numerical
 analysis

31 1 Introduction

32 Railway transportation provides safe, economical, and comfortable transport for either passengers or

33 goods [1, 2]. An important component of railway systems are railway sleepers. The main functions of

- 34 railway sleepers are distributing axle loads from rails to the substructures of railway systems and
- bolding the rails at the proper gauge [1, 3]. Prestressed concrete railway sleeper is the most commonly
- 36 used type of sleeper around world with its economic cost, long service life, and sound structural
- 37 performance. However, prestressed concrete sleepers can still be damaged due to flexural, shear, or

38 bond stresses. In long-term service, fatigue load heavily influences progressive cracking. Fatigue

39 failure can be defined as structural failure below the stress limit under cyclic loading [4]. A worldwide

40 survey of most critical concrete sleeper failures was conducted by W. Ferdous, A. Manalo (2014), and

41 the results indicated fatigue is one of the top 5 most critical causes of failure for concrete sleepers [5].

Fatigue failure can be defined as a failure happens below the stress limit of a material when it has been applied to repeated loads. In concrete sleepers, crushing or cracking under compressive stress can be

45 applied to repeated loads. In concrete steepers, crushing of cracking under compressive stress can be 44 considered ordinarily as the failure criterion to define fatigue state in design process. A global trend

45 that the increases in traffic flow and train tonnage cause accumulated damage due to cyclic loads,

46 which might cause railway sleeper failure in long-term service life.

47 The permissible stress or allowable stress design concept for prestressed concrete sleepers is used in 48 many countries [6]. This method is based on the permissible stress of materials and a load factor is 49 used to increase the static axle load to incorporate dynamic effects [7]. However, the permissible stress 50 method may underestimate the material strength, while dynamic loads are not considered thoroughly. 51 A concept based on a probabilistic model of the load resistance, limit state method, is developed for 52 railway sleepers. It applies the magnitude of factors may be varied so that they may be used either with 53 the plastic conditions in the ultimate state or with the more elastic stress range at working loads [8]. 54 Ultimate limit state is the railway sleepers must be able to withstand the loads against failure. Fatigue 55 limit state is a concrete sleeper services under cyclic loading for years and the accumulated damage

56 progressively reaches failure [9].

57 Previous research by Wakui, H and Okuda, H (1997) found cracking of concrete sleepers developed 58 rapidly due to repeated impact loads, and it was suggested a fatigue limit state should be included in 59 concrete sleeper design [10]. Many experimental and analytical investigations regarding dynamic 60 behaviour of prestressed concrete railway sleepers have been conducted by Kaewunruen, S and 61 Remennikov, A.M, (2007, 2014, 2013) and their research provides amount of data for analysis of cyclic loads and fatigue damage [11-13]. The fatigue loading tests were carried out by Rantala et al (2018) to 62 analyse the fatigue properties of the sleepers and the effect of fatigue on their stiffness [14]. Šimonová 63 64 et al presented a study on the influence of the age and level of concrete fatigue on prestressed railway sleeper [15]. Maekawa, K. et al (2006) conducted the 3D fatigue analysis which predicts the fatigue 65 life of reinforced concrete slabs [16]. Zhang, C. et al (2019) developed 3D concrete T-shape girder in 66 order to investigate the fatigue performance of the carriageway plates of bridge [17]. These studies 67 68 provide a fundamental understanding of fatigue behaviour in concrete structures.

69 In this research, it is possible to better understand the fatigue life of the railway sleepers associated 70 with various service conditions. On this ground, there is clearly a need to carry out an investigation of 71 the fatigue life simulation. This article presents a numerical simulation of the fatigue life of prestressed 72 concrete sleepers. Finite element analysis was conducted to investigate the life cycle of prestressed 73 concrete sleepers. Chinese Type III prestressed concrete sleepers were used to study fatigue life. A 74 physical model of the sleeper is established and validated. The results show the effect of typical support 75 conditions, various impact loads, and different track stiffnesses on service life of the prestressed 76 concrete sleeper. A theoretical fatigue life assessment method is also introduced in order to validate the numerical model. The findings presented in this study aim to develop a fatigue life assessment 77 78 model, which will eventually help railway engineers to better estimate the service life of railway 79 sleepers under various conditions.

80 2 Fatigue life assessment method

81 2.1 Damage accumulation method

82 Previous research has presented the fatigue life assessment method of concrete sleepers [9, 18]. The

83 theoretical assessment method is based on the damage accumulation method and extending Miner's

rule, and can be used to evaluate the fatigue life under constant amplitude cyclic loads. For each cyclic

load, the critical tensile stress of the prestressing tendons of the prestressed concrete sleeper can be calculated and the fatigue life for the cyclic load can be determined. The cumulative damage index is

86 calculated and the fatigue life for the cyclic load can be de87 given by:

$$\sum D_i = \sum_i \frac{n(\Delta \sigma_i)}{N(\Delta \sigma_i)} \tag{1}$$

89 where $n(\Delta \sigma_i)$ is the applied number of cycles at a stress range $\Delta \sigma_i$; $N(\Delta \sigma_i)$ is the resisting number 90 of cycles at a stress range $\Delta \sigma_i$.

- 91 The S-N curve is the relationship between the magnitude of an alternating stress and the number of
- 92 cycles until there is failure for a selected material. The maximum applied number of cycles at an
- alternating stress can be determined and it is expressed as failure cycles (fatigue life) [19, 20]. The S-
- 94 N curve for prestressing steel is shown in **Figure 1** and the failure cycles of prestressing steel under
- 95 cyclic loads can be estimated by:



Figure 1.	S-N curve	for prestressing	ng steel	[19]

98

96

97

88

99 If $(\Delta \sigma > \Delta \sigma_{N^*})$

$$\log N_f = \log N^* - k_1 [\log(\Delta \sigma) - \log(\Delta \sigma_{N^*})]$$
⁽²⁾

100

102

101 If $(\Delta \sigma < \Delta \sigma_{N^*})$

$\log N_f = \log N^* + k_2 [\log(\Delta \sigma_{N^*}) - \log(\Delta \sigma)]$ (3)

103 where $\Delta \sigma$ is the stress range of the prestressing tendons; $\Delta \sigma_{N^*}$ is the stress range at N^* cycles; k_1 , 104 k_2 are the stress exponents. **Table 1** illustrates parameters of prestressing steel S-N curve. (Note: this 105 S-N curve is able to define the type of prestressing tendons used in this research)

S-N curve of prestressing steel used for		Stress	exponent	$\Delta \sigma_{N^*}$ at N^*
	N^*	k_1	k_2	cycles (Mpa)

Pre-tensioning	10 ⁶	5	9	185

127

Table 1. Parameters of S-N curve for prestressing steel [19]

107 2.2 Fatigue life assessment

108 To assess fatigue life, the cracking progression of the railway sleeper needs to be analysed. In the initial 109 stage, no cracking appears and the concrete stress at the bottom fibre can be calculated by:

110
$$\sigma_{cF}^{b} = \frac{nA_{ps}\sigma_{se}}{A_{t}} + \frac{nA_{ps}\sigma_{se}e}{I_{t}}y_{t}$$
(4)

111 where σ_{se} is the effective stress per prestressing tendon; A_{ps} is the cross-section area of a tendon; *e* 112 is the eccentricity; *n* is the number of tendons; A_t is the transformed area of the sleeper; I_t is the 113 inertia moment of transformed section before cracking; y_t is the distance of the centroidal axis of 114 transformed area from soffit.

115 The cracking moment is calculated by:

116
$$M_{cr} = I_t \frac{\sigma_{cF}^b + f_{cf}}{y_t}$$
(5)

117 where f_{cf} is the tensile strength of the sleeper.

118 As the cracking progresses, the neutral axis of the sleeper cross-section will change. Therefore, the 119 distance y_{CG} from the centre gravity of the effective transformed area to the top of the compressed 120 area needs to be determined by:

121
$$y_{CG} = root \left[\left[S_{pcII} - n_e A'_{p3} \left(h - y_{cg} - d_3 \right) - n_e A'_{p2} \left(h - y_{cg} - d_2 \right) - n_e A'_{p1} \left(h - y_{cg} - d_1 \right) \right], y_{cg} \right]$$
(6)

where the S_{pcII} is the first moment at the bottom fibre after cracking; A'_{pi} is the total area of the prestressing tendons at layer i; d_i is the distance from the prestressed tendons at layer i to the bottom of tension area; n_e is the modular ratio.

125 The effective transformed section can be estimated using the transformed area of sleeper cross-section 126 A_{tII} :

$$A_{tII} = A_{cII} + n_e A_p \tag{7}$$

128 where A_{cII} is the effective concrete area of sleeper cross-section.

129 The moment of inertia of the cracking section is given by:

130
$$I_{cr} = I_{ccr} + n_e A'_{p3} (h - y_{cg} - d_3)^2 + n_e A'_{p2} (h - y_{cg} - d_2)^2 + n_e A'_{p1} (h - y_{cg} - d_1)^2$$
(8)

131 where I_{ccr} is the inertia moment of effective centroid. (Note: the selected prestressed concrete sleeper

has three layers of prestressing tendons, the equation (6)-(8) can be also applied for other cross-sectionof prestressed concrete sleepers.)

134 The effective inertia moment in the lifetime is calculated by:

135
$$I_{ef} = I_{cr} + (I_t - I_{cr}) \left(\frac{M_{cr}}{M_{max}}\right)^3$$
(9)

136 where I_t is the inertia moment of transformed section before cracking; M_{cr} is the cracking moment; 137 M_{max} is the maximum bending moment at the section under cyclic loads.

138
$$\Delta \sigma_{pt1} = n_e \frac{M_{max} - M_{min}}{I_{ef}} (h - y_{cg} - d_1)$$
(10)

139 where M_{min} is the minimum bending moment at the section under cyclic loads.

140 Using the output value of $\Delta \sigma_{pt1}$, the failure cycles of the prestressing tendons under constant cyclic 141 loading can be estimated by Equation (2) or (3).

142 **3** Fatigue life simulation method

143 3.1 Fatigue analysis type in finite element method

144 In this study, ANSYS Workbench has been used to investigate the fatigue life of prestressed concrete sleepers. The aim of fatigue analysis in this simulation is to ascertain capability of a material to survive 145 146 from the cyclic loads during service life. In general, fatigue analysis can be categorised into Strain Life, 147 Stress Life, and Fracture Mechanics. The Strain Life method focuses on crack initiation where the strain can be directly measured. This method deals with relatively low cycles (less than 10^5 cycles). 148 Fracture Mechanics is used to determine crack growth. The time from crack initiation to the crack 149 150 growing to a critical size can be calculated using this method. Sometimes, Fracture Mechanics (crack 151 growth) is used with the Strain Life method (crack initiation) to determine total fatigue life.

In this research, the Stress Life method is adopted. This method is typically characterised by an empirical S-N curve as part of material definition of the sleeper model and modified by a variety of factors dealing with relatively high cycles (more than 10^5 cycles). It should be noted that fatigue life results using the stress life method show the available life for the given fatigue analysis without considering crack propagation or fracture. Therefore, the output results represent the cycles until the sleeper will fail due to fatigue [21, 22].

158 3.2 Cyclic loading type

159 The Constant amplitude, Proportional loading is applied in this research shown in Figure 2. Loading 160 is of constant amplitude because only one set of FE stress results along with a loading ratio is required 161 to calculate the alternating and mean values. Loading is proportional since only one set of FE results are needed (principal stress axes do not change over time). Common types of constant amplitude 162 163 loading are fully reversed (apply a load, then apply an equal and opposite load; a load ratio of -1) and 164 zero-based (apply a load then remove it; a load ratio of 0). Since loading is proportional, looking at a 165 single set of FE results can identify critical fatigue locations. Likewise, since there are only two loadings, no cycle counting or cumulative damage calculations need to be done [21, 22]. 166



168

184

186

Figure 2. Constant amplitude load

169 3.3 Equivalent alternating stress

In analysis of fatigue using the Stress Life method, the S-N curve is required to relate the fatigue to the stress state. Therefore, the equivalent alternating stress is used to query the S-N curve to determine the fatigue life. The equivalent alternating stress is the stress used to query the fatigue S-N curve after accounting for fatigue loading type, mean stress effects, multiaxial effects, and any other factors in fatigue analysis. The equivalent alternating stress can be determined as following steps:

- 175 1. Calculate the alternating mean stress tensor.
- 176 2. Collapse alternating and mean stress from tensor to scalar using selected stress component.
- Calculate the Equivalent Alternating Stress using the desired empirical stress theory, as specified
 by the Mean Stress Theory property of the Fatigue Tool object.

179 Several empirical options can be selected including Gerber, Goodman, and Soderberg theories as 180 shown below. According to these theories, the value reported as the equivalent alternating stress is used

- to query the fatigue life from the S-N curve. The equivalent alternating stress is the last calculated
- quantity before determining the fatigue life [21, 22].

183 *Gerber Equation:*

$$\frac{\sigma_{Alternating}}{S_{Endurance\ Limit}} + \left(\frac{\sigma_{Mean}}{S_{Ultimate\ Strength}}\right)^2 = 1 \tag{11}$$

185 Goodman Equation:

$$\frac{\sigma_{Alternating}}{S_{Endurance\ Limit}} + \frac{\sigma_{Mean}}{S_{Ultimate\ Strength}} = 1 \tag{12}$$

187 Soderberg Equation:

188
$$\frac{\sigma_{Alternating}}{S_{Endurance\ Limit}} + \frac{\sigma_{Mean}}{S_{Yield\ Strength}} = 1 \tag{13}$$

189 3.4 Fatigue life results

The simulation results of fatigue life are represented the number of cycles until the railway sleeper fails due to fatigue. In a Stress Life analysis with constant amplitude, if the equivalent alternating stress is lower than the lowest alternating stress defined in the S-N curve, the life at that point will be used [21, 22].

194 4 Prestressed concrete sleeper details

- 195 Chinese Type III prestressed concrete sleeper is used to study fatigue life. Material properties and 196 section details are indicated in **Figure 3** and **Table 2** with the following parameters:
- 197 (1) Track gauge: 1435mm
- 198 (2) Concrete strength: C60
- 199 (3) Prestressing tendons: 10No. of 7mm dia. (tendon area: 384mm²)
- 200 (4) Prestressing force: 420kN (pre-tensioning)



201



202



Figure 3. Chinese Type III prestressed concrete sleeper geometric details

Material properties	Basic variables	Value
Concrete	Mean compressive strength	65MPa
	Modulus of elasticity	33GPa
Prestressed wire	Yield strength	1570MPa

Modulus of elasticity

200GPa

204**Table 2.** Material properties

205 **5** Finite element model of the prestressed concrete sleeper

The finite element model was developed to study fatigue life. The sleeper model used in this study is made up of 3D solid elements shown in **Figure 4**. The finite element model is composed of railway sleeper, prestressed tendons, and ballast. The concrete of sleeper is modelled as solid elements with most of these elements being 10-node tetrahedron elements, while the prestressed tendons are modelled as beam elements. The ballast is also modelled as solid elements. The material constitutive model of the FE sleeper model follows the experimental data presented in Section 5.1. It should be noted that the S-N curve used for prestressing tendons of the sleeper model is assumed in terms of **Table 1**.

213 The engineering properties used in the FE modelling are illustrated in **Table 3**. These properties were 214 selected because they were identical to a particular type of concrete sleeper manufactured in China. In 215 the model, concrete, prestressing tendons, and ballast (block support) are considered to be well adhere. The No-Separation contact type is used to simulate the constraints between concrete and prestressing 216 217 tendons, which the bond slip and bursting are not considered. The contact type between the sleeper and 218 ballast is Rough, which there is no sliding allowed. The bottom interface of tensionless ballast is set as 219 fixed support and edges of ballast layer are set as free. The remaining boundary conditions at both rail 220 seats have been set as hinges where the longitudinal and lateral displacements are restrained. In the 221 simulation of prestressing force transfer, the Thermal Condition is used to define the temperature in 222 the tendons, which the thermal load can be regarded as the prestressing force. After the prestressing 223 force transferred, the cyclic load can be applied to the FE sleeper model for analysing fatigue life of 224 the prestressed concrete sleeper. It should be noted that the size of ballast model is 225 3200mm×900mm×300mm, which the selection follows the sleeper density with 1308 sleepers per km. 226 The thickness of ballast layer is usually 250mm to 300mm, thus the 300mm thickness of ballast is 227 accepted.



228

229

Figure 4. Finite element model of the Chinese Type III prestressed concrete sleeper

Parts N	Iodulus of elasticity	Density	Poisson's ratio
	(MPa)	(kg/m^3)	
Sleeper	33000	2400	0.23
Ballast	1500	1800	0.20
Tendons	200000	9800	0.30

Table 3. Material properties used in the FE model

231 5.1 Model validation

To validate the model, the static capacity test of Chinese Type III prestressed concrete sleeper is used to verify structural and material properties of the FE sleeper model [23, 24]. An experimental programme as conducted at Beijing Jiaotong University (apparatus shown in **Figure 5**) used to validate the FE sleeper model. In the experiment, the loading jack was applied to a rubber plate located just above the surface of the centre of the sleeper. The loading level increases up to 140kN. The DIC (digital

237 imagine correlation) was selected to obtain load-deflection response.





Figure 5. Apparatus of the static capacity test of sleeper





Figure 6. Static capacity test simulation for FE sleeper model validation

In order to validate the quality of the FE sleeper model, the numerical results have been compared with experimental results from the static capacity test in previous research [26]. **Figure 6** shows the static capacity simulation for FE sleeper model material and structural validation. **Figure 7** presents the comparison between the FE analysis and experimental results. The results are found to be in very good agreement with the static capacity test. The maximum difference between the experimental and numerical results is only 5.99%.





Figure 7. Load-deflection response between FE results and experimental results

250

6 Results of the fatigue life prediction

251 Section 2 introduces the theoretical fatigue life assessment method. The cyclic loads (dynamic loads) 252 between 55kN to 365kN are chosen to calculate the fatigue life. Each dynamic load is assumed as 253 constant which can calculate only one fatigue life for both numerical and theoretical assessment method. 254 The output results represent the sleeper will fail under the constant dynamic load after output number 255 of cycles. The simulation of fatigue life refers to the results of theoretical fatigue life assessment. Figure 8 presents the cyclic loads setting. The pressure is applied at the rail seat area and the magnitude 256 range of cyclic loads (dynamic loads) is following theoretical calculation in order to compare the results. 257 258 The S-N curve used in predicting fatigue life is shown in Table 1 of Section 2. The comparison of 259 numerical and theoretical fatigue life results is demonstrated in Table 4 and Figure 9. From Table 4, 260 it can be seen that the errors between numerical and theoretical results range from 0.03% to 35.13%. The average error is 13.03%. From Figure 9, it is seen that the FE analysis results are quite similar to 261 262 the theoretical fatigue life results. In order to validate the numerical fatigue model, the experimental data is also used to compare with numerical and theoretical results. Parvez and Foster have conducted 263 264 fatigue experiments of prestressed concrete sleepers to observe failure cycles [25, 26]. Two specimens are chosen from fatigue experiments to calculate the average of the failure cycles presented in Table 265 5. Based on their test results, the comparison between experimental, theoretical, and numerical results 266 267 is shown in Table 6. A good correlation between the numerical, theoretical, and experimental results 268 provides a reliable method for further parametric study.



	Life cycles	Life cycles	
Load (kN)	(numerical)	(theoretical)	Error %
55	3.47E+10	4.19E+10	20.59
65	1.54E+10	1.70E+10	10.31
75	7.71E+09	7.24E+09	6.19
85	3.43E+09	3.23E+09	5.95
95	1.68E+09	1.50E+09	10.80
105	8.79E+08	7.21E+08	17.94
120	3.76E+08	3.59E+08	4.39
130	1.73E+08	1.85E+08	6.70
140	8.48E+07	9.77E+08	15.14
150	4.38E+07	5.30E+07	20.95
160	2.39E+07	2.95E+07	23.41
175	1.41E+07	1.68E+07	18.75
185	8.62E+06	9.74E+06	13.03
195	5.41E+06	5.77E+06	6.66
205	3.48E+06	3.48E+06	0.03
215	2.30E+06	2.14E+06	6.92
225	1.76E+06	1.33E+06	24.21
245	1.21E+06	8.04E+05	33.36
260	8.49E+05	5.61E+05	33.89
270	6.79E+05	4.45E+05	34.45
280	5.48E+05	3.56E+05	35.13
290	3.44E+05	2.86E+05	16.92
305	2.34E+05	2.31E+05	1.30
315	1.88E+05	1.88E+05	0.09
325	1.52E+05	1.54E+05	1.00
335	1.23E+05	1.26E+05	2.76
345	1.00E+05	1.05E+05	4.37
355	85186	86773	1.86
365	72829	72364	0.64

Figure	8.	Setup	for	fatigue	life	simu	lation

Table 4. Comparison of theoretical and numerical fatigue life results





274

Figure 9. Comparison between numerical and theoretical fatigue life results

Specimen ID	Failure cycles	А	verage	Standard Deviation
SF2-a SF3-a	773793 1018787	8	896290 1732	
Table 5. Experimental failure cycles [25]				
	F	ailure cycles		
Experimental result	Theoretical result	Deviation ratio %	Numerical resu	ult Deviation ratio %
896290	889577	0.75	849000	5.28

275

Table 6. Comparison between experimental, theoretical, and numerical results

276 **7** Parametric study

The comparison between the theoretical and model results implies that the fatigue life under various dynamic loads can be predicted and simulated use the FE model. Therefore, the validated FE model can be utilised to analyse the fatigue life under other critical conditions.

280 7.1 Support conditions

In a conventional track system, railway sleepers are usually laid on ballast and subgrade. It is usually assumed there is full contact between sleeper and ballast for analysis and design purposes. However, voids and hanging support conditions could occur and can cause problems to both sleeper and track system. Previous studies [27, 28] indicate 5 typical sleeper/ballast contact patterns used to analyse how fatigue life is affected by the support conditions. The sleeper/ballast contact patterns are illustrated in **Figure 10**. Full contact between sleepers and ballast is typically assumed for analysis and design 287 purposes. The voids and pockets in the sleeper/ballast contact interface form between sleepers and the

ballast underneath that could cause problems to both the sleepers and the track system as a whole. The

selected support patterns are practical concerns in actual railway track problems. In the simulation, the

290 positive reaction at rail-seat and the negative reaction at mid-span are considered and the fatigue life

- depends on the minimum of output. The ratio of the void length and sleeper length is controlled to
- 292 observe the change of fatigue life under different support patterns. The simulation has been conducted
- for each pattern. Loads of between 55kN to 325kN are applied in the FE model.



294

Figure 10. Sleeper/ballast contact patterns: (a) central void, (b) single hanging, (c) double hanging, (d) triple hanging, (e) side-central voids

297

298 7.1.1 Central void

The central void is a void which forms at the centre of the sleeper and expands symmetrically in both directions. The ratio of the central void length to the sleeper length is given:

$$301 \qquad \alpha_c = \frac{L_c}{L} \tag{11}$$

- 302 where L_c is the central void length; L is the sleeper length.
- 303 The ratio α_c of 30.77% (800/2600), 38.46% (1000/2600), 46.15% (1200/2600), 53.85% (1400/2600)
- 304 were respectively simulated to analyse fatigue life. Figure 11 shows the simulation setup and stress
- 305 distribution contour. In the FE model, the length of central void area can be adjusted and expanded in
- both directions. Figure 12 indicates the results of fatigue life in the central void support condition.



309

Figure 11. Simulation of the central void support condition

250 00

1000.00 (mm)

750.00



310



- 312 Figure 11 shows the maximum stress occurs at bottom rail-seat area, which the minimum fatigue life
- 313 should happen at rail-seat area. The effect of the central void on the fatigue life of the prestressed
- 314 concrete sleeper is shown in **Figure 12**. The results show that central void length of less than 1000mm
- 315 (ratio α_c less than 38.46%) is unlikely to influence the fatigue life in comparison with full contact
- pattern, while the change of fatigue life is just around 2%. The central void length increases to 1200mm
- (ratio α_c =46.15%), and the fatigue life falls by 51% on average for loads of between 55kN to 325kN.
- 318 When the central void length increases to 1400mm (ratio $\alpha_c = 53.85\%$), the fatigue life reduces by 83%. 319 It is obvious that the fatigue life is incensitive when the small void ratios from 0 to 40% are used
- 319 It is obvious that the fatigue life is insensitive when the small void ratios from 0 to 40% are used.
- 320 7.1.2 Single hanging
- The single hanging is a void that forms at the one end of the sleeper and grows incrementally to the other end of the sleeper. The ratio of the single side void length to the sleeper length is given as:

$$323 \qquad \alpha_s = \frac{L_s}{L} \tag{12}$$

324 where L_s is the side void length; L is the sleeper length.

The ratio α_s of 11.54% (300/2600), 15.38% (400/2600), 19.23% (500/2600), 23.08% (600/2600) were respectively simulated to analyse the fatigue life. **Figure 13** shows the simulation setup and stress distribution contour. In the FE model, the length of the single side void area can be adjusted and expanded in the other direction. **Figure 14** indicates the results of fatigue life in the single hanging support condition.





Figure 13. Simulation of the single hanging support condition





Figure 14. Comparison of the single hanging pattern results

335 Figure 13 shows the maximum stress occurs at bottom rail-seat area, which the minimum fatigue life 336 should happen at rail-seat area. Figure 14 presents the effect of the one end hanging condition on the fatigue life of the prestressed concrete sleeper. The fatigue life seems to be significantly affected by 337 338 this support condition. Single hanging void lengths ranging from 300mm to 600mm are investigated. 339 The fatigue life decreases when single hanging void length increases. When the side hanging length is 340 300mm (ratio $\alpha_s = 11.54\%$), the fatigue life reduces by 35.98% in average. The difference between 400mm (ratio $\alpha_s = 15.38\%$), 500mm (ratio $\alpha_s = 19.23\%$) voids and the full contact pattern are 76.89% 341 342 and 98.04% respectively. When the side hanging length increases to 600mm (ratio $\alpha_s=23.08\%$), the 343 fatigue life reaches half of the full contact support pattern which is a 99.90% change.

344 7.1.3 Double hanging

The double hanging is when voids are present at both ends of the sleeper. The ratio of the double side void length to the sleeper length is given as:

347
$$\alpha_d = \frac{L_{dL}}{L}, \beta_d = \frac{L_{dR}}{L}$$
 (13)

348 where L_{dR} , L_{dL} are the void length at right and left sides; L is the sleeper length.

The ratio α_d , β_d of 11.54% (300/2600), 15.38% (400/2600), 19.23% (500/2600), 23.08% (600/2600) are respectively simulated to analyse the fatigue life. **Figure 15** shows the simulation setup and stress distribution contour. In the FE model, the length of each side void area can be adjusted and expanded in each direction. The void length on both sides is assumed equal. **Figure 16** indicates the results of fatigue life in the double hanging support condition.



356

Figure 15. Simulation of the double hanging support condition





358

Figure 16. Comparison of the double hanging pattern results

Figure 15 shows the maximum stress occurs at bottom rail-seat area, which the minimum fatigue life should happen at rail-seat area. The changes in fatigue life of the prestressed concrete sleeper for varying lengths of double hanging voids are illustrated in **Figure 16**. The results are similar to the single hanging support pattern. Single hanging void lengths from 300mm to 600mm are investigated. The fatigue life decreases as single hanging void length increases. With the side hanging length at 300mm (ratio α_d =11.54%), the fatigue life reduces by 36.71% on average. When the side hanging

- length increases to 600mm (ratio α_d =23.08%), the fatigue life only reaches half of the full contact support pattern.
- 367 7.1.4 Triple hanging

The triple hanging is when there are voids at both ends and a pocket in the middle of the sleepers. The ratio of the triple side void length to the sleeper length is given as:

370
$$\alpha_t = \frac{L_t}{L}, \beta_t = \frac{L_{tc}}{L}$$
(14)

371 where L_t is the void length at sides; L_{tc} is the void length at the centre; L is the sleeper length.

372 The ratio α_t , β_t of 11.54%/30.77% (300/2600; 800/2600), 19.23%/38.46% (500/2600; 1000/2600),

373 23.08%/30.77% (600/2600; 800/2600), 23.08%/46.15% (600/2600; 1200/2600) are respectively

374 simulated to analyse the fatigue life. **Figure 17** shows the simulation setup and stress distribution

- 375 contour. In the FE model, the length of each end void and centre void area can be adjusted and expanded.
- 376 Only symmetrical cases are considered in this research. **Figure 18** indicates the results of fatigue life
- in the triple hanging support condition.





Figure 17. Simulation of triple hanging support condition





Figure 18. Comparison of the triple hanging pattern results

383 Figure 17 shows the maximum stress occurs at bottom rail-seat area, which the minimum fatigue life 384 should happen at rail-seat area. The fatigue life of the voided prestressed concrete sleeper under triple void contact pattern is demonstrated in Figure 18. It is found that a small void ratio (α_t , β_t of 385 11.54%/30.77% (300mm/2600mm; 800mm/2600mm)) does not influence the fatigue life very much. 386 387 In comparison with α_t , β_t of 19.23%/38.46% (500mm/2600mm; 1000mm/2600mm) and α_t , β_t of 23.08%/30.77% (600mm/2600mm; 800mm/2600mm), side voids have more sensitive effects than 388 central voids. To observe the changes in large side hanging with different central voids, the α_t , β_t of 389 23.08%/30.77% (600mm/2600mm; 800mm/2600mm) and 23.08%/46.15% (600mm/2600mmm; 390 1200mm/2600mm) are investigated. At loads of between 55kN and 160kN, the large central voids of 391 392 triple hanging support pattern have less fatigue life. However, when the large loads are applied (more 393 than 215kN), the fatigue life becomes very similar to each other. This demonstrates that poor support 394 conditions under large dynamic loads can easily result in failure on the prestressed concrete sleepers.

395 7.1.5 Side-central voids

The side-central voids are formed at the edge hanging and central void at the same time. The ratio of the side-central voids length to the sleeper length is given as:

$$398 \qquad \alpha_{s-c} = \frac{L_s}{L}, \beta_{s-c} = \frac{L_c}{L}$$
(15)

399 where L_{dR} , L_{dL} are the void length at right and left sides; L is the sleeper length.

400 The ratio α_{s-c} , β_{s-c} of 11.54%/30.77% (300/2600; 800/2600), 19.23%/38.46% (500/2600; 401 1000/2600), 23.08%/30.77% (600/2600; 800/2600), 23.08%/46.15% (600/2600; 1200/2600) are 402 respectively simulated to analyse the fatigue life. **Figure 19** shows the simulation setup and stress 403 distribution contour. In the FE model, the length of edge and central void area can be adjusted and 404 expanded. The void length at both sides is assumed equal. **Figure 20** indicates the results of fatigue 405 life in the side-central void support condition.



408

Figure 19. Simulation of side-central void support condition





410

Figure 20. Comparison of the side-central void pattern results

411 **Figure 19** shows the maximum stress occurs at bottom rail-seat area, which the minimum fatigue life 412 should happen at rail-seat area. The changes in the fatigue life of the prestressed concrete sleepers due 413 to side and central voids are illustrated in **Figure 20**. This contact pattern can be considered to be a 414 combination of central void and single hanging. In this support pattern, small voids (α_{s-c} , β_{s-c} of 415 11.54%/30.77% (300mm/2600mm; 800mm/2600mm)) do not have much influence on the fatigue life 416 with the change being just 35.14%. From **Figure 20**, it can be seen that the same side voids but different

417 central voids (α_t , β_t of 23.08%/30.77% (600mm/2600mm; 800mm/2600mm) and 23.08%/46.15%

418 (600mm/2600mm; 1200mm/2600mm)), the results of the fatigue lives are very similar. The results
419 also demonstrate the side voids have more influence than central voids.

420 7.2 Impact load distribution

The FE model analyses the fatigue lives under various magnitude of dynamic loads. The output results of fatigue lives are under a constant dynamic load. In the field, the railway sleepers could experience a wide range of dynamic loads. This section analyses the fatigue life under different impact load distributions. In this section, the support condition is set as full contact pattern.

- 425 A comprehensive investigation of actual impact loads was conducted by Leong, J and Kaewunruen, S
- 426 (2007, 2007) [7, 28]. The frequency of occurrence of impact loads per year has been recorded, as shown
 427 in Figure 21. Over 1 year, data were obtained from two Teknis Wheel Condition Monitoring stations
- 428 located on different heavy-haul mineral lines in Australia. A total of nearly six million passing wheels
- 429 were measured. A number of 1,609,712 passing wheels measured for a year from unit trains with 26
- 430 tons to 28 tons axle loads were used in fatigue life analysis. The analysis of field measurement indicates
- 431 over 96% of wheels created impact loads less than 50kN. However, there were still more than 40,000
- 432 passing wheels higher than 140kN. From **Table 4** above, it can be seen that large loads can significantly
- 433 cause a reduction of fatigue life. Therefore, the effect of impact load distribution needs to be studied.
- The impact load distribution data is applied in the FE model. MATLAB is used to generate an impact
- 435 load distribution and inputted into the numerical model.

The dashed line shown in **Figure 21** demonstrates the impact load distribution. The slope of the dashed line controls the impact load magnitude range and volume of occurrence of impact loads (the total

- 438 cycles need to be close to field measurement). In this section, two cases of the impact load distribution
- 439 are investigated and the field measurement data is regarded as the control group. Case 1 is the range of 440 impact load up to 410kN with low volume of occurrence. Case 2 shows the impact load range up to
- 441 240kN but low volume of occurrence. The impact load distribution of Case 1 and Case 2 are shown in
- 442 **Figure 22** and **Table 7**.





444

Figure 21. Field measurement of impact loads per year



445 446

(a) Case 1



⁴⁵⁰

 Table 7. Impact load distributions

From **Table 10**, it can be seen that the total passing wheels per year for Case 1 and Case 2 are 1,609,711 and 1,609,751 respectively. They are very close to field measurement cycles (1,609,751 cycles) to ensure the distributions can be compared reasonable. Small impact loads (less than 50kN) for both Case 1 and Case 2 follow field data because small impact loads don't have significant effect on the fatigue life.

456 The total cycles (passing wheels) per year for Case 1, Case 2, and Field data are assumed following 457 Table 7 and keep the same every year, the damage index in each year can be determined. According 458 to Miner's rule (Equation (1)), the fatigue life in years can be calculated. The outcome of the fatigue 459 life with impact load distributions is shown in Table 8. There is not much difference in fatigue life 460 between Case 1, Case 2, and the field measurement data as 96% of total cycles are small impact loads. From Table 8, it can be seen that the result of Case 1 is 14.28% less than field measurement data 461 462 whereas the result of Case 2 is 8.51% higher than field measurement data. The results demonstrate greater large impact load occurrence can lead to the reduction of the service life of prestressed concrete 463 464 sleepers. Table 4 shows that the fatigue life is up to 4.19E+10 cycles under constant 55kN dynamic load which also indicates the large impact loads need to be controlled for increasing the service life of 465 466 railway sleepers.

Impact load distribution	Life cycles	Years
Case 1	8.57E+08	32
Case 2	1.09E+09	37
Field data	1.00E+09	35

Table 8. Results of the fatigue life under different impact load distributions

468 7.3 Track stiffness

Track stiffness can influence the dynamic behaviour of trains, bearing capacity, track geometry, and
 service life of sleepers. This section discusses the effect of track stiffness on the fatigue life of
 prestressed concrete sleepers. In this section, the support condition is set as full contact pattern.

472 Previous research conducted by Martin, X et al (2010) [29] has investigated three levels of track 473 stiffness ('soft', 'normal', and 'stiff') according to the static approach. The track properties are shown 474 in **Table 9**. In their research, 100kN wheel load is applied. These track stiffness properties are input 475 into the FE model to analyse the performance of the railway sleeper. The simulation results for 'soft',

476 'normal', and 'stiff' track are presented in **Table 10**.

	Track A: 'soft'	Track B: 'normal'	Track C: 'stiff'
Stiffness of ballast/subgrade	10kN/mm	50kN/mm	100kN/mm
Rail displacement	3.16mm	1.28mm	0.58mm

477

Table 9. Track	stiffness	properties
----------------	-----------	------------

Track type	Life cycles	Years
Soft	1.42E+09	42
Normal	1.08E+09	37
Stiff	9.63E+08	34

478

Table 10. Results of the fatigue life in different track stiffness

The results from **Table 13** indicate stiffer track have shorter life cycles. In comparison with these three types of tracks, the fatigue life of 'soft' track is 31.54% more than 'normal' track while that of 'stiff' track is 10.69% less than 'normal' track. The results show that high track stiffness increases dynamic load in the wheel-rail reaction and causes the reduction of life cycles. However, these results do not indicate stiffer tracks have worse performance than softer tracks because the parametric study only considers how track stiffness influences fatigue life. In actual track systems, relatively high track stiffness can provide sufficient track resistance and reduce deflections.

486 8 Conclusion

487 This paper aimed to develop a numerical model of prestressed concrete sleepers to effectively simulate 488 the fatigue life and performance of railway sleepers under cyclic loads. The objective was to better 489 understand mechanisms of fatigue problems of prestressed concrete sleepers by using an analytical 490 model. To investigate fatigue life, the stress life method was chosen to develop a numerical model. This method is based on an empirical S-N curve and modified by a variety of factors. The theoretical 491 assessment method is also presented in order to validate the numerical model. The model predictions 492 493 for fatigue life successfully matched with theoretical results. Based on the obtained results of this study, 494 the key findings are revealed as follows:

- Generally, dynamic loads significantly affect fatigue life. Fatigue life of prestressed concrete
 sleepers is inversely proportional to the magnitude of dynamic loads. This phenomenon indicates
 the high impact loads need to be controlled.
- Based on Stress Life method of fatigue analysis type, a numerical analysis of fatigue life has been conducted using FEM. The previous theoretical fatigue life assessment method is used for validation of the numerical fatigue life model.
- In numerical study, the S-N curve is used to define the alternating stress for calculating fatigue life.
 The equivalent alternating stress is determined by empirical stress theory.
- Statistical analysis of the fatigue loads from field measurement and their fatigue performance on
 the prestressed concrete sleeper.
- Development of reasonable numerical fatigue life model can be used on prestressed concrete
 sleepers. However, in practice, dynamic loads could be various due to different contact conditions.
 Therefore, the results of fatigue life only considering constant load are the limit to this research.
- On this point, it is important to note that rail/wheel irregularity needs to be considered in further study.

510 Parametric studies investigate the support conditions, impact load distribution, and track stiffness, 511 which influence the fatigue life of railway sleeper. In general, the railway sleepers in conventional 512 tracks are considered to have an ideal contact between the sleeper and ballast. However, in some 513 situations problems could result in voids and pockets formed in the sleeper/ballast contact. Five support 514 conditions are investigated. The stress distributions of the sleepers are analysed in order to determine 515 the minimum fatigue life of each support patterns. In the support condition study, the positive moment 516 at rail-seat is more critical than negative moment at midspan, which means the minimum fatigue life 517 usually occurs at bottom rail-seat area. Central void support patterns do not influence the fatigue life 518 much. In addition, regular tamping activities typically concentrate the ballast reaction below the rail 519 seats, which is similar to central void support pattern. When the voids occur at the side of prestressed 520 concrete sleepers, the life cycles reduce. With the side voids increasing, the life cycles decrease sharply. 521 In actual life, the full support pattern is favorable for the rail seat sections but could be unfavorable for 522 the centre section. Therefore, it is necessary to conduct track inspection to ensure well supported 523 conditions. The results of impact load distributions indicate a greater proportion of high impact loads 524 affect service life. Small proportions (1.1%) of high impact loads can cause 14% life cycle reduction. 525 If the dynamic loads can be controlled no more than 70kN, the performance of railway sleepers is 526 improved. The investigation of track stiffness reveals stiffer track can decrease the life cycles. High 527 track stiffness results in increases of dynamic loads in wheel-rail reaction as well as sleepers, which 528 causes the reduction of life cycles. It should be noted that the experimental programs are suggested to 529 conduct in future research, which can be compared with theoretical and numerical results for more 530 accurate prediction.

531 This article demonstrates a reliable approach for evaluating the fatigue life of prestressed concrete 532 sleepers. The parametric study provides design flexibility and choices to engineers. The outcome of 533 this paper will enhance the safety of the track component, and railway sleeper manufacturers could use 534 the model to assess their product designs.

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