A note on radio wave propagation

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A note on radio wave propagation in relation to Westward Ho! Musings on Mathematics and Mechanics

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In [1], the rigorous mathematical achievements of George Neville Watson are highlighted as his most well-known. However, for academics in other disciplines and people in industry, past and present, this viewpoint is debatable. Notably, although an obituary of Watson by John Macnaghten Whittaker [2] highlights his contribution to the understanding of long wavelength radio wave propagation within the atmosphere of the earth, this is not considered in [1].

The motivation for this article stems from the final two paragraphs in [1], where Watson is pitied for his over-valuation of rigor and supposed lack of appreciation of 'reasonable mathematical' (non-rigorous) arguments. By contextualising Watson's contribution to the theory of long wavelength radio wave propagation [3] and [4], the reader is encouraged to form their own opinion on the matter.

In 1901 [5, p.19-23], Guglielmo Marconi observed long wavelength radio waves emitted in Cornwall were received in Newfoundland (which are over 3,500 km apart). Consequently, between 1901-1918 leading mathematicians and theoretical physicists of the era, based explanations for the phenomena on diffraction of the radio waves by, and roughly along, the surface of the ground/water (referred to as surface-diffraction theory). Notable figures who attempted to explain the phenomena via these means included: in the UK, Hector Munro MacDonald, John William Nicholson and Augustus Edward Hough Love; in France, Henri Poincaré; in the German state, Jonathan Zenneck and Arnold Sommerfeld; as well as a number of their protégés. The mathematical models that arose from this physical notion, derived by those above (and their supporters), had solutions which agreed qualitatively with Marconi's observation (albeit not all related radio propagation phenomena observed by practitioners at the time). The idea of surface-diffraction as an explanation for the phenomena likely arose, in part, from the mathematical study of optics, where a wealth of techniques had been recently developed, which could be applied to the problem.

Only in 1911, once the US Navy had recognised the potential of long-range wireless communication, would reliable data appear to test their theories quantitatively. Experiments were commissioned by the United States National Bureau of Standards and conducted by Louis Austin who published the data, as well as that found from a subsequent study in [6]. Moreover, using statistical techniques, Austin and his collaborator Louis Cohen, illustrated an empirical rule, from then onward, known as the Austin-Cohen formula. This rule described the intensity of long wavelength radio waves as a function of antennae height of the transmitter and receiver (h_t and h_r), wavelength of the radio wave (λ), distance between the antennae (d) and current in the antennae (I_t and I_r), namely

$$I_r = (4.25)\left(\frac{I_t h_t h_r}{d\lambda}\right) e^{-(0.0015)d/\lambda^{1/2}}. \qquad (1)$$

In comparison, theoretical predictions of those mentioned in the previous paragraph, were (up to details avoided here for simplicity) approximately of the form (with wavelength and distance varying and other variables remaining fixed)

$$I_r \propto \frac{1}{\lambda} e^{-cd/\lambda^{1/3}} \qquad (2)$$

for various constants c.

Following publication of the Austin-Cohen formula, after a spirited trans-Atlantic academic debate on the consistency of surface-diffraction theory, it became evident that quantitatively, theoretical predictions, represented in (2), for the attenuation of I_r based on diffraction alone did not agree with experimental observations, represented in (1) [5, p.81-84].

Alternatively, not long after Marconi's 1901 announcement, another idea to explain the phenomena was proposed independently by Oliver Heaviside and Arthur Edwin Kennelly. The idea was based on the existence of a reflecting (refracting) layer of ions in the outer atmosphere which with the earth's surface bound a roughly spherical shell shaped region of the atmosphere in which radio waves could propagate. Notably, in relation to Stigler's law of eponymy,[1] the idea of a conducting region of the outer atmosphere (which would refract electromagnetic waves) goes back further to studies of terrestrial magnetism by Carl Friedrich Gauss, Arthur Schuster and Balfour Stewart (see [5, p.91] or [7, p.36]). We know this layer today to be part of the ionoshpere, namely the Kennelly-Heaviside layer, but the existence of the layer was

[1]Stigler's law of eponymy states that scientific discoveries attributed to particular individuals, are typically not named after their original discoverer(s). Stigler's law itself conforms to this law.

not justified until the 1920s (to a sufficient standard to award a Nobel prize) in the experimental work of Edward Victor Appleton (see [5, Part 2], and notably [8]). However, within academic literature, this idea was not particularly popular pre-1918, and most academic articles related to long wavelength radio wave propagation within the atmosphere ignored it.

However the notion of a reflecting region in the outer atmosphere became popular amongst those who used the associated radio technology, in particular, following a publication of Henry William Eccles in 1912 [9], which supposed a conducting outer layer of the atmosphere that reflected, and an inner layer that refracted, radio waves within the earth's atmosphere. Eccles argued that this region of the outer atmosphere was responsible for various observed phenomena associated with radiowave propagation, including Marconi's observation [7]. Although Eccles proposed an estimate for the height of such a region of the atmosphere (between 100-200km in altitude), he did not formulate a mathematical model to describe whether the physical principles he advocated for explained Marconi's observation or the Austin-Cohen formula. Consequently, in the mid-1910s, experimental physicists and radio engineers also debated the topic in the pages of *The Electrician* [7, p.47-48], arising in further confusion regarding a justification for Marconi's observation.

Meanwhile, during the late 1910s in the Netherlands, Balthasar van der Pol was researching radio wave propagation within the atmosphere for his doctoral thesis. Notably, van der Pol observed the debate regarding surface-diffraction theory and the confusion this had caused in radio practitioner literature. Consequently, van der Pol wrote to Watson requesting his support to resolve the surface-diffraction theory debate. We note here, that the debate concerned not only a lack of consistency with the Austin-Cohen formula, but also those of a more subtle mathematical nature (insufficient rigor in mathematics in the associated literature related to (2)), i.e. the inconsistency of (1) and (2) was potentially due to a lack of accuracy in approximation.

In [3], Watson responded by, rigorously, as stated in [5], establishing that a boundary value problem arising from a mathematical model for the propagation of radio waves, derived from Maxwell's equations, which used a surface-diffraction modelling assumption at the earth's surface, and no reflection/refraction in the outer atmosphere, had a solution that was inconsistent with the Austin-Cohen formula (i.e. establishing (2) for solutions to the model). However, Watson also concluded that an ionising region in the upper atmosphere, necessarily "plays a dominant role" in long wavelength radio wave propagation within the atmosphere. Watson's paper, effectively ended the notion, which lasted nearly two decades, that surface-diffraction alone could theoretically address the long wavelength radio wave propagation problem [5].

To establish this result, Watson introduced a transformation, today known as a Watson transform, which generated a rapidly convergent series to approximate the solution to the boundary value problem (at specific points of the domain). The principal innovation was to transform an explicit series representation of the solution to the boundary value problem (valid everywhere but which converged slowly) into a contour integral in the complex plane (using Cauchy's residue theorem). Subsequently, a deformation of the contour was made to convert the slowly converging series into a rapidly converging series amenable to analysis and computation (which did not necessarily converge everywhere), but which formed an asymptotic expansion in the large ka limit, with k and a representing the wavelength of the radio wave and radius of the earth respectively. Concerning long wavelength radio wave propagation within the atmosphere, Watson took $ka \approx 8000$ from Love's previous work.

Watson's key ideas described in the previous paragraph, are illustrated with clarity in [10] via their application to an analogous illustrative problem, and further contextualised in [11]. Moreover, subtleties associated with rigorous aspects of the contour deformation in Watson's argument (omitted in [3]) which affect the convergence of the resulting series are illustrated in [10]. Returning to the motivation for this article, via the aforementioned transform, we see that Watson recognised that a rigorous slowly convergent series is less practically useful, for this type of problem, than a rapidly convergent (but not necessarily rigorously derived, see [11, p.301]) series amenable to computation.

However, in [4], Watson also highlighted the potential importance of an "ionising layer" within the earth's atmosphere in relation to long wavelength radio wave propagation. Specifically, Watson considered a simple mathematical model for long wavelength radio wave transmission within the earth's atmosphere that was motivated by Heaviside and Eccles. Notably, along with the aforementioned modelling considerations used in [3], this new model included a "reflective boundary condition" that encapsulated an ionised region of the earth's outer atmosphere. The corresponding solution to the associated mathematical problem agreed well with the

Austin-Cohen formula and was consistent with Eccles predictions of the altitude of such a reflecting layer. Furthermore, it is noteworthy that this mathematical model was the first to incorporate the reflection (or refraction) of radio waves by the outer atmosphere.

Following Watson's groundbreaking work which incorporated surface-diffraction with reflection from the atmosphere (see [7, Table 1.1] to contextualise this contribution), experiments were conducted and analysed by Captain Henry Joseph Round and Thomas Lydwell Eckersley of the Marconi company, respectively, which were quantitatively consistent with Watson's model [12].

However, unfortunately for Watson, in the 1920s, it also became clear (primarily via non-academic means) that short wavelength, rather than long wavelength radio waves were not only cheaper to produce, but could also travel (and be clearly received across) comparable distances. Consequently, commercial and academic interest in long-wavelength radio wave propagation within the earths atmosphere rapidly diminished. In addition, due to the lack of an explanation for finer details of the ionosphere illustrated by Appleton (skip zones, refraction of short wave radio waves, practical irrelevance of Austin-Cohen formula) Watson's model was subsumed relatively quickly.

Nevertheless the idea encapsulated in the Watson transform, which gave a solution representation usable for practical purposes, remained important to academics and engineers who developed the theory of atmospheric refraction of electromagnetic waves in the decades that followed [5, p.101-108][2]. Moreover, the mathematical model in [4] directly influenced radio sounding experiments of Eckersley which directly influenced Appleton's experimental studies of the ionosphere.

Returning again to the motivation for this article, it seems fair to say that mathematical modelling of physical phenomena constitutes a pursuit of reason. Relatedly, although it was Watson's appreciation of rigor which meant he could devise a transformation that would remain useful (see [11]), it was precisely Watson's appreciation of reason, that motivated him to introduce an unverified physical phenomena (which was subsequently verified) to the mathematical modelling of long wavelength radio wave propagation within the atmosphere of the earth.

Penultimately, we consider a general issue associated with brief articles about mathematicians or scientists who have produced a variety of work. Simply, it is difficult to highlight everything in a brief article. For instance, in Robert Alexander Rankin's obituary of Watson [13], contributions to various aspects of pure mathematics are highlighted and radio waves are not explicitly mentioned. However, focus is drawn to Watson's competency in numerical computation and the "great demand" during the 1939-1945 period, for Watson's tome on Bessel functions [14], which notably contains not only theoretical investigations, but "extensive tables", likely for practical use, by scientific establishments around the world. However, in comparison, in Whittaker's obituary of Watson [2], almost a page is dedicated to Watson's contribution to the theory of radio wave propagation.

Finally, returning to [1], as in [13], it appears that it is reasonable to summarise Watson's mathematical works without considering, in detail, radio wave propagation. However, the notion that Watson was not "reasonable" (in the sense of Marc Kac [1], as stated therein) seems to be contrary to Watson's contribution to the theory of long wavelength radio wave propagation. We also note that Watson's extensive numerical computations highlighted in [14], as well as numerous other works, could alternatively have been used to demonstrate this lack of consistency.

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