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# Securitization and Aggregate Investment Efficiency\*

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## Abstract

This paper studies the efficiency of competitive equilibria in economies where the expansion of investment is facilitated by securitization. We show that the use of securitization is generally associated with constrained inefficient aggregate investment, thereby potentially justifying regulatory intervention in markets for securitized assets. We examine the effectiveness of two real-world policy instruments to address this inefficiency: ex-ante capital / leverage requirements, as well as skin-in-the game (retention) requirements. We find that leverage/capital restrictions can increase welfare in our environment, but that forcing originators to hold additional skin-in-the game is not welfare improving.

**JEL codes:** D52, D53, E44, G18, G23.

**Keywords:** Securitization, pecuniary externalities, financial frictions, macroprudential regulation, fire-sales, incomplete markets, retention requirements, skin-in-the-game.

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# 1 Introduction

Securitization refers to the process by which financial intermediaries transform illiquid assets, such as corporate loans or mortgages, into marketable securities. As a result, securitization allows loans which were traditionally held to maturity by originators, to be sold (at least in part) as securitized assets to outsiders. Typically, a special purpose vehicle (SPV) is set up by a sponsoring financial intermediary, which purchases a pool of assets from the sponsor and/or other originators. These purchases are financed by sales of asset-backed securities (ABS) to institutional investors, that are backed by the cash-flows from the pool. One of the appeals of ABS is that they can be designed to suit institutional investors' preferences for relatively safe assets.

The rise of this originate-to-distribute model has led to significant alterations in global capital markets and the nature of financial intermediation. In the US for example, non-agency ABS outstanding grew from approximately \$11.3 billion in 1986 to over \$1.36 trillion in 2015, and coincided with the increase of specialty non-bank lenders such as Countrywide Financial. Despite a number of flaws in the securitization process, highlighted in the fallout of the financial crisis of 2007-2009, growth in securitization is generally viewed as having increased overall credit availability and lowered the cost of credit in advanced economies.<sup>1</sup>

Financial intermediaries value securitization because it allows them to alter the size and composition of their balance sheets when markets for the underlying assets are incomplete. Specifically, motivated by standard liquidity / risk-management considerations or binding regulatory requirements, intermediaries can use securitization to shed asset risk. This allows them to re-deploy capital towards alternative, possibly more profitable, investment opportunities.

It is well known from the theoretical literature that competitive equilibria can exhibit inefficient aggregate investment levels when markets are incomplete.<sup>2</sup> Thus, while securitization may raise investment through enhanced risk-sharing, it may cause over-investment, and as such the welfare implications of securitization are unclear. The main contribution of this paper is to show that aggregate investment is always constrained inefficient in economies whenever securitization is useful in expanding investment.

We develop a dynamic general equilibrium model of investment with three periods ( $t = 0, 1, 2$ ). In period 0, risk-neutral borrowers with limited capital, whom we refer to as inter-

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<sup>1</sup>For surveys on securitization, see [Gorton and Metrick \(2012\)](#), and/or [Segoviano et al. \(2015\)](#). We provide evidence on the growth of ABS and demand from institutional investors in Appendix ??, for both the United States and Europe. See also [Acharya and Schnabl \(2010\)](#) and [Claessens et al. \(2012\)](#) for discussions on the myriad factors behind the growth of ABS.

<sup>2</sup>See for example [Lorenzoni \(2008\)](#). Generally speaking, opening additional markets does not necessarily increase overall welfare, a result first established by [Hart \(1975\)](#), and generalized in [Elul \(1995\)](#).

intermediaries, finance risky investment projects by creating and selling safe debt (ABS) backed by the investments' cash-flows. Investment returns are subject to both idiosyncratic and aggregate risk. In creating safe debt, intermediaries are limited by their capital and the riskiness of returns as they retain residual risk. Projects may succeed early (period 1) yielding complete returns, or partly succeed late (period 2) yielding partial returns, or fail late returning nothing. The overall fraction of intermediaries' projects that succeed is uncertain, and depends on the underlying state of the economy.

By engaging in securitization at period 0, individual intermediaries can shed idiosyncratic risk, thereby increasing their capacity to create and issue safe debt. Importantly, and in line with empirical evidence, we assume that frictions in the securitization process limit the amount of idiosyncratic risk that intermediaries can shed. This is modeled in the form of a skin-in-the-game constraint, which arises from a standard moral hazard problem in which originators require incentives to undertake costly screening effort. Thus, while ex-ante homogeneous, intermediaries will differ at period 1 as returns on their individual investments may be early or late, with the degree of heterogeneity dependent upon the amount of securitization at  $t = 0$ .<sup>3</sup>

At period 1, we assume those with early returns (early types) always have sufficient funds to service their debt obligations and invest in new opportunities. On the other hand, if there is a recession late types require outside funding to undertake new investment opportunities and service debt. Financial frictions rule out state-contingent contracts at period 0, and new borrowing at period 1 (investor preferences rule out default). To raise funds, late types sell their late investments to early types via a spot market (fire-sale). However, they may be constrained in their ability to raise funds, in which case they will have to forgo new positive NPV investments (credit-crunch).

The extent to which late borrowers are constrained will depend directly on their investment and securitization decisions at time 0, and indirectly through the prevailing asset price, which is a function of both the aggregate funds early types have (demand), and the aggregate quantity of assets for sale (supply). Crucially, as atomistic intermediaries do not anticipate the impact of their period 0 decisions on the asset price at period 1, a pecuniary externality arises that results in socially excessive investment and securitization at time 0 whenever late types are constrained at period 1. In fact, we show that the reduction in return variability via securitization is only valuable when late types are constrained, which is precisely when aggregate investment in the economy is constrained inefficient.

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<sup>3</sup>Intermediation here may be viewed as market finance, which is often also referred to as shadow banking. We abstract from regulatory arbitrage motives for securitization, see for example [Acharya et al. \(2013\)](#). We also ignore tax considerations as discussed in [Han et al. \(2015\)](#).

To better understand the role of securitization in our model, note that a reduction in the variability of asset returns via securitization mitigates the impact of financial frictions by moving funds from early to late types at period 1. This substitutes for contingent contracts or direct borrowing, which might otherwise provide such transfers but are assumed to be unavailable. As a result, intermediaries can create more safe debt for investors by increasing pledgeable income when asset returns are late. This is the standard partial equilibrium view of how more securitization can lead to higher leverage and investment.<sup>4</sup>

Our framework is novel in that it highlights an aspect of securitization previously unstudied; that securitization also affects spot market prices by altering the distribution of cash in the market. Specifically, with more securitization at period 0, demand for assets at time 1 declines since the funds of early types are reduced. On the supply side, late types require less funds and have more assets to sell. Overall, we show there is a reduction in the price of assets at period 1, creating a transfer from late to early types and thereby exacerbating the financial frictions and the pecuniary externality.

Another key contribution of this paper is that it provides a framework to analyze the welfare implications of policies designed to curb excessive leverage and limit securitization in the financial sector. We show that leverage restrictions, akin to those outlined in the Basel III reforms, can be welfare improving when the competitive equilibrium is characterized by over-investment. Importantly, since securitized lending is at the heart of the (mostly unregulated) shadow banking sector, our results suggest broader regulation of the financial industry may be valuable.

It is plausible that welfare could be improved by recently enacted policies forcing originators to hold more skin-in-the-game than the laissez-faire level, since securitization affects leverage indirectly. For example, the retention requirements specified in the U.S. Dodd-Frank Act and the European Capital Requirements Directive. These policies are interesting not only because they are a part of the current policy discussion, but because they also apply to both regulated and unregulated entities. It is also reasonable to assume that they require significantly less information than direct restrictions on the balance sheet, such as capital or leverage constraints. In our model, the total effect of forcing more skin-in-the-game can be decomposed into a direct effect and a price effect. There is a direct tightening of the constraints on late intermediaries, which reduces the resources available to intermediaries in a bad state of the world, causing them to reduce leverage ex-ante. On the other hand, reduced aggregate investment increases the price of assets in a fire-sale, which in turn increases the returns to intermediaries in the fire-sale and results in increased leverage. The

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<sup>4</sup>See for example [DeMarzo \(2005\)](#), [Coval et al. \(2009\)](#), [Gorton and Metrick \(2012\)](#), and [Kiff and Kisser \(2014\)](#).

direct effect is obvious and provides an intuitive rationale for tightening constraints as a means to reduce excessive investment. However, this is undone by the price effect, leaving the negative impact of the tighter constraint to dominate. Thus, the rationale for policies to increase skin-in-the-game beyond the laissez-faire level as a means to reduce excessive leverage cannot rely solely on partial equilibrium arguments.

## Related Literature

This paper presents a novel model of a fire-sale induced credit-crunch. Unlike some previous models, such as [Lorenzoni \(2008\)](#) and [Stein \(2012\)](#), both demand (total cash held by early types) and supply (total assets for sale held by late types) in the fire-sale are endogenously determined in our framework. Endogenizing cash in the market is necessary to examine the ex-ante risk-sharing function of securitization. Thus, we are able to show that more securitization simultaneously increases the quantity of assets available for sale, while at the same time reducing funds available on the demand side. As a result, our model captures situations where outside liquidity is endogenously very limited, as we might expect in a severe financial crisis.<sup>5</sup>

Our paper is most closely related to [Gennaioli et al. \(2013\)](#), where the main role of securitization is also to pool idiosyncratic risk, allowing the financial sector to increase investment through higher leverage.<sup>6</sup> Our framework is a generalization of their benchmark model with rational expectations. The key difference is that we assume the existence of frictions in the securitization process that limit risk-sharing, which together with financial frictions lead to constrained inefficient competitive equilibria in our model. [Gennaioli et al. \(2013\)](#) focus on a different inefficiency associated with securitization; namely the inability of investors to recognize aggregate risk. However, under rational expectations, the use of securitization in [Gennaioli et al. \(2013\)](#) is completely efficient, whereas in our environment this is not the case. In fact, securitization is privately valuable in our environment *only* when aggregate investment is constrained inefficient.

There is a growing literature on securitization, which for the most part focuses on security design and the contractual features arising from asymmetric information. For example, [DeMarzo and Duffie \(1999\)](#) and [DeMarzo \(2005\)](#) examine security design problems in static settings whereas [Hartman-Glaser et al. \(2012\)](#) focus on a dynamic problem. [Hanson and Sunderam \(2013\)](#) consider a security design problem incorporating endogenous information acquisition by investors. [Shleifer and Vishny \(2010\)](#) develop a novel model of securitization

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<sup>5</sup>For example, in the financial crisis of 2007-2009, the lack of outside capital was partly evidenced by the US government's decision to rescue institutions through mergers rather than seeking recapitalizations.

<sup>6</sup>[Diamond \(1984\)](#) first showed risk-pooling by financial intermediaries can increase investment and welfare.

where intermediaries sell assets to maximize fee revenue from intermediation. In contrast, this paper focuses on the general equilibrium effects of securitized lending and the associated welfare implications.

[Gale and Gottardi \(2015\)](#) also examine the impact of pecuniary externalities on ex-ante capital structure of intermediaries. Debt is preferable to equity in their model due to tax advantages, and they identify the impact of fire-sales by bankrupt firms on the ex-ante capital structure. Our paper differs in that we abstract from tax advantages of debt and the possibility of default in equilibrium, while focusing instead on the role of securitization on ex-ante capital structure. [Ahnert et al. \(forthcoming\)](#) emphasize a different mechanism to expand investment when investors are infinitely risk-averse, namely encumbering assets for safe debt-holders. [Daley et al. \(forthcoming\)](#) also examine the efficacy of skin-in-the-game requirements. In a partial equilibrium environment, they find that asset retentions may be decreasing in the information publicly available regarding the quality of securities issued by banks.

Finally, this paper is related to the extensive literature on pecuniary externalities which arise from incomplete markets. This literature goes back to the seminal work of [Hart \(1975\)](#), [Diamond \(1980\)](#), [Stiglitz \(1982\)](#), [Geanakoplos and Polemarchakis \(1986\)](#) and [Greenwald and Stiglitz \(1986\)](#). In our application, market incompleteness precludes individuals from equalizing marginal returns to investment. This is similar to the type of friction studied in [Shleifer and Vishny \(1992\)](#), [Gromb and Vayanos \(2002\)](#), [Caballero and Krishnamurthy \(2001\)](#), [Allen and Gale \(2004\)](#), [Lorenzoni \(2008\)](#), [Farhi et al. \(2009\)](#), [Davila et al. \(2012\)](#), and [He and Kondor \(2016\)](#), among others.<sup>7</sup> This paper shows how pecuniary externalities may result in inefficient investment when the securitization process is plagued by frictions. Thus, we link the literature on investment with incomplete markets and asset securitization. This allows us to study welfare and examine policies in a well-understood framework.

## 2 Model

We consider a three-period economy ( $t = 0, 1, 2$ ), populated by a measure one of both investors and intermediaries.

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<sup>7</sup>[Krishnamurthy \(2010\)](#) and [Brunnermeier and Oehmke \(2013\)](#) survey the literature on aggregate implications of financial frictions. [Davila \(2015\)](#) provides a discussion of several key papers in this literature, including the impact of various modeling assumptions on welfare analysis. In his terminology, we model a “terms-of-trade” externality.

## 2.1 Investor’s Problem

Investors are endowed with wealth  $w$  at  $t = 0$ , and have preferences:

$$U = E_0 \left[ c_0 + \beta \min_{\omega \in \Omega} \{c_{2,\omega}\} \right], \quad (1)$$

where  $c_0$  is consumption at  $t = 0$ ,  $c_{2,\omega}$  is consumption at period 2 in state  $\omega \in \Omega$ , and  $\beta \in (0, 1)$  is a discount factor. These preferences for investors are similar to those in [Stein \(2012\)](#) and [Gennaioli et al. \(2013\)](#), and capture evidence that a large class of investors that purchase ABS, such as pension or mutual funds, have a strong desire for safety.<sup>8</sup> Investors do not have direct access to investment opportunities, but can save by purchasing safe debt claims (it is straightforward to show that investors would not purchase risky assets from intermediaries at the equilibrium). We denote investors’ purchases of safe debt (savings) by  $B$ , which pays a gross return  $r$  per unit at  $t = 2$ .<sup>9</sup> Debt purchases are chosen to maximize (1), subject to the following budget constraints at  $t = 0, 2$ :

$$c_0 + B \leq w, \quad (2)$$

$$c_{2,\omega} \leq rB \quad \forall \omega \in \Omega. \quad (3)$$

## 2.2 Intermediary’s Problem

Intermediaries are risk-neutral and endowed with  $k$  resources at  $t = 0$ . For simplicity, we assume that intermediaries do not discount the future. As a result, they are indifferent between consumption at  $t = 0, 1, 2$  so without loss of generality, we assume intermediaries consume only in the final period.

Each intermediary has access to risky investment projects at  $t = 0$ , which can either succeed and return  $R_0$ , or fail and return nothing. Projects may succeed early at  $t = 1$ , or late at  $t = 2$ , and returns are subject to both idiosyncratic and aggregate risk. The probability of success at  $t = 1$  is identical and independent across intermediaries and varies with the aggregate state  $\omega$  at period 1, where  $\omega \in \Omega \equiv \{g, b\}$ . The state  $g$  captures a “good” or growth state where intermediaries’ projects are successful, whereas  $b$  captures a “bad” state where relatively few intermediaries’ projects succeed. The probability that the good state occurs at  $t = 1$  is  $q_1$ , while the probability of the bad state is  $1 - q_1$ . We assume that

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<sup>8</sup>See [Bernanke et al. \(2011\)](#), as well as [Stein \(2012\)](#) and [Gennaioli et al. \(2013\)](#) for discussions on the desire for safety by institutional investors that represent savers. A demand for safety can also be interpreted as a convenience yield if there is a demand for the use of ABS as collateral in sale and repurchase agreements; see [Gorton and Metrick \(2012\)](#).

<sup>9</sup>The use of short-term safe debt generates qualitatively similar results in our environment.



all projects succeed early if the state is good at  $t = 1$ , whereas only a fraction  $\alpha < 1$  succeed early if the state is bad.

The return on remaining projects in the bad state is a function of ex-ante intermediary screening effort. Specifically, when such effort is undertaken, remaining projects provide a fraction  $\theta < 1$  of their original return upon success. In the absence of such effort, we assume that remaining projects provide no return at all (i.e., they completely fail at time 1).<sup>10</sup> We take screening effort of an intermediary to be private information, the non-pecuniary costs of which are given by  $\xi I$  where  $\xi > 0$  is the marginal cost and  $I$  is the level of investment.

The probability that remaining projects succeed at  $t = 2$  depends on the aggregate state  $\omega$  at  $t = 2$ , which may also be either good or bad. The probability of success at  $t = 2$  is equal to 1 if the good state occurs, or 0 if the bad state is realized. The probability of the good state at  $t = 2$ , conditional on the good state at  $t = 1$  is  $q_{2,g}$ , while the probability is  $q_{2,b}$ , conditional on the bad state at  $t = 1$ . Thus, the expected gross unit returns on period 0 investments when intermediaries exert screening effort can be written as

$$E_{\pi} R_0 \equiv [q_1 + (1 - q_1)(\alpha + (1 - \alpha)q_{2,b}\theta)] R_0, \quad (4)$$

while without effort, returns are  $[q_1 + (1 - q_1)\alpha] R_0$ . From equation (4), we see that projects which do not succeed at  $t = 1$  become “impaired” in two ways, since the return upon success is reduced by  $(1 - \theta)R_0$ , and the probability of failure increases from  $(1 - q_1)(1 - \alpha)(1 - q_{2,b})$  to  $1 - q_{2,b}$ . The following assumption serves to ensure that undertaking screening effort is worthwhile.

**ASSUMPTION 1.** (*Value of screening effort at  $t = 0$* )

$$(1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 > \xi. \quad (5)$$

Each intermediary also has access to new risky investment projects at  $t = 1$ , that either succeed or fail at  $t = 2$ . The gross return per unit of investment is  $R_1$  in the case of success and zero otherwise. We assume that returns on  $t = 1$  investments are perfectly correlated across intermediaries, thus they all succeed if the state is good at  $t = 2$  or they all fail if the state is bad at  $t = 2$ . Assuming that new investments are not subject to any idiosyncratic risk is not crucial, but allows us to focus on one round of securitization. Also, this implies that returns on new investments in the bad state at  $t = 1$  are perfectly correlated with

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<sup>10</sup>If we interpret intermediary investments as mortgages for example, then a bad state not only results in more failures, but the underlying homes may also be worth less. Alternatively,  $\theta$  may capture early failures within an intermediary’s portfolio of mortgages. Such early failures were characteristic of sub-prime mortgages issued in the United States prior to the recent crisis.

returns on existing investments. This captures evidence of increased correlation across asset returns in bad times.

Investment at any period requires intermediaries to incur non-pecuniary costs  $C(I)$ , above and beyond any screening costs. We assume  $C(\cdot)$  is a strictly convex function such that  $C', C'' > 0$  and  $C(0) = C'(0) = 0$ . We interpret these costs as effort required to find and maintain investments, and assume these costs are small enough to ensure investment at  $t = 1$  is always socially worthwhile.

**ASSUMPTION 2.** (*Positive NPV opportunities at  $t = 1$* )  $q_{2,\omega}R_1 - C'(R_0(k + w)) > 1 \forall \omega$ .

This assumption allows for a credit-crunch at  $t = 1$ , which is necessary for the existence of a constrained inefficient equilibrium in our model. We discuss the importance of this assumption following Proposition 2.

At the start of  $t = 0$ , each intermediary invests  $I_0$  while holding  $Y_0$  in cash. Investments and reserves are financed with intermediary wealth  $k$  and through funds raised by issuing claims to investors. They issue long-term risk-less debt  $D$  at  $t = 0$ , promising a gross return  $r$  at  $t = 2$ .

Intermediaries may also sell cash flows associated with  $S_0 \leq I_0$  units of their own investment. As we show below, asset sales will be constrained by the moral hazard associated with unobservable intermediary screening effort, necessitating a “skin-in-the-game” requirement akin to [Holmström and Tirole \(1997\)](#).<sup>11</sup>

Along with sales, intermediaries can purchase cash flows  $T_0$  from other intermediaries. We interpret  $T_0$  as the cash-flows derived from a pool of all other intermediaries’ assets. Although an intermediary’s own projects have the same expected payoffs per unit as the pool of other intermediaries’ assets, due to diversification the latter bear no idiosyncratic risk, only aggregate risk. This is important because this diversification allows intermediaries to increase pledgeable cash-flows in the bad state when their asset returns may be late. In other words, cash-flows from the pool of assets  $T_0$  provide better collateral than the cash-flows from the intermediary’s own investment  $I_0$ . Such collateral is valuable when frictions limit the ability of intermediaries to raise funds.

The purchases and sales of cash-flows by intermediaries are interpreted as a standard form of securitization. Each intermediary can be viewed as creating a “special-purpose vehicle” (SPV) that purchases a pool of intermediary assets and issues ABS. In this interpretation, intermediaries’ retain the most junior “tranche” (equity) in the SPV while the senior “tranche”

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<sup>11</sup>Skin-in-the-game can arise as an optimal contractual arrangement due to informational asymmetries between originators of securities and outsiders, as modeled in [DeMarzo and Duffie \(1999\)](#). This can arise in other interpretations, such as [Gorton and Pennacchi \(1995\)](#) and [Pennacchi \(1988\)](#), where this type of structure arises to address moral hazard. [Cerasi and Rochet \(2014\)](#) also provide a model of securitization of this type, in which banks hold an equity tranche to maintain proper incentives.

(safe debt) is sold to investors with a commitment by the intermediary to provide the SPV with a liquidity guarantee of  $rD$ . For ease of exposition, we refer to the junior tranche as ABS and the senior tranche as debt, however these can both be interpreted as ABS. Securitization in our model thus amounts to pooling idiosyncratic risk across intermediaries.<sup>12</sup>

The decisions of intermediaries at  $t = 1$  consist of investing in new opportunities, purchasing or selling securitized assets from other intermediaries, selling cash flows against their own  $t = 0$  investments that have not yet been realized, or holding cash. In the good state, all intermediaries are identical as all  $t = 0$  investments succeed and are realized early. As a result, there is no motive for trade, and each intermediary makes  $I_{1,g}$  new investments and holds  $Y_{1,g}$  in cash. In the bad state, early intermediaries have relatively more funds available for investment, and thus intermediaries with late returns may have access to relatively profitable investment opportunities that cannot be exploited. We denote early types by  $e$  and late types by  $l$ . Early intermediaries invest an amount  $I_{1,b}^e$  at  $t = 1$ , while late intermediaries invest  $I_{1,b}^l$ .

Funds may be transferred between intermediaries through the exchange of assets or via the sale of remaining cash-flows on  $t = 0$  investment. Importantly, given investors' preferences, intermediaries have no other means to generate funds from outsiders, and we assume that funds can only be raised from other intermediaries via asset sales.<sup>13</sup>

**ASSUMPTION 3.** (*Market Incompleteness*) *Intermediaries cannot write state-contingent contracts, and can not directly borrow and lend to each other at  $t = 1$ .*

The missing markets we assume provide a role for securitized lending in our model, since the existence of contingent securities at period 0 or frictionless borrowing at period 1 eliminates the value in securitizing assets. These types of constraints on financing are the subject of a vast literature which has highlighted a number of possibilities.<sup>14</sup> Ruling out *all* borrowing at  $t = 1$  is unnecessary, but significantly reduces complexity. We relax this assumption in Section 4.2.1 and show that the qualitative results of the model remain.

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<sup>12</sup>We do not distinguish between originating intermediaries, sponsoring intermediaries and “special-purpose vehicles” (SPV). Ignoring the difference between the latter two is not vital since these are typically artificial constructs, operating according to a set of pre-specified rules. We ignore the difference between the former two for tractability. A detailed discussion of the process can be found in [Gorton and Metrick \(2012\)](#).

<sup>13</sup>As  $t = 1$  investments may yield nothing and investors are infinitely risk averse, they never lend additional funds at time 1. This can be relaxed without altering the qualitative nature of our results, as long as the possibility that late intermediaries may be financially constrained is retained.

<sup>14</sup>For example, limits to borrowing may be justified by the presence of asymmetric information as in [Stiglitz and Weiss \(1981\)](#), limited commitment following [Kehoe and Levine \(1993\)](#), or moral hazard as in [Gorton and Pennacchi \(1995\)](#) and [Acharya and Viswanathan \(2011\)](#). Although the limit on borrowing is not endogenous in our environment, it is straightforward to generalize the model in this way and retain the general inefficiency results characterized below in expression (23), though the comparative static results are significantly more complex for this case.

Denote early intermediaries' period 1 purchases of securitized assets by  $T_{1,b}^e$ . Late intermediaries' sales of securitized assets are  $-T_{1,b}^l$ , while sales of remaining cash-flows on  $t = 0$  assets are  $S_{1,b}^l$ . Cash holdings of early and late intermediaries are denoted  $Y_{1,b}^e$  and  $Y_{1,b}^l$  respectively. We now formally define the intermediary's problem, which is to choose  $I_0$ ,  $S_0$ ,  $T_0$ ,  $D$ ,  $Y_0$ ,  $I_{1,g}$ ,  $I_{1,b}^e$ ,  $I_{1,b}^l$ ,  $T_{1,b}^e$ ,  $T_{1,b}^l$ ,  $S_{1,b}^l$ ,  $Y_{1,g}$ ,  $Y_{1,b}^e$ , and  $Y_{1,b}^l$  to maximize expected profits at  $t = 2$  which we denote  $\Pi_0$ .

$$\Pi_0 = q_1 \cdot \Pi_{1,g} + (1 - q_1) \cdot (\alpha \Pi_{1,b}^e + (1 - \alpha) \Pi_{1,b}^l) - \xi I_0 - C(I_0) - rD, \quad (6)$$

where

$$\begin{aligned} \Pi_{1,g} &= q_{2,g} R_1 I_{1,g} + Y_{1,g} - C(I_{1,g}), \\ \Pi_{1,b}^e &= q_{2,b} (R_1 I_{1,b}^e + \theta R_0 [(1 - \alpha) T_0 + T_{1,b}^e]) + Y_{1,b}^e - C(I_{1,b}^e), \\ \Pi_{1,b}^l &= q_{2,b} (R_1 I_{1,b}^l + \theta R_0 [I_0 - S_0 + (1 - \alpha) T_0 + T_{1,b}^l - S_{1,b}^l]) + Y_{1,b}^l - C(I_{1,b}^l). \end{aligned}$$

The first term in (6),  $\Pi_{1,g}$ , is expected profit at  $t = 1$  in the good state. In the good state, recall that all  $t = 0$  projects succeed early and the proceeds are either re-invested in new opportunities, with gross expected returns  $q_{2,g} R_1 I_{1,g} - C(I_{1,g})$ , or held as reserves,  $Y_{1,g}$ . Note that as intermediaries are all identical in this case, and each has sufficient funds to repay investors, there is no trade.

The second term in (6) is expected profit at period 2 in the bad state. Expected profits are a weighted sum of early and late types' profits,  $\Pi_{1,b}^e$  and  $\Pi_{1,b}^l$  respectively. Profits for early types consist of expected returns on new investment,  $q_{2,b} R_1 I_{1,b}^e - C(I_{1,b}^e)$ , reserves carried into period 2,  $Y_{1,b}^e$ , and late returns on securitized assets purchased either at  $t = 0$  or  $t = 1$ ,  $\theta R_0 [(1 - \alpha) T_0 + T_{1,b}^e]$ . On the other hand, profits for late types consist of investment returns,  $q_{2,b} R_1 I_{1,b}^l - C(I_{1,b}^l)$ , reserves  $Y_{1,b}^l$ , and late returns on assets not sold,  $\theta R_0 [I_0 - S_0 + (1 - \alpha) T_0 + T_{1,b}^l - S_{1,b}^l]$ .

Finally, the last three terms in (6) capture the costs of effort,  $\xi I_0$ , costs of investment in the initial period,  $C(I_0)$ , and debt repayment,  $rD$ . Intermediaries maximize (6) subject to

the following set of constraints:

$$(\lambda_0) I_0 + p_0(T_0 - S_0) + Y_0 \leq k + D, \quad (7)$$

$$(\lambda_{1,g}) I_{1,g} + Y_{1,g} \leq R_0(I_0 + T_0 - S_0) + Y_0, \quad (8)$$

$$(\lambda_{1,b}^e) I_{1,b}^e + p_1 T_{1,b}^e + Y_{1,b}^e \leq R_0(I_0 - S_0) + \alpha R_0 T_0 + Y_0, \quad (9)$$

$$(\lambda_{1,b}^l) I_{1,b}^l + p_1(T_{1,b}^l - S_{1,b}^l) + Y_{1,b}^l \leq \alpha R_0 T_0 + Y_0, \quad (10)$$

$$(\mu_{1,S}) S_0 + S_{1,b}^l \leq (1 - a)I_0, \quad (11)$$

$$(\mu_{1,T}) 0 \leq T_{1,b}^l + (1 - \alpha)T_0, \quad (12)$$

$$(\eta_{1,g}) rD \leq Y_{1,g}, \quad (\eta_{1,b}^l) rD \leq Y_{1,b}^e, \quad (\eta_{1,b}^e) rD \leq Y_{1,b}^l. \quad (13)$$

The solution to the intermediary problem is characterized in Appendix A, where the Lagrange multipliers associated with each constraint are given in brackets above. Inequality (7) is the budget constraint at  $t = 0$ , which requires investment costs, net purchases at price  $p_0$  and reserves be no greater than equity and debt. Expressions (8)-(10) are the budget constraints at  $t = 1$  in the good state, and for the early and late intermediaries in the bad state respectively. Early intermediaries projects have been successful, resulting in  $R_0(I_0 - S_0)$  more funds than late types. They can use the returns from their individual investments, along with securitized assets, to purchase assets from late ones at a price  $p_1$  or invest in new opportunities and reserves. Late intermediaries use returns from securitized assets, plus funds raised from asset sales, to finance new investment and reserves. Constraint (12) ensures that securitized asset sales are feasible (we ignore the analogous constraints on early types since they are never binding in equilibrium). The last set of constraints (13) are the intermediaries' collateral constraints that ensure debt is always repaid.<sup>15</sup> Constraint (11) requires further discussion and is explained below. The timing of actions is depicted in Figure 1.

## Moral hazard and skin-in-the-game

The definition of the intermediary problem outlined above assumes screening effort is undertaken, and that there is a skin-in-the-game constraint, namely expression (11). It follows from Assumption 1 that all things equal, buyers prefer that screening effort be undertaken by sellers. Thus, in a symmetric equilibrium all intermediaries' must choose to undertake such effort. They will do this precisely when  $\Pi_0$ , defined in (6), is at least as large as profits

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<sup>15</sup>In our environment, the long term contracts between intermediaries and investors are renegotiation-proof. To see this, note that we could simply re-interpret the contracts as short-term, which are then always rolled over at  $t = 1$  in equilibrium. However, the inability to commit to long term contracts may be a potentially important source of inefficiency in a more general environment.

when no screening effort is undertaken.<sup>16</sup> The latter is defined in the following expression.

$$\tilde{\Pi}_0 \equiv q_1 \Pi_{1,g} + (1 - q_1) (\alpha \Pi_{1,b}^e + (1 - \alpha) (\Pi_{1,b}^l - q_{2,b} \theta R_0 (I_0 - S_0 - S_{1,b}^l))) - C(I_0) - rD. \quad (14)$$

Therefore,  $\Pi_0 \geq \tilde{\Pi}_0$  reduces to constraint (11), where  $a > 0$  is given by

$$a = \frac{\xi}{(1 - q_1)(1 - \alpha)q_{2,b}\theta R_0}. \quad (15)$$

This says that an intermediary may not be able to sell all of its investments, as otherwise they do not have the incentive to exert screening effort. Indeed, in the next section we establish that this constraint can bind at the equilibrium, and that intermediaries will always be required to hold at least fraction  $a$  of their own investments.<sup>17</sup>

In this paper, skin-in-the-game requirements arise from intermediaries' aversion to effort. This type of constraint may also arise in a richer risk-shifting environment, as in [Acharya and Viswanathan \(2011\)](#). Both imply a similar capital structure whereby intermediaries retain a portion of the underlying risk in the form of an equity stake to ensure they have sufficient incentives to behave, while the remaining cash-flows are sold off to other intermediaries. Nevertheless, there are important implications of a richer environment. Generally, the corresponding skin-in-the-game requirements may be a function of ex-ante intermediary choices such as investment and effort, rather than simply being a constant fraction of initial investment as in (15) above. Thus, these would entail a richer but more complex set of effects in response to changes in aggregates which we have precluded in our environment for tractability.

### 3 Equilibrium

In this section we characterize the competitive market equilibrium. The intermediary problem is to choose investment, reserves, trade and debt levels to maximize expected profits subject to budget, collateral, sales, and investors' participation constraints. The investor problem is to choose how much debt and securities issued by intermediaries to purchase (if any), and savings to maximize expected utility of consumption subject to budget constraints. The price of debt,  $r$ , and the prices of securities  $p_0, p_1$ , are taken as given by intermediaries

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<sup>16</sup>We formulate the incentive-constraint such that  $\tilde{\Pi}_0$  represents the supremum of the set of all such values. This upper bound occurs when the seller is able to obtain the price  $p_0$  for the assets, even in the absence of undertaking effort. The use of this upper bound on the value to shirking ensures that the skin-in-the-game threshold (15) is sufficient to ensure effort for all out-of-equilibrium beliefs.

<sup>17</sup>We discuss the implications when the incentive constraint does not bind in equilibrium in Section 4.2.2.



- (a)  $B = D$  (market for debt at  $t = 0$ ),
- (b)  $T_0 = S_0$  (market for assets at  $t = 0$ ),
- (c)  $\alpha T_{1,b}^e = (1 - \alpha)(S_{1,b}^l - T_{1,b}^l)$  (market for assets at  $t = 1$ ).

We now describe the optimal decisions of investors and intermediaries given prices, and then show how market clearing determines equilibrium prices.

### 3.1 Optimal Decisions of Investors

Investors save by purchasing claims from intermediaries. Since investors value risky assets at their lowest possible realization, they are priced out of the market for risky assets by intermediaries. More specifically, given that all risky investments may fail and return nothing, investors value these at 0 while intermediaries value them at their net present values. As a result, investors purchase only the risk-free debt issued by intermediaries if their break-even condition on funds lent,  $r \geq \beta^{-1}$ , is satisfied. This participation constraint places a lower bound on the equilibrium interest rate with investors willing to supply funds inelastically as long as this condition is met. We summarize investor behavior in the Lemma below:

**LEMMA 1.** *Investors demand only safe debt, where* 
$$B = \begin{cases} 0, & \text{if } r < \beta^{-1} \\ [0, w], & \text{if } r = \beta^{-1} \\ w, & \text{if } r > \beta^{-1} \end{cases} .$$

### 3.2 Optimal Decisions of Intermediaries

The following assumption allows us to focus on non-trivial equilibria in which intermediaries have incentives to borrow at  $t = 0$ .

**ASSUMPTION 4.** (*Positive Leverage*)

$$\begin{aligned} E_\pi R_0 - \beta^{-1} - \xi - C'(k) \\ > q_1(R_0 - \beta^{-1})(q_{2,g}R_1 - 1 - C'(R_0k)) + (1 - q_1)(1 - \alpha)\beta^{-1}(q_{2,b}R_1 - 1). \end{aligned}$$

Assumption 4 ensures that the marginal benefit of debt is positive when  $D = 0$ , which holds when  $t = 0$  investment returns are sufficiently high, and establishes the following result.

**LEMMA 2.** *In equilibrium, intermediaries do not hold cash reserves at  $t = 0$  (i.e.,  $Y_0 = 0$ ), and  $0 < D < w$  when investor wealth is sufficiently high.*



*Proof.* See Appendix B. □

It is never optimal for intermediaries to finance reserve holdings at  $t = 0$  via debt. To see this, note that for every unit of debt raised at  $t = 0$ , intermediaries must generate  $r - 1 \geq \beta^{-1} - 1 > 0$  at  $t = 1$  to service this additional unit of debt. Given Assumption 4, some debt is always valuable so that leverage is not zero. Returns to borrowing eventually become negative however, due to the convexity of investment costs, and thus there is a limit to the amount of resources intermediaries can absorb. We assume throughout the paper that  $w$  is sufficiently large to guarantee that all wealth is not absorbed, i.e.,  $D < w + k$ .<sup>18</sup>

We now consider the optimal reserve holdings, trade and investment decisions by intermediaries at period 1, taking as given prices and period 0 decisions. First, note that neither returns from new investments at  $t = 1$ , nor late returns on  $t = 0$  investments can be pledged to repay investors at  $t = 2$ . This is because investors value these pledges at the lowest possible return, which is zero. Thus, intermediaries must carry reserves equal to at least  $rD$  into period 2 to ensure that debt is repaid irrespective of the state at  $t = 1$ .

Consider trade between intermediaries at  $t = 1$  when the state at is good. In this case, intermediaries are identical as all  $t = 0$  investments succeed early, and hence there is no trade. Thus, intermediaries simply set aside the required reserves and invest the remainder in new opportunities since these are always worthwhile, from Assumption 2. Hence,  $I_{1,g} = R_0 I_0 - rD$ .

In the bad state, intermediaries differ at period 1 in that the proportion  $\alpha$  of intermediaries projects are successful. These early types receive the full return on the fraction of  $t = 0$  investments that were not securitized. The fraction  $1 - \alpha$  of late types do not receive early returns on their own investments. Due to securitization, all intermediaries also receive a fraction of the early returns from other intermediaries' projects.

For a given  $p_1$ , early types can use their funds to either invest in new opportunities or purchase assets from late types. The amount of new investment,  $I_{1,b}^e$ , equates the marginal return to investment with the marginal return on purchasing assets. The former is simply  $q_{2,b}R_1 - C'(I_{1,b}^e)$ , while the latter is  $q_{2,b}\theta R_0/p_1$ , as  $q_{2,b}\theta R_0$  is the net present value on  $t = 0$  investments in the bad state. Note that for higher values of  $p_1$ , the return on purchasing assets is lower and therefore more investment is undertaken and fewer assets are purchased by early types. Analogously, for lower values of  $p_1$ , early types purchase more assets, and invest less. The investment and sales decisions by late types involve a similar trade-off.

By selling  $t = 0$  assets, late types forgo the returns, but can increase new investment and/or generate reserves required to service debt. Sales consist of securitized assets on hand,  $-T_{1,b}^l$ , as well as any of their own investments which were not sold at  $t = 0$ ,  $S_{1,b}^l$ . Late types

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<sup>18</sup>This does not affect the qualitative nature of the results and is done to simplify the welfare analysis.

may be constrained if they run out of assets to sell, in which case the multipliers for incentive and sales constraints will bind ( $\mu_{1,S}, \mu_{1,T} > 0$ ).

**LEMMA 3.** *Investment, sales and purchases of assets, and cash reserves in the bad state at  $t = 1$  are as follows:*

*Early types:*

$$\begin{aligned} I_{1,b}^e &: q_{2,b}R_1 - C'(I_{1,b}^e) = \frac{q_{2,b}\theta R_0}{p_1}, \\ T_{1,b}^e &= \frac{R_0(I_0 - S_0) + \alpha R_0 T_0 - rD - I_{1,b}^e}{p_1}, \\ Y_{1,b}^e &= rD. \end{aligned}$$

*Late types:*

$$\begin{aligned} I_{1,b}^l &: q_{2,b}R_1 - C'(I_{1,b}^l) = \frac{q_{2,b}\theta R_0}{p_1} + \frac{\mu_{1,T}}{(1 - q_1)(1 - \alpha)}, \\ S_{1,b}^l - T_{1,b}^l &= \min \left[ \frac{I_{1,b}^l + rD - \alpha R_0 T_0}{p_1}, (1 - \alpha)T_0 + (1 - a)I_0 - S_0 \right], \\ Y_{1,b}^l &= rD. \end{aligned}$$

*Proof.* See Appendix B. □

As can be seen from Lemma 3, if  $\mu_{1,T} > 0$ , equilibrium investment levels will differ across types at  $t = 1$ . As a result, when late types are constrained at  $t = 1$ , intermediaries always find it optimal to securitize as much as possible.

**LEMMA 4.**  $\mu_{1,T} > 0 \iff I_{1,b}^e > I_{1,b}^l > 0 \iff \mu_{1,S} > 0$ . *Moreover, when constrained, intermediaries prefer to securitize as much as possible at  $t = 0$ , i.e.,  $S_0 = (1 - a)I_0$ , and  $S_{1,b}^l = 0$ .*

*Proof.* See Appendix B. □

Late intermediaries may only trade assets at  $t = 1$  to generate funds for investment. If late intermediaries cannot raise sufficient funds,  $I_{1,b}^e > I_{1,b}^l$ . Furthermore, when  $\mu_{1,T} > 0$ , securitized assets are worth more than individual investments, since they provide relatively more resources to late types who value them more. As result, being constrained at  $t = 1$  means that intermediaries will securitize to the extent possible at  $t = 0$ .

Finally, we describe the optimal choice of investment at  $t = 0$ , which is determined by

the following first order condition on  $I_0$ :

$$E_\pi R_0 - r + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'(I_{1,g})) + (1 - q_1)\alpha(R_0 - r)(q_{2,b}R_1 - 1 - C'(I_{1,b}^e)) \\ + (1 - q_1)(1 - \alpha)(-r)(q_{2,b}R_1 - 1 - C'(I_{1,b}^l)) + \mu_{1,S}(1 - a) = \xi + C'(I_0). \quad (16)$$

The marginal return to a unit of investment at  $t = 0$ , given that  $D > 0$  is  $E_\pi R_0 - r$  plus the marginal returns from re-investing early returns at  $t = 1$ . When the state at  $t = 1$  is good, each additional unit of  $I_0$  (financed by one unit of  $D$ ) generates  $R_0 - r$  units of resources at  $t = 1$  that can be reinvested for a net expected return  $q_{2,g}R_1 - 1 - C'(I_{1,g})$ . Similarly, when the state at  $t = 1$  is bad, another unit of  $I_0$  using borrowed funds generates  $R_0 - r$  units of resources for the early types and  $-r$  units for the late types. These can be reinvested at net returns of  $q_{2,b}R_1 - 1 - C'(I_{1,b}^e)$  and  $q_{2,b}R_1 - 1 - C'(I_{1,b}^l)$ . If the sales constraint binds at  $t = 0$ , increasing  $I_0$  provides an additional benefit: it raises by  $(1 - a)$  units the quantity of assets late types can sell at  $t = 1$ , thereby mitigating the financial frictions intermediaries face at  $t = 1$ . Moreover, the value of relaxing the sales constraint ( $\mu_{1,S}$ ) depends on the anticipated asset price  $p_1$ . A higher price lowers this value while a lower price raises it. Thus, when intermediaries are constrained, securitization affects the level of ex-ante investment through both the level of  $a$  and the price  $p_1$ . The optimal choice of  $I_0$  simply equates the marginal benefit of investment, the left hand side of (16), with its marginal cost,  $\xi + C'(I_0)$ .<sup>19</sup>

### 3.3 Market Clearing

From the optimal choices of investors and intermediaries, we can infer that  $r$  must satisfy the following bounds  $E_\pi R_0 - \xi - C'(k) \geq r \geq \beta^{-1}$ . Since demand for debt is downward sloping and supply is perfectly elastic at a price of  $\beta^{-1}$ , we have  $r = \beta^{-1}$  in equilibrium.

Consider the  $t = 0$  market for securitized assets. It is shown in Lemma 4 that when constrained at  $t = 1$ ,  $S_0 = T_0 = (1 - a)I_0$ . When unconstrained, intermediaries are indifferent over their choices of  $T_0$  and  $S_0$ . Regardless of the choices of  $T_0$  and  $S_0$ , any candidate equilibrium price  $\tilde{p}_0$  must clear the market, and thus  $S_0(\tilde{p}_0) - T_0(\tilde{p}_0) = 0$ . Inspecting the intermediaries' problem, it is clear that  $\tilde{p}_0$  has no effect on the budget, since all agents are identical and net purchases are zero. Thus optimal choices are determined by the first order conditions from the intermediaries' problem at a given  $p_1$ , which are provided in Appendix

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<sup>19</sup>If intermediaries incur losses at  $t = 1$ , these will be borne by their equity which would then place an upper bound on  $I_0$ . In such a case, investment at  $t = 1$  is zero, which is never true if the intermediary is constrained as shown in the proof of Lemma 4. The constrained case is the main focus of the paper and thus we ignore the possibility that intermediary equity is entirely wiped out at  $t = 1$ .

A. The  $t = 0$  price that clears the market satisfies

$$p_0 = \frac{a\mu_{1,S}^* + \xi + C'(I_0^*)}{\lambda_0^*} + 1. \quad (17)$$

In an unconstrained equilibrium,  $p_0 = (\xi + C'(I_0^*))/\lambda_0^* + 1$  is simply the marginal cost of time 0 investment. When constrained,  $p_0$  reflects the fact that securitized assets are relatively more valuable in this type of equilibrium, as they provide more resources to late types in the bad state of the world.

We now consider the determination of  $p_1$ . From the optimal choices of intermediaries described in Lemma 3, we focus on prices in the range  $q_{2,b}\theta R_0 \geq p_1 \geq \theta R_0/R_1$ . To understand these bounds, note that if  $p_1$  were to exceed the conditional return on assets, early types would not be willing to purchase them, since they can always invest in new projects that earn positive profit. Thus, at the equilibrium, assets will only trade at fire-sale prices (i.e. below NPV). On the other hand, if  $p_1$  is below  $\theta R_0/R_1$ , early types do not make any new investments as buying up cheap assets is more profitable (and thus late types do not invest either).<sup>20</sup>

The following proposition ensures the existence of a unique constrained equilibrium, which is the main focus of subsequent analysis.

**PROPOSITION 1.** *Given Assumptions 1-4, a symmetric competitive equilibrium exists. The equilibrium may be constrained such that  $\mu_{1,T} > 0$ , in which case it is unique.*

*Proof.* See Appendix B. □

A sufficient condition for the existence of a constrained equilibrium is provided in the proof of Proposition 1. Intuitively, late types are constrained in equilibrium when screening costs are low, the value of  $t = 0$  assets is relatively low in the bad state, conditional on undertaking effort (i.e.  $\theta$  is small),  $t = 1$  returns are high and/or when there is more heterogeneity across intermediaries at  $t = 1$ . Most importantly, the extent to which intermediaries can securitize assets also determines if late types will be constrained, and to what extent.

## 4 Welfare

In this section, we examine the efficiency of allocations at the competitive equilibrium. It is instructive to begin by characterizing the first-best allocation, as it helps to clarify the role of market incompleteness and securitization in the subsequent analysis.

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<sup>20</sup>We ignore the possibility that there is zero aggregate investment at  $t = 1$ . In this case, the equilibrium is unconstrained (see Lemma 4), and  $I_0$  is simply determined by the collateral constraints.

## 4.1 First-Best

We consider a planner that maximizes total expected surplus in the economy subject to the participation of investors. We focus on the point of the first-best frontier at which investors receive a utility of  $w$ , the level they achieve in the market outcome. This is the simplest way to compare the allocations achieved by a planner, with those that result in the competitive equilibrium. Assuming that the planner places an equal weight on all intermediaries, the planner's problem is to choose *aggregate* quantities  $I_0, D, Y_0, I_{1,g}, I_{1,b}^e, I_{1,b}^l, Y_{1,g}, Y_{1,b}$  to maximize welfare  $\Pi^P$  where

$$\begin{aligned} \Pi^P = & q_1 [q_{2,g}R_1I_{1,g} - C(I_{1,g}) + Y_{1,g}] + (1 - q_1) [\alpha(q_{2,b}R_1I_{1,b}^e - C(I_{1,b}^e)) \\ & + (1 - \alpha)(q_{2,b}R_1I_{1,b}^l - C(I_{1,b}^l)) + Y_{1,b}] + (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0I_0 - \xi I_0 - C(I_0) - rD, \end{aligned} \quad (18)$$

subject to the participation constraint of investor,  $r = 1/\beta$ , and the following budget and debt repayment constraints:

$$(\lambda_0) I_0 + Y_0 \leq k + D, \quad (19)$$

$$(\lambda_{1,g}) I_{1,g} + Y_{1,g} \leq R_0I_0 + Y_0, \quad (20)$$

$$(\lambda_{1,b}) \alpha I_{1,b}^e + (1 - \alpha)I_{1,b}^l + Y_{1,b} \leq \alpha R_0I_0 + Y_0, \quad (21)$$

$$(\eta_{1,g}) rD \leq Y_{1,g}, \quad (\eta_{1,b}) rD \leq Y_{1,b}. \quad (22)$$

We refer to the solution of this problem as the first-best, the salient features of which are outlined in the proof of the following result.

**LEMMA 5.** *In the first-best, investment is equalized across intermediaries at  $t = 1$  in every state.*

*Proof.* See Appendix B. □

The planner maximizes the profits of all intermediaries jointly subject to a single budget constraint in each state. This implies that investment by late and early types is always equated at  $t = 1$  since investment technologies across intermediaries are identical and exhibit decreasing returns to scale. This first-best outcome can also be achieved in a decentralized competitive equilibrium when intermediaries can write contingent contracts at  $t = 0$  to transfer resources from early to late types at  $t = 1$ . This is shown in the proof of Lemma 5. It is also straightforward to show that the first-best obtains in the case where  $a = 0$ , in which case securitization fully completes markets.

## 4.2 Incomplete Markets, Securitization and Efficiency

When the aggregate state or individual type information is not ex-ante contractible and borrowing ex-post is infeasible, markets are incomplete and late intermediaries that have insufficient funds to repay debt and invest are forced to generate funds via asset sales on the spot market. Securitization changes the distribution of returns at  $t = 1$ , moving resources from early to late types, thereby substituting for contingent contracts and limiting the need for asset sales. This is precisely how securitization substitutes for missing markets in our environment.

To understand the nature of the inefficiency associated with securitization in the competitive equilibrium, it is necessary to establish a welfare benchmark. Consider a planner subject to the same market restrictions as intermediaries. Such a planner cannot directly re-allocate funds from early to late types at  $t = 1$  in the bad state. Instead, the planner must rely on asset sales to achieve re-allocations across types. As a result, the planner cannot always equalize marginal returns to investment across types at  $t = 1$  as in the first-best allocation. Thus, a second-best planning problem is nearly identical to the intermediaries', except that the planner's choices at  $t = 0$  reflect aggregate quantities and thus the planner can fully account for any price effects that the choice of these aggregates have on  $t = 1$  decisions. As in Section 4.1, we fix investor utility to that which obtains in the market equilibrium, which constrains the price of debt to  $r = 1/\beta$ .

Rather than directly compare the competitive equilibrium with the second-best allocation, we take an alternative approach to establish the inefficiency of the market equilibrium. Consider a perturbation of aggregate investment at  $t = 0$ , at the competitive equilibrium, such that the change is equal across intermediaries. Such a perturbation is financed by borrowing from investors and thus raises leverage of the intermediation sector, but does not change  $r$ . Using the envelope theorem, the change in welfare from such a perturbation is:

$$\frac{d\Pi_0}{dI_0} = \frac{dp_1}{dI_0} \cdot \alpha T_{1,b}^{e*} \left( \frac{\lambda_{1,b}^{l*}}{1-\alpha} - \frac{\lambda_{1,b}^{e*}}{\alpha} \right) = \frac{dp_1}{dI_0} \left[ \frac{\mu_{1,T}^*}{p_1^*} (1-\alpha)(1-a)I_0^* \right]. \quad (23)$$

If  $d\Pi_0/dI_0 \neq 0$ , the equilibrium is constrained inefficient and a planner could engineer a Pareto improvement, even when subject to the same market incompleteness as intermediaries. Equation (23) captures a price effect which represents the difference between the individuals' first order condition on  $t = 0$  investment and that from the second-best planner's problem.<sup>21</sup> This price effect is non-zero when the marginal return on investment differs

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<sup>21</sup>The perturbation generally affects both prices  $p_0$  and  $p_1$ , however changes in  $p_0$  have no impact on time 0 intermediaries at the equilibrium, since each has net securitized assets purchases of zero.

across early and late types, captured here by the difference in the date 1 multipliers on the intermediaries' budget constraints in the bad state, weighted by the relevant population sizes. This is true precisely when late types are constrained in their ability to raise funds at  $t = 1$ , i.e.,  $\lambda_{1,b}^l/(1 - \alpha) - \lambda_{1,b}^e/\alpha > 0 \iff \mu_{1,T} > 0$ . We summarize the above discussion in the following result.

**PROPOSITION 2.** *If late intermediaries are constrained in their ability to raise funds, i.e.,  $\mu_{1,T} > 0$ , then the competitive equilibrium is constrained inefficient, in that a planner facing the same constraints as the private market can engineer a Pareto improvement. Moreover, assets are securitized if and only if aggregate investment is constrained inefficient.*

*Proof.* See Appendix B. □

The intuition behind the inefficiency is as follows. Generally, individual investment decisions at  $t = 0$  impact the price of assets at  $t = 1$  in the bad state. In the unconstrained case, price changes represent a redistribution of resources across intermediaries, which are irrelevant for welfare as they are risk-neutral. When  $\mu_{1,T} > 0$  however, the price of assets in the bad state at  $t = 1$  affects the ability of late types to raise funds, which in turn impacts aggregate investment at  $t = 1$ . Since atomistic intermediaries do not take into account the effect of their  $t = 0$  investment decisions on the  $t = 1$  asset price, there is a pecuniary externality that renders the competitive equilibrium inefficient. Moreover, securitization is only valuable when the competitive equilibrium is inefficient, as this is precisely when intermediaries are constrained and value the additional ex-ante insurance that securitization provides.

Assumption 2 ensures that all intermediaries have access to positive NPV investments at  $t = 1$ . This is crucial for the existence of constrained inefficient equilibria characterized by over-investment. To see this, consider the case in which late intermediaries exhaust their investment opportunities at  $t = 1$ . In this case, they are not constrained and therefore no pecuniary externality exists. Now consider the other extreme case in which early intermediaries have no investment opportunities. In this case, early types value assets sold by late types at NPV or  $\theta R_0/R_1$ . Therefore, even though late intermediaries may be constrained in their ability to raise funds, no pecuniary externality can exist as changing  $t = 1$  resource allocations cannot raise the price any further.

The following proposition characterizes the link between frictions in the securitization process and the efficiency of the competitive equilibrium.

**PROPOSITION 3.** *When frictions associated with the securitization process are relatively small, i.e.,  $\xi$  is small, the competitive market equilibrium is first-best. When frictions associated with securitization are sufficiently large, i.e.,  $\xi$  is large, the competitive equilibrium*

is constrained inefficient. Formally, there exists an  $\bar{a} \in (0, 1)$ , such that for  $a < \bar{a}$ , the competitive market equilibrium is first-best, while for  $a > \bar{a}$ , late types are constrained and the competitive equilibrium is constrained inefficient.

*Proof.* See Appendix B. □

To understand this result, recall that the extent to which late intermediaries are constrained depends on  $a$ . When a large enough proportion of assets can be securitized, the late types will not be constrained in raising funds at  $t = 1$ . In this case, the marginal return to  $t = 1$  investment is equalized across intermediaries and thus  $t = 0$  decisions reflect the full social cost and benefit of investment (i.e., the allocation is first-best). In the extreme case in which  $a = 0$ , this is obvious since all assets can be securitized and thus intermediaries have the same resources at  $t = 1$ . On the other hand, if the frictions associated with securitization are sufficiently large, the competitive equilibrium is constrained inefficient. Importantly, in this case the competitive market equilibrium is not only inefficient relative to the first-best, but the second-best as well. Thus, micro level frictions in the securitization process lead to constrained inefficient investment at the macro level. We further characterize the nature of the inefficiency as follows.

**COROLLARY 1.** *When the competitive market equilibrium is constrained inefficient, time 0 investment may be either too large or too small relative to the second-best. If  $\alpha R_0 - 1/\beta > 0$ , there is under-investment, while  $1/\beta - \alpha R_0 > a(1 - \alpha)R_0$  is sufficient for over-investment.*

*Proof.* See Appendix B. □

From Equation (23), it is clear that the sign of the inefficiency is determined entirely by the sign of  $dp_1/dI_0$ . To determine the sign of the price effect, note that  $\alpha R_0 - 1/\beta$  is the marginal change in aggregate cash in the market at  $t = 1$  in the bad state that arises from an additional unit of  $I_0$ . If this is positive, an increase in  $I_0$  increases cash in the market and puts upward pressure on the asset price, thus making  $dp_1/dI_0$  positive. The price impact from investment at  $t = 0$  on cash in the market at  $t = 1$  is not considered by atomistic individuals and as a result  $I_0$  is too small from a social perspective. Analogously, if  $\alpha R_0 - 1/\beta$  is sufficiently negative, more investment at  $t = 0$  reduces cash in the market in the bad state of the world and this reduces the fire-sale price and results in over-investment.

#### 4.2.1 Permitting Borrowing Between Intermediaries at $t = 1$

Assumption 3 is crucial to the inefficiency described in (23). If intermediaries could pledge all cash flows to lenders at  $t = 1$ , the resulting equilibrium would be efficient. Completely



ruling out borrowing between intermediaries at  $t = 1$  is helpful to ease exposition, and we show in this section that this can be relaxed without affecting the qualitative results of the model.

Consider a contingent contract in which the lender receives a gross per-unit return  $\rho$  when the borrowers' investment is successful and nothing otherwise. This is not restrictive since both parties are risk-neutral. Importantly, as above we assume that the borrower must exert effort to ensure asset quality. When screening effort is exerted, the probability of success conditional on the good state at time 2 is  $p_H$ , where  $0 < p_H < 1$ . Alternatively, it is only  $p_H - \phi$  (where  $0 < \phi < p_H$ ). As above, we assume the cost of screening effort at time 1 is proportional to investment,  $\xi I_{1,b}^l$ , and that the fall in asset quality is sufficiently large that lenders only wish to lend when effort has been undertaken.

Given the above, we write the following incentive-constraint at time 1, which limits the amount  $B$  that can be borrowed.

$$q_{2,b}p_H (R_1 I_{1,b}^l - \rho B) - \xi I_{1,b}^l \geq q_{2,b}(p_H - \phi) (R_1 I_{1,b}^l - \rho B) \quad (24)$$

$$\implies \rho B \leq \left( R_1 - \frac{\xi}{q_{2,b}\phi} \right) I_{1,b}^l \quad (25)$$

Note that early types are both the purchasers of securitized assets as well as lenders, and that risk-neutrality ensures  $R_0/p_1 = \rho$ .

As in the case with asset sales only, we can perturb aggregate investment at the equilibrium. Denote the multiplier on the incentive constraint (25) by  $\mu_{1,b}$ .

$$\frac{d\Pi^*}{dI_0} = \frac{dp_1}{dI_0} \left[ \frac{\mu_{1,b}^*}{p_1^*} (1 - \alpha)(1 - a)I_0^* + \mu_{1,b}^* B^* \frac{\rho^*}{p_1^*} \right]. \quad (26)$$

Equation (26) is identical to (23), except for the additional non-negative term  $\mu_{1,b}^* B^* \rho^* / p_1^*$  inside the square brackets. The price effects of changes in  $I_0$ , which also affects  $\rho$  through the arbitrage condition in this case, have an impact on welfare whenever late types are constrained at the equilibrium. Thus, while borrowing at  $t = 1$  may have quantitative implications, the externality is completely analogous since the incentive constraint binds if and only if the sales constraint binds.

#### 4.2.2 Permitting Loss of Asset Quality

Throughout the paper, we focus on equilibria in which intermediaries' undertake screening effort to improve asset quality. Nevertheless, here we briefly examine the case in which the incentive constraints arising from the moral hazard problem in period 0 are violated,

despite Assumption 1. Clearly, any equilibrium where incentive constraints do not hold is characterized by the prevalence of low quality assets. Moreover, such an equilibrium is constrained efficient and is marked by the absence of fire-sales even in the bad state.

To better understand this, note that from Assumption 1, screening effort is worthwhile for the individual intermediary that retains all of its investment. Of course, if a sufficient fraction of the assets are sold and effort cannot be observed, this is no longer true. In such a case, all assets are of low quality and all assets are securitized in equilibrium. As a result, there is no heterogeneity and no trade at time 1, and thus no pecuniary externality. Regardless, this case is clearly not efficient, since effort is assumed to be worthwhile.

The primitives under which this type of equilibrium might obtain can be ascertained from the expression in Assumption 1, namely  $(1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 > \xi$ . The left hand side represents the marginal value of screening effort, and the right hand side the marginal costs. The values of these parameters also determine the threshold value of skin-in-the-game  $a$ , as described in (15). If the returns to effort were sufficiently reduced (i.e., the left hand side) relative to  $\xi$ , the value of  $a$  increases which serves makes the type of equilibrium described here more likely to arise.<sup>22</sup>

### 4.3 Over-investment and Securitization

In this section we focus on over-investment, the case in which the competitive equilibrium is characterized by excessively high leverage ex-ante and low asset prices ex-post. We show below that increasing the amount of securitization that is feasible will further reduce prices in a fire-sale, thus it is reasonable to think that more securitization could be undesirable relative to the second-best. To formalize this type of argument, rewrite constraint (11) as  $S_0 + S_{1,b}^l \leq (1 - a - \Delta)I_0$ , where  $\Delta = 0$ . We consider the impact of a tightening of the skin-in-the-game constraint on the intermediary at the competitive equilibrium by increasing  $\Delta$  from an initial value of zero.

$$\left. \frac{d\Pi_0}{d\Delta} \right|_{\Delta=0} = -\mu_{1,S}^* I_0^* + \alpha T_{1,b}^{e*} \left( \frac{\lambda_{1,b}^{l*}}{1 - \alpha} - \frac{\lambda_{1,b}^{e*}}{\alpha} \right) \frac{dp_1}{d\Delta}. \quad (27)$$

We first note that this derivative is zero in the unconstrained case. This is because investment at early and late types is identical when unconstrained. When constrained, the first term in the expression above is a direct effect, which is always negative and captures the fact that an increase in skin-in-the-game restricts the ability of late intermediaries to generate funds

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<sup>22</sup>Establishing existence is straightforward and analogous to the proof of the unconstrained case in Proposition 1.

at  $t = 1$ .

The second term in (27) is an indirect effect, which captures the change in the equilibrium price due to changes in the distribution of cash-in-the-market at  $t = 1$ . The sign of the indirect effect is determined by  $dp_1/d\Delta$ , since the bracketed term in (27) is strictly positive in a constrained equilibrium. If the price effect is negative, then a smaller value of skin-in-the-game, i.e., more securitization, improves welfare unambiguously. On the other hand, if the price effect is positive and dominates the direct effect, less securitization leads to higher welfare. We show in the proof of Proposition 4 below that  $dp_1/d\Delta$  is in fact positive in the over-investment case, but that the total effect is generally negative.

**PROPOSITION 4.** *In the over-investment case,  $dp_1/d\Delta > 0$  and  $d\Pi_0/d\Delta < 0$  at the equilibrium, i.e., welfare is increasing in the extent of securitization.*

*Proof.* See Appendix B. □

At a given price, an increase in skin-in-the-game reduces investment at  $t = 0$ . This is because a higher value of skin-in-the-game changes the distribution of cash in the market for a given  $I_0$ , resulting in a greater proportion of returns for early types that value cash less in a constrained equilibrium. On the other hand, a reduction in  $I_0$  puts upward pressure on the price of assets at  $t = 1$ . This price effect makes assets more valuable, encouraging higher  $I_0$ , and opposing the direct effect described above. We are left with a somewhat counterintuitive conclusion; that more securitization leads to more investment/leverage, which is welfare improving *even when investment/leverage is excessive*.

## 5 Policy

The most recent financial crisis has resulted in significant regulatory reforms worldwide, a number of which were directed at markets for securitized assets. The most important being the Dodd-Frank Act in the United States, and the regulatory reform initiatives across the European Union.<sup>23</sup> There are many subtleties, and a number of differences in the reforms across jurisdictions. However, we can group these into two broad categories: retention and transparency requirements, and we focus on the former.

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<sup>23</sup>The Dodd-Frank Wall Street Reform and Consumer Protection Act was signed July 21, 2010, but continues to adapt new rules, put in place by a number of regulatory agencies to implement its (broad) goals. In the European Union, securitization reform arises from various sections of the Basel II and III Accords, the Capital Requirements Directive, as well as a variety of others. As a part of the “Action Plan on Building a Capital Markets Union”, the European Commission published draft regulations on September 2015, which outline significant changes (if implemented) to the way in which securitization markets function in Europe.

Retention requirements, set to 5% skin-in-the-game in the US and EU reforms, can be interpreted as imposing a higher level of skin-in-the-game in our environment.<sup>24</sup> We have shown in Proposition 4 that increases in skin-in-the-game beyond the laissez-faire level are welfare-reducing in a constrained equilibrium, and have no welfare implications in the unconstrained case (the regulation would not be binding). This is true despite the fact that increased skin-in-the-game serves to reduce socially excessive leverage and raises prices in a fire-sale.

Our previous discussion applies to policies that are directly targeted at securitization. However, a number of regulatory reforms enacted as part of the Basel III agreement are also designed to limit excessive investment and/or leverage in the financial sector. One such reform is a restriction on leverage that is independent of asset risk. In our model, as ex-ante investment may be inefficient, a direct leverage restriction (or capital requirement as the two are equivalent in our model), can improve welfare. However, this type of regulation presents a number of practical difficulties that arise from asymmetric information between intermediaries and regulators.<sup>25</sup> Perhaps more importantly, many participants in securitization markets do not fall under the regulatory umbrella, i.e., are shadow banks. Thus, unlike the policies discussed above, tying the hands of regulated entities through leverage and/or capital controls would likely result in more resources flowing into the shadow sector. This would seem to be a major hurdle facing regulators going forward and represents an important area of further research. As a summary of our results, we highlight the policy implications of the model in the following Proposition.

**PROPOSITION 5.** *When the competitive equilibrium is characterized by over-investment:*

- *Welfare decreases if skin-in-the-game requirements exceed the laissez-faire level.*
- *Direct leverage restrictions, when imposed on all intermediaries, are welfare increasing.*

These results follow directly from the analysis in Section 4. The first point is simply a restatement of Proposition 4. To see the second point, note that a reduction in leverage in our framework is equivalent to a perturbation that reduces the initial investment  $I_0$ , for

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<sup>24</sup>Article 122a of the European Capital Requirements Directive and Section 941 of the U.S. Dodd-Frank Act both require a five percent minimum retention rate by securitizers or originators, with exceptions for various types of underlying assets. Notably, “qualified” residential mortgage backed securities, which are backed by loans that meet a specific underwriting criteria. Recently, the DC federal court of Appeals also decided that these requirements ought not apply to managers raising capital to purchase Collateralized Loan Obligations.

<sup>25</sup>For instance, identical portfolios can have significantly different levels of leverage depending on the accounting treatment of derivatives (GAAP versus IFRS).

all intermediaries.<sup>26</sup> Equation (23) characterizes the welfare impact of a change in leverage, which is negative in the case of over-investment, as shown in the proof to Corollary 1.

Our model also makes a number of empirical predictions. First, (17) implies that high prices for securitized assets today, that is the divergence of prices for securitized assets from fundamentals, foreshadows an inefficient fire-sale and a credit-crunch in the future. This is consistent with the behavior of prices for ABS before and after the recent crisis. Furthermore, if returns on securitized assets in a fire-sale are below the cost of financing at the margin, the equilibrium is characterized by over-investment. Conversely, if returns on securitized assets exceed financing costs in a fire-sale, the equilibrium will exhibit under-investment. This is most easily seen from the conditions for the existence of these different equilibria, namely equations (90) and (92).

## 6 Conclusion

We study the efficiency of competitive equilibria in an incomplete markets economy where securitization improves risk-sharing, and thus can raise overall investment. We find that by its very nature, securitization has unintended consequences that render the competitive market equilibrium inefficient. Specifically, while the additional insurance offered by securitization can increase overall investment, it may also lead to a pecuniary externality whereby the resulting expansion in investment is socially excessive. A regulatory intervention that directly limits leverage can mitigate this over-investment problem, as long as it can be applied to banks and non-banks. On the other hand, forcing issuers of securitized assets to hold more skin-in-the-game than the laissez-faire level is not welfare improving.

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<sup>26</sup>Importantly these restrictions ought to be imposed on both banking and non-banking firms.

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## A Solution to the Intermediary Problem

The necessary conditions for the intermediary problem, given in (6)-(13), are as follows:

$$I_0 : [(1 - q_1)(1 - \alpha)q_{2,b}\theta + \lambda_{1,g} + \lambda_{1,b}^e]R_0 + \mu_{1,S}(1 - a) - \lambda_0 - \xi - C'(I_0) \leq 0 \quad (28)$$

$$T_0 : (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 + (\lambda_{1,g} + (\lambda_{1,b}^e + \lambda_{1,b}^l)\alpha)R_0 + \mu_{1,T}(1 - \alpha) - \lambda_0 p_0 \leq 0 \quad (29)$$

$$S_0 : - (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 - (\lambda_{1,g} + \lambda_{1,b}^e)R_0 - \mu_{1,S} + \lambda_0 p_0 \leq 0 \quad (30)$$

$$D : - r + \lambda_0 - r(\eta_{1,g} + \eta_{1,b}^e + \eta_{1,b}^l) \leq 0 \quad (31)$$

$$I_{1,g} : q_1(q_{2,g}R_1 - C'(I_{1,g})) - \lambda_{1,g} \leq 0 \quad (32)$$

$$I_{1,b}^e : (1 - q_1)\alpha(q_{2,b}R_1 - C'(I_{1,b}^e)) - \lambda_{1,b}^e \leq 0 \quad (33)$$

$$T_{1,b}^e : (1 - q_1)\alpha q_{2,b}\theta R_0 - \lambda_{1,b}^e p_1 \leq 0 \quad (34)$$

$$I_{1,b}^l : (1 - q_1)(1 - \alpha)(q_{2,b}R_1 - C'(I_{1,b}^l)) - \lambda_{1,b}^l \leq 0 \quad (35)$$

$$T_{1,b}^l : (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 - \lambda_{1,b}^l p_1 + \mu_{1,T} = 0 \quad (36)$$

$$S_{1,b}^l : - (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 + \lambda_{1,b}^l p_1 - \mu_{1,S} \leq 0 \quad (37)$$

$$Y_0 : - \lambda_0 + \lambda_{1,g} + \lambda_{1,b}^e + \lambda_{1,b}^l \leq 0 \quad (38)$$

$$Y_{1,g} : q_1 - \lambda_{1,g} + \eta_{1,g} = 0 \quad (39)$$

$$Y_{1,b}^e : (1 - q_1)\alpha - \lambda_{1,b}^e + \eta_{1,b}^e = 0 \quad (40)$$

$$Y_{1,b}^l : (1 - q_1)(1 - \alpha) - \lambda_{1,b}^l + \eta_{1,b}^l = 0 \quad (41)$$

First consider the intermediaries' problem at  $t = 1$ , taking prices  $r$ ,  $p_0$ ,  $p_1$ , and quantities  $I_0$ ,  $S_0$ ,  $T_0$  and  $D$  as given. Since intermediaries have access to positive NPV projects at  $t = 1$  (Assumption 2), it follows from (32), (33), and (35) that the budget constraints bind:  $\lambda_{1,g}$ ,  $\lambda_{1,b}^e$ ,  $\lambda_{1,b}^l > 0$ . This implies that intermediaries hold exactly the amount of cash needed to repay debt, and surplus funds are invested. Formally, combining (32)-(35) with (39)-(41) implies  $\eta_{1,g}$ ,  $\eta_{1,b}^e$ ,  $\eta_{1,b}^l > 0$ , and thus

$$Y_{1,g} = Y_{1,b}^e = Y_{1,b}^l = rD. \quad (42)$$

We now pin down investment and trade. If the state at  $t = 1$  is good, then all intermediaries are identical with resources  $R_0 I_0 - rD$ . There is no motive for trade since investment opportunities are identical, thus all intermediaries invest  $I_{1,g} = R_0 I_0 - rD$ . If the state at  $t = 1$  is bad, early intermediaries' purchase assets at price  $p_1$  according to (34). Combining this with (33) and the corresponding budget constraint we obtain:

$$I_{1,b}^e : q_{2,b}R_1 - C'(I_{1,b}^e) = \frac{q_{2,b}\theta R_0}{p_1}, \quad (43)$$

$$T_{1,b}^e = \frac{R_0(I_0 - S_0) + \alpha R_0 T_0 - rD - I_{1,b}^e}{p_1}. \quad (44)$$

If late intermediaries can raise sufficient funds to repay  $D$  and to invest at  $t = 1$ , the sales constraint does not bind and thus  $\mu_{1,T} = 0$ . In this case, the marginal returns on investment between early and late types are equalized and  $\alpha \lambda_{1,b}^e = (1 - \alpha) \lambda_{1,b}^l$ , so that  $I_{1,b} \equiv I_{1,b}^e = I_{1,b}^l$ .

When  $\mu_{1,T} > 0$ , late intermediaries are financially constrained at  $t = 1$ . Hence,  $I_{1,b}^e > I_{1,b}^l$  as late types can not generate enough funds to equalize investments. Using (35) and (36) we can then write

$$I_{1,b}^l = \min \left\{ I_{1,b}^e, (1-a)(\alpha R_0 + p_1(1-\alpha))I_0 - rD \right\} \quad (45)$$

$$S_{1,b}^l - T_{1,b}^l = \min \left\{ \frac{I_{1,b}^l + rD - \alpha R_0 T_0}{p_1}, (1-\alpha)(1-a)I_0 \right\}. \quad (46)$$

Using (29), (30), and (33)-(36) the multipliers can be written:

$$\mu_{1,T} = p_1(1-q_1)(1-\alpha) (C'(I_{1,b}^e) - C'(I_{1,b}^l)), \quad (47)$$

$$\mu_{1,S} = \frac{\mu_{1,T}}{p_1} ((1-\alpha)p_1 + \alpha R_0). \quad (48)$$

To solve for the optimal choices at  $t = 0$ , note that the budget constraint binds, so that  $\lambda_0 > 0$ . Using (38) and (31), it is clear that if debt is positive (Lemma 2), then no cash is held. We can then recover the quantity of debt via the budget constraint for given  $I_0$ ,  $S_0$  and  $T_0$  so that:

$$Y_0 = 0, \quad (49)$$

$$D = I_0 + p_0(T_0 - S_0) - k. \quad (50)$$

Using the above results, the optimal choices of  $I_0$ ,  $S_0$  and  $T_0$  are then characterized by the corresponding first-order conditions (28)-(30) when  $\mu_{1,T} = 0$ . When late intermediaries are constrained  $S_0 = (1-a)I_0$ , as shown in Lemma 4, and  $I_0$ ,  $T_0$  are characterized by (28) and (29).

## B For Online Publication: Proofs

### Proof of Lemma 2

*Proof.* To show that intermediaries hold no cash at  $t = 0$ , combine (31) with (39)-(41) to obtain  $\lambda_0 \leq r(\lambda_{1,g} + \lambda_{1,b}^e + \lambda_{1,b}^l)$ . Substituting this into (38) gives  $(1-r)(\lambda_{1,g} + \lambda_{1,b}^e + \lambda_{1,b}^l) < 0$  as  $r \geq \beta^{-1} > 1$  and thus  $Y_0 = 0$ .

We now show that  $D > 0$ , which is true as the marginal return to debt exceeds the cost at  $I_0 = k$ , where  $I_0$  is characterized by (16). Setting  $\mu_{1,S} = I_{1,b}^l = (q_{2,b}R_1 - 1 - C'(I_{1,b}^e)) = 0$ ,  $I_{1,g} = R_0k$ , and  $r = \beta^{-1}$ , yields the expression in Assumption 4, which is sufficient for  $D > 0$ . Finally, to show that  $D < w + k$  for sufficient  $w$ , rewrite the necessary condition for an optimal  $I_0$ , evaluated at  $I_0 = w + k$ .

$$E_\pi R_0 + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'((R_0 - \beta^{-1})(w + k) + R_0k)) + (1 - q_1)\alpha(R_0 - r)(q_{2,b}R_1 - 1 - C'(I_{1,b}^e)) + (1 - q_1)(1 - \alpha)(-r)(q_{2,b}R_1 - 1 - C'(I_{1,b}^l)) + \mu_{1,S}(1 - a) \leq r + \xi + C'(w + k) \quad (51)$$

Setting  $I_{1,b}^e = 0$ , and  $(1 - q_1)(1 - \alpha)(-r)(q_{2,b}R_1 - 1 - C'(I_{1,b}^l)) = 0$  gives:

$$E_\pi R_0 - r + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'((R_0 - \beta^{-1})(w + k) + R_0k)) + (1 - q_1)\alpha(R_0 - r)(q_{2,b}R_1 - 1) + \mu_{1,S}(1 - a) \leq r + \xi + C'(w + k). \quad (52)$$

If (52) holds, then (51) must hold as well. From the first order conditions, we have  $\mu_{1,S}(1 - a) = (1 - q_1)(1 - \alpha)[q_{2,b}R_1 - C'(I_{1,b}^l)]p_1 - q_{2,b}\theta R_0(1 - a)$ . Setting price at the upper bound  $p_1 = q_{2,b}\theta R_0$ , and  $I_{1,b}^l = 0$ , we bound  $\mu_{1,S}(1 - a) \leq (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0[q_{2,b}R_1 - 1](1 - a)$ . Note that the bound on  $\mu_{1,S}$  is not a function of  $w$ , while the other terms on the left hand side of (52) are strictly decreasing in  $w$ , and the right hand side is strictly increasing. Therefore, (52) must be strict for sufficiently large  $w$ . □

### Proof of Lemma 3

*Proof.* The expressions for  $I_{1,b}^e$ ,  $T_{1,b}^e$ ,  $I_{1,b}^l$ , and  $S_{1,b}^l - T_{1,b}^l$  are derived directly from the first-order conditions of the intermediaries' problem in Appendix A. Cash holding at time 1 is determined by the debt repayment constraints, which are all binding. To show this, combine (32) and (39) to obtain:

$$\eta_{1,g} = \lambda_{1,g} - q_1 \geq q_1(q_{2,g}R_1 - 1 - C'(I_{1,b}^e)) > 0,$$

(33) and (40) to obtain  $\eta_{1,b}^e \geq (1 - q_1)\alpha(q_{2,b}R_1 - 1 - C'(I_{1,b}^e)) > 0$ , and (35) and (41) to get  $\eta_{1,b}^l \geq (1 - q_1)(1 - \alpha)(q_{1,b}R_1 - 1 - C'(I_{1,b}^l)) > 0$ , where all inequalities follow from Assumption 2. □

## Proof of Lemma 4

*Proof.* Equations (33) and (34) yield

$$C'(I_{1,b}^e) = q_{2,b} \left( R_1 - \frac{\theta R_0}{p_1} \right). \quad (53)$$

Similarly, using (35) and (36) we have

$$C'(I_{1,b}^l) = q_{2,b} \left( R_1 - \frac{\theta R_0 + \frac{\mu_{1,T}}{(1-q_1)(1-\alpha)}}{p_1} \right). \quad (54)$$

Hence,  $\mu_{1,T} = 0 \Leftrightarrow I_{1,b}^e = I_{1,b}^l$ . Moreover, when  $\mu_{1,T} > 0$ ,  $C'(I_{1,b}^l) < C'(I_{1,b}^e) \implies I_{1,b}^e > I_{1,b}^l$  as  $C(\cdot)$  is convex. To show that investment must be positive in the constrained case, suppose  $I_{1,b}^l = 0$  and  $\mu_{1,S} = 0$ . From (35) we have  $\lambda_{1,b}^l = (1 - q_1)(1 - \alpha)q_{2,b}R_1$ , and thus from (37) we have  $\mu_{1,S} = (1 - q_1)(1 - \alpha)q_{2,b}(p_1 - \theta R_1)$ . Therefore  $p_1 > \theta R_1 > q_{2,b}\theta R_1$ , which implies the price exceeds NPV and thus  $I_{1,b}^l$  cannot be zero.

Adding (29), and (30) and dividing through by  $\alpha(1 - \alpha)$ , we obtain

$$\left[ \frac{\lambda_{1,b}^l}{1 - \alpha} - \frac{\lambda_{1,b}^e}{\alpha} \right] R_0 + \frac{\mu_{1,T}}{\alpha} \leq \frac{\mu_{1,S}}{\alpha(1 - \alpha)}. \quad (55)$$

Now note that the square-bracketed term is proportional to  $\Delta C \equiv C'(I_{1,b}^e) - C'(I_{1,b}^l)$  which is strictly positive when  $\mu_{1,T} > 0$  as  $I_{1,b}^e > I_{1,b}^l$ . Hence,  $\mu_{1,T} > 0 \implies \mu_{1,S} > 0$ . To show the reverse, assume  $\mu_{1,S} > 0$ . This implies that the marginal benefit of increasing  $S_0 + S_{1,b}^l$  is strictly positive. In other words, using the left-hand sides of (30) and (37) we have:

$$\begin{aligned} & - (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 - (\lambda_{1,g} + \lambda_{1,b}^e)R_0 - \mu_{1,S} + \lambda_0 p_0 \\ & - (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 + \lambda_{1,b}^l p_1 - \mu_{1,S} = 0. \end{aligned} \quad (56)$$

Using (36) and (29) we have:

$$\left[ \frac{\lambda_{1,b}^l}{1 - \alpha} - \frac{\lambda_{1,b}^e}{\alpha} \right] R_0 + \frac{\mu_{1,T}}{\alpha} \geq \frac{\mu_{1,S}}{\alpha(1 - \alpha)}. \quad (57)$$

Note that the LHS is always non-negative and strictly positive only when  $\mu_{1,T} > 0$ . Since the LHS is bounded below by a positive number as  $\mu_{1,S} > 0$ , it must be the case that  $\mu_{1,T} > 0$ . Finally, to show that  $S_0 = (1 - a)I_0$ , and  $S_{1,b}^l = 0$ , we add (28) and (30) to obtain  $[\lambda_0(p_0 - 1) - C'(I_0)]/a = \mu_{1,S}$ , when investment and sales are non-negative. This implies that  $p_0 > 1 + C'(I_0)/\lambda_0$ . Hence, it is profitable to invest and then sell assets at  $t = 0$ . Thus,  $S_0 = (1 - a)I_0$ , and therefore  $S_{1,b}^l = 0$  as there are no assets to sell at period 1.

□

## Proof of Proposition 1

*Proof.* Begin with the equilibrium at  $t = 1$ . At a price equal to NPV ( $p_1 = q_{2,b}\theta R_0$ ), demand is zero because early types have access to positive NPV investments, while supply is positive for the same reason and thus excess demand is negative. At  $p_1 = \theta R_0/R_1$ , demand is positive since buying assets dominates investment for early types, while supply is zero, and thus excess demand is positive. We conclude that an equilibrium in the asset market at  $t = 1$  exists since excess demand, characterized in (58), is a continuous function. Note that  $q_{2,b}\theta R_0 > \theta R_0/R_1 = q_{2,b}\theta R_0/q_{2,b}R_1$ , since  $q_{2,b}R_1 > 1$  by assumption.

Given arbitrary feasible  $t = 0$  choices  $I_0, T_0, S_0$  and  $D$ , excess demand at  $t = 1$  is

$$z(p_1) = \max \left\{ \frac{\alpha R_0(I_0 + T_0 - S_0) - rD - I_{1,b}}{p_1}, \alpha \frac{R_0(I_0 - S_0) + \alpha R_0 T_0 - rD - I_{1,b}^e}{p_1} - (1 - \alpha)^2 T_0 \right\}, \quad (58)$$

where optimal  $t = 1$  investment is characterized by (33)-(36) and we denote  $I_{1,b} = I_{1,b}^l = I_{1,b}^e$  in the unconstrained case. For prices at which sales are not constrained, we have

$$z'(p_1) = -\frac{dI_{1,b}}{dp_1} - \frac{\alpha R_0(I_0 + T_0 - S_0) - rD - I_{1,b}}{p_1^2} < 0, \quad (59)$$

where the inequality follows from  $dI_{1,b}/dp_1 = q_{2,b}\theta R_0/C''(I_{1,b}^e)p_1^2 > 0$  and the fact that  $\alpha R_0(I_0 + T_0 - S_0) - rD - I_{1,b} \geq 0$ , since aggregate investment cannot exceed aggregate resources at  $t = 1$ . When constrained,

$$z'(p_1) = -\alpha \frac{dI_{1,b}^e}{dp_1} + T_{1,b}^e < 0, \quad (60)$$

where  $dI_{1,b}^e/dp_1 = \frac{q_{2,b}\theta R_0}{C''(I_{1,b}^e)p_1^2} > 0$ . Since excess demand is monotone in either case, the equilibrium at  $t = 1$  is unique.

We now consider  $t = 0$ . From (29) and (30), it is clear that for sufficiently low  $p_0$ ,  $S_0 = 0$  and  $T_0 \geq 0$ . Alternatively, for sufficiently high  $p_0$ ,  $T_0 = 0$ , while  $S_0 \geq 0$ . Since excess demand is continuous, there exists an equilibrium  $p_0$  that clears the market at  $t = 0$ . From the first order conditions on the intermediary problem we have

$$p_0 = \frac{a\mu_{1,S}^* + \xi + C'(I_0^*)}{\lambda_0} + 1, \quad (61)$$

for non-negative values of  $T_0$  and  $S_0$ . In the unconstrained case,  $p_0 = (\xi + C'(I_0))/\lambda_0 + 1$ , which is simply the marginal cost of investment. This price must obtain at the equilibrium since there is no collateral motive for securitized assets, which implies that supply is zero for any price lower and demand is zero for any price higher. In this case,  $p_0 = (\xi + C'(I_0))/\lambda_0 + 1$  and any value of  $S_0 = T_0 \in [0, (1 - a)I_0]$  represents an equilibrium. If the equilibrium is constrained, the price is determined by (61) and  $S_0 = T_0 = (1 - a)I_0$ , by Lemma 4. To pin

down the value of  $I_0^*$ , note that in equilibrium  $S_0^* = T_0^*$  so that from (28) we have that  $I_0^*$  is characterized as follows:

$$E_\pi R_0 + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'(I_{1,g}) + (1 - q_1)(\alpha R_0(q_{2,b}R_1 - 1 - C'(I_{1,b}^e)) - r(q_{2,b}R_1 - 1 - C'(I_{1,b}^l)) + (1 - q_1)\alpha r(C'(I_{1,b}^e) - C'(I_{1,b}^l)) + \mu_{1,S}(1 - a) = r + \xi + C'(I_0). \quad (62)$$

The solution to this equation exists due to Assumption 4 and is unique as the second-order condition follows from  $C''(\cdot) > 0$ . In the unconstrained case, the optimal choice of  $I_0$  is independent of  $S_0$  and  $T_0$  and is uniquely characterized by the following first-order condition:

$$E_\pi R_0 + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'(I_{1,g})) + (1 - q_1)(\alpha R_0 - r)(q_{2,b}R_1 - 1 - C'(I_{1,b})) = r + \xi + C'(I_0). \quad (63)$$

Finally, we show that parameters exist such that the equilibrium may be constrained. Posit an unconstrained equilibrium. From (33)-(36), we have

$$p_1 = \frac{q_{2,b}\theta R_0}{q_{2,b}R_1 - C'(I_{1,b})}, \quad (64)$$

where  $I_{1,b} = \alpha R_0 I_0 - rD$  must hold to satisfy the aggregate resource constraint at the equilibrium. Substituting (64) into (10) gives aggregate supply

$$(1 - \alpha)(S_{1,b}^l - T_{1,b}^l) = (1 - \alpha) \cdot \frac{\alpha R_0(I_0 - T_0)(q_{2,b}R_1 - C'(I_{1,b}))}{q_{2,b}\theta R_0}. \quad (65)$$

If parameters exist such that (65) exceeds  $(1 - \alpha)^2(1 - a)I_0$ , then the equilibrium can not be unconstrained. Using  $I_0 - T_0 \geq aI_0$ ,  $k + w > I_0$ , and simplifying yields the following sufficient condition for aggregate supply to exceed  $(1 - \alpha)^2(1 - a)I_0$ :

$$\frac{\alpha a}{q_{2,b}\theta(1 - a)(1 - \alpha)} \cdot [q_{2,b}R_1 - C'(\alpha R_0(k + w) - \beta^{-1}w)] > 1. \quad (66)$$

Since  $q_{2,b}R_1 - C'(\alpha R_0(k + w) - \beta^{-1}w) > 1$  due to Assumption 2, we can further simplify to obtain the following sufficient condition:

$$\left(\frac{\alpha}{1 - \alpha}\right) \left(\frac{a}{1 - a}\right) > q_{2,b}\theta. \quad (67)$$

Substituting  $a = \frac{\xi}{(1 - q_1)(1 - \alpha)q_{2,b}\theta R_0}$ , we have

$$\frac{\alpha}{1 - \alpha} > q_{2,b}\theta \left(\frac{(1 - q_1)(1 - \alpha)q_{2,b}\theta R_0}{\xi} - 1\right). \quad (68)$$

□

## Proof of Lemma 5

*Proof.* We first outline the salient features of the solution to the planning problem described in (18)-(22). It is straightforward to show that all budget and debt repayment constraints bind, that no cash reserves are held at  $t = 0$  and the interior solution is unique since  $-C''(\cdot) < 0$ . Combining the first order conditions on  $t = 1$  investment in the bad state gives

$$(1 - q_1)(q_{2,b}R_1 - C'(I_{1,b}^e)) = (1 - q_1)(q_{2,b}R_1 - C'(I_{1,b}^l)) \implies I_{1,b}^e = I_{1,b}^l = I_{1,b}. \quad (69)$$

Furthermore,

$$I_{1,g} = (R_0 - r)I_0 + rk, \quad (70)$$

$$I_{1,b} = (\alpha R_0 - r)I_0 + rk, \quad (71)$$

where we have used  $D = I_0 - k$ . Investments at  $t = 0$  and  $t = 1$  are thus related as follows:

$$\begin{aligned} E_\pi R_0 - r + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'(I_{1,g})) + (1 - q_1)(\alpha R_0 - r)(q_{2,b}R_1 - 1 - C'(I_{1,b})) \\ = \xi + C'(I_0). \end{aligned} \quad (72)$$

The planner equates the marginal cost of investment at  $t = 0$  with the marginal benefit of investment across the states at  $t = 1$ , where investments are equalized across intermediary types in the low state.

## Contingent Securities

We now introduce contingent securities traded at  $t = 0$ , conditional on individual type at  $t = 1$  when the state is bad (there is no gains from trading securities that pay off when the state is good since all intermediaries are identical in this state). Further, there is no motive for trade at  $t = 1$  and we ignore this possibility. The security pays the owner one unit, conditional on the bad state if no early returns have been realized. Denote by  $\zeta_l$  and  $\zeta_e$  the quantities of this security purchased or sold, and  $\rho_0$  the corresponding price. The intermediaries' problem is:

$$\begin{aligned} \Pi_0 = q_1[q_{2,g}R_1I_{1,g} - C(I_{1,g}) + Y_{1,g}] + (1 - q_1)(\alpha[q_{2,b}R_1I_{1,b}^e - C(I_{1,b}^e) + Y_{1,b}^e] \\ + (1 - \alpha)[q_{2,b}R_1I_{1,b}^l - C(I_{1,b}^l) + Y_{1,b}^l]) + (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0I_0 - \xi I_0 - C(I_0) - rD, \end{aligned} \quad (73)$$

subject to:

$$(\lambda_0) I_0 + \rho_0(\zeta_l - \zeta_e) + Y_0 \leq k + D, \quad (74)$$

$$(\lambda_{1,g}) I_{1,g} + Y_{1,g} \leq R_0I_0 + Y_0, \quad (75)$$

$$(\lambda_{1,b}^e) I_{1,b}^e + Y_{1,b}^e \leq R_0I_0 - (1 - \alpha)\zeta_e + Y_0, \quad (76)$$

$$(\lambda_{1,b}^l) I_{1,b}^l + Y_{1,b}^l \leq -\alpha\zeta_l + Y_0, \quad (77)$$

$$(\eta_{1,g}) rD \leq Y_{1,g}, \quad (78)$$

$$(\eta_{1,b}^e) rD \leq Y_{1,b}^e, \quad (79)$$

$$(\eta_{1,b}^l) rD \leq Y_{1,b}^l. \quad (80)$$



All budget constraints bind,  $Y_0 = 0$ , and all collateral constraints will bind, so that  $Y_{1,g} = Y_{1,b}^e = Y_{1,b}^l = rD$ . We can then re-write the objective and obtain the following necessary optimality conditions:

$$I_0 : \lambda_{1,g}(R_0 - r) + \lambda_{1,b}^e(R_0 - r) + \lambda_{1,b}^l(-r) - \xi = 0, \quad (81)$$

$$I_{1,g} : q_1 [q_{2,g}R_1 - C'(I_{1,g})] = \lambda_{1,g}, \quad (82)$$

$$I_{1,b}^e : (1 - q_1)\alpha [q_{2,b}R_1 - C'(I_{1,b}^e)] = \lambda_{1,b}^e, \quad (83)$$

$$I_{1,b}^l : (1 - q_1)(1 - \alpha) [q_{2,b}R_1 - C'(I_{1,b}^l)] = \lambda_{1,b}^l, \quad (84)$$

$$\zeta_l : q_1(q_{2,g}R_1 - C'(I_{1,g}))(-rp_0) + \lambda_{1,b}^e(-rp_0) + \lambda_{1,b}^l(\alpha - rp_0) = 0, \quad (85)$$

$$\zeta_e : q_1(q_{2,g}R_1 - C'(I_{1,g}))(rp_0) + \lambda_{1,b}^e(rp_0 - (1 - \alpha)) + \lambda_{1,b}^l(rp_0) = 0. \quad (86)$$

Using (85) and (86), we obtain  $\lambda_{1,b}^e/\alpha = \lambda_{1,b}^l/(1 - \alpha)$ , which substituted into the first order conditions for  $t = 1$  investment yield  $I_{1,b}^e = I_{1,b}^l = I_{1,b}$ . Thus, trading securities at  $t = 0$  permits agents to equate the marginal returns to investment at  $t = 1$ . Write the foc for  $I_0$  after re-arranging terms as follows:

$$\begin{aligned} E_\pi R_0 - r + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'(I_{1,g})) + (1 - q_1)(\alpha R_0 - r)(q_{2,b}R_1 - 1 - C'(I_{1,b})) \\ = \xi + C'(I_0). \end{aligned} \quad (87)$$

This is identical to (72), hence the market allocation when Arrow securities are traded is Pareto efficient.  $\square$

## Proof of Proposition 2

*Proof.* Differentiating the Lagrangian for the intermediaries' problem with respect to aggregate  $I_0$  yields

$$\frac{d\Pi_0}{dI_0} = \frac{dp_1^*}{dI_0} \left[ \frac{\mu_{1,T}^*}{p_1^*} (1 - \alpha)(1 - a)I_0^* \right], \quad (88)$$

using the market clearing condition  $\alpha T_{1,b}^{e*} = -(1 - \alpha)T_{1,b}^{l*}$ . This is equation (23) in Section 4.2, which we provide here for convenience. Since the price effect in (88) is non-zero, the equilibrium is constrained inefficient whenever  $\mu_{1,T} > 0$ .

To show that intermediaries value securitization only when the competitive equilibrium is inefficient, note that when unconstrained,  $\mu_{1,S} = \mu_{1,T} = 0$ , and  $I_{1,b}^e = I_{1,b}^l = I_{1,b}$ . In this case the allocation in the competitive equilibrium coincides with the planning solution. To see this, note that in equilibrium  $S_0^* = T_0^*$ , as the securitization market must clear at  $t = 0$ . This implies that  $I_0^* = D^* + k$ , where  $I_0^*$  is pinned down by the first-order condition (16). When intermediaries are not constrained, this condition is identical in equilibrium to (72), the corresponding condition for the planner. Importantly, the determination of  $I_0^*$  is independent of  $p_1^*$ ,  $S_0^*$  and  $T_0^*$  as  $I_{1,g}^* = R_0 I_0^* - \beta^{-1} D^*$ ,  $I_{1,b}^* = \alpha R_0 I_0^* - \beta^{-1} D^*$ . Thus, securitization has no value when intermediaries are unconstrained as its use cannot improve welfare. When intermediaries are constrained, securitization has value as  $I_0^*$  is then indirectly a function of  $p_1$  via (16) as  $t = 1$  investment is a function of  $p_1$ .  $\square$

### Proof of Proposition 3

*Proof.* It is straightforward to show that a constrained equilibrium is not possible for a sufficiently small value of  $a$ . In an unconstrained equilibrium,  $S_0$ ,  $S_{1,b}^l$  and  $I_0$  are not functions of  $a$ . Thus, assuming  $S_0 > 0$ , the sales constraint  $S_0 + S_{1,b}^l \leq (1-a)I_0$  implies that there is a unique  $0 < \bar{a} < 1$  such that the intermediary is constrained whenever  $a > \bar{a}$ .  $\square$

### Proof of Corollary 1

*Proof.* From (23),  $\text{sign}(d\Pi_0^*/dI_0) = \text{sign}(dp_1^*/dI_0)$ . Multiplying the market clearing condition at  $t = 1$  by  $p_1^*$  and noting that  $T_{1,b}^{l*} = -(1-\alpha)T_0^* = -(1-\alpha)(1-a)I_0^*$  due to the binding sales constraint implies that  $p_1\alpha T_{1,b}^{e*} = p_1(1-\alpha)^2(1-a)I_0^*$ . Differentiating with respect to  $I_0$  noting that  $S_0^* = T_0^*$  gives:

$$\frac{dp_1^*}{dI_0} = \frac{\alpha((a + \alpha(1-a))R_0 - 1/\beta) - (1-\alpha)^2(1-a)p_1^*}{\alpha \frac{q_{2,b}\theta R_0}{C'''(I_{1,b}^{e*})p_1^{*2}} + (1-\alpha)^2(1-a)I_0^*}. \quad (89)$$

The denominator is strictly positive since  $\frac{q_{2,b}\theta R_0}{C'''(I_{1,b}^{e*})p_1^{*2}} > 0$  as  $C''(\cdot) > 0$ . Thus, the sign of  $dp_1^*/dI_0$  is determined solely by the numerator. First, we characterize parameters to support the over-investment case. This case occurs when

$$p_1^* > \frac{\alpha((a + \alpha(1-a))R_0 - 1/\beta)}{(1-\alpha)^2(1-a)}. \quad (90)$$

Assume that (67) holds so that in equilibrium  $\mu_{1,T} > 0$ . Then, a sufficient condition for over-investment is that  $a + \alpha(1-a) \leq \frac{1}{\beta R_0} \alpha R_0 + (1-\alpha)aR_0 \leq 1/\beta$  which ensures the LHS of (90) is 0. As  $p_1^* \geq \theta R_0/R_1 > 0$  in equilibria with positive investment at  $t = 1$ , the condition ensures over-investment in equilibrium.

Under-investment occurs when the inequality in (90) is reversed. From (65) we have the following upper bound on the price in a constrained equilibrium:

$$p_1^* < \bar{p}_c = \frac{\alpha a R_0}{(1-\alpha)(1-a)}. \quad (91)$$

Therefore, a sufficient condition for under-investment is

$$\frac{\alpha a R_0}{(1-\alpha)(1-a)} < \frac{\alpha((a + \alpha(1-a))R_0 - 1/\beta)}{(1-\alpha)^2(1-a)} \iff \alpha R_0 > 1/\beta, \quad (92)$$

which is the expression in Corollary 1. Note that as (67) and the conditions for over- and under-investment must hold simultaneously, the former may be viewed as restrictions on  $R_0$  or  $\beta$ .  $\square$

## Proof of Proposition 4

*Proof.* Differentiating welfare with respect to  $\Delta$  at the optimum we have

$$\frac{d\Pi_0^*}{d\Delta}\Big|_{\Delta=0} = -\mu_{1,S}I_0^* - \frac{dp_1^*}{d\Delta} [\lambda_{1,b}^{e*}T_{1,b}^{e*} + \lambda_{1,b}^{l*}(T_{1,b}^{l*} - S_{1,b}^{l*})] \quad (93)$$

$$= \frac{\mu_{1,T}^*I_0^*}{p_1^*} \left[ (1-\alpha)(1-a)\frac{dp_1^*}{d\Delta} - \alpha R_0 - (1-\alpha)p_1^* \right], \quad (94)$$

where we have used (48), and  $T_0^* = S_0^* = (1-a-\Delta)I_0^*$ . We ignore the price effects arising from changes in  $p_0^*$ , since intermediaries have net zero sales at  $t=0$ , and thus changes in  $p_0^*$  do not alter the budget and in turn do not affect investment or welfare. The  $t=1$  market-clearing condition,  $\alpha T_{1,b}^{e*} = -(1-\alpha)T_{1,b}^{l*}$  can be expressed as:

$$\alpha \frac{((a+\alpha(1-a))R_0 - 1/\beta)I_0^* + \beta^{-1}k - I_{1,b}^{e*}}{p_1^*} = (1-\alpha)^2(1-a-\Delta)I_0^*. \quad (95)$$

Denote  $M = (1-\alpha)^2(1-a)p_1 - \alpha((a+\alpha(1-a))R_0 - \beta^{-1})$ , where  $M > 0$  is the condition for over-investment derived in the proof of Corollary 1. Differentiating both sides of (95) with respect to  $\Delta$  and solving for  $dp_1^*/d\Delta$  we have:

$$\frac{dp_1^*}{d\Delta}\Big|_{\Delta=0} = \frac{\alpha R_0 + (1-\alpha)p_1^*}{(1-\alpha)(1-a)} \cdot \left[ \frac{(1-\alpha)(1-a) \left( (1-\alpha)I_0^* - \frac{M}{\alpha R_0 + (1-\alpha)p_1^*} \frac{\partial I_0^*}{\partial \Delta} \right)}{M \frac{\partial I_0^*}{\partial p_1} + \frac{\alpha q_{2,b}\theta R_0}{p_1^{*2}C''(I_{1,b}^{e*})} + (1-\alpha)^2(1-a)I_0^*} \right], \quad (96)$$

where we have used  $\frac{dI_{1,b}^{e*}}{d\Delta} = \frac{q_{2,b}\theta R_0}{p_1^{*2}C''(I_{1,b}^{e*})} \frac{dp_1^*}{d\Delta}$  and  $\frac{dI_0^*}{d\Delta} = \frac{\partial I_0^*}{\partial \Delta} + \frac{\partial I_0^*}{\partial p_1} \frac{dp_1^*}{d\Delta}$ . Define the partial derivatives as follows:

$$\frac{\partial I_0^*}{\partial \Delta} = \frac{(\alpha R_0 + (1-\alpha)p_1)(1-q_1)(1-\alpha) (C'(I_{1,b}^{l*}) - C'(I_{1,b}^{e*}) + C''(I_{1,b}^{l*})M_l I_0^*)}{q_1(R_0-r)^2C''(I_{1,g}^*) + (1-q_1)(1-\alpha)C''(I_{1,b}^{l*})M_l^2 + C''(I_0^*)}, \quad (97)$$

$$\frac{\partial I_0^*}{\partial p_1} = \frac{-\frac{M_e(1-q_1)\alpha q_{2,b}\theta R_0}{p_1^2} + (1-q_1)(1-\alpha)^2(1-a)(q_{2,b}R_1 - C'(I_{1,b}^{l*}) - C''(I_{1,b}^{l*})M_l I_0^*)}{q_1(R_0-r)^2C''(I_{1,g}^*) + (1-q_1)(1-\alpha)C''(I_{1,b}^{l*})M_l^2 + C''(I_0^*)}, \quad (98)$$

$$= \frac{\frac{\mu_{1,T}(1-a)(1-\alpha)}{p_1} + \frac{M\lambda_{1,b}^{l*}}{\alpha p_1} - C''(I_{1,b}^{l*})M_l I_0^*}{q_1(R_0-r)^2C''(I_{1,g}^*) + (1-q_1)(1-\alpha)C''(I_{1,b}^{l*})M_l^2 + C''(I_0^*)}, \quad (99)$$

where

$$M_e = aR_0 + \alpha(1-a)R_0 - \beta^{-1}, \quad (100)$$

$$M_l = (1-a)(\alpha R_0 + (1-\alpha)p_1) - \beta^{-1}. \quad (101)$$

Then,  $d\Pi_0^*/d\Delta > 0$  when the following is true

$$\frac{dp_1^*}{d\Delta} > \frac{\alpha R_0 + (1-\alpha)p_1^*}{(1-a)(1-\alpha)} \iff \Omega \equiv \left[ \frac{(1-\alpha)(1-a) \left( (1-\alpha)I_0^* - \frac{M}{\alpha R_0 + (1-\alpha)p_1^*} \frac{\partial I_0^*}{\partial \Delta} \right)}{M \frac{\partial I_0^*}{\partial p_1} + \frac{\alpha q_{2,b}\theta R_0}{p_1^{*2}C''(I_{1,b}^{e*})} + (1-\alpha)^2(1-a)I_0^*} \right] > 1. \quad (102)$$

Using  $q_{2,b}R_1 - \frac{q_{2,b}\theta R_0}{p_1^*} = C'(I_{1,b}^{e*})$  from the first order conditions, rewrite  $\Omega$  as

$$\Omega = \frac{\epsilon}{\epsilon + \phi}, \quad (103)$$

where

$$\epsilon = (1 - \alpha)^2(1 - a) \left[ I_0^* + \frac{(1 - q_1)M}{\delta} \left( C'(I_{1,b}^{e*}) - C'(I_{1,b}^{l*}) - C''(I_{1,b}^{l*})M_l I_0^* \right) \right], \quad (104)$$

$$\phi = \frac{q_{2,b}\theta R_0}{p_1^*} \left( \frac{(1 - q_1)(1 - \alpha)^2(1 - a)M}{\delta} + \frac{\alpha}{p_1^* C''(I_{1,b}^{e*})} - \frac{MM_\epsilon(1 - q_1)\alpha}{\delta p_1^*} \right), \quad (105)$$

$$\delta = (1 - q_1)(1 - \alpha)C''(I_{1,b}^{l*})M_l^2 + C''(I_0^*) > 0. \quad (106)$$

Manipulation gives

$$\phi = \frac{(1 - q_1)q_{2,b}\theta R_0}{p_1\delta} \left( \frac{M^2}{p_1} + \frac{\alpha\delta}{p_1 C''(I_{1,b}^{e*})} \right) > 0. \quad (107)$$

$M_l < 0$  in the over-investment case. To see this, note that  $M_l$  is strictly increasing in  $p_1$ . Evaluating  $M_l$  at the maximum constrained price  $\bar{p}_c = a\alpha R_0/(1 - \alpha)(1 - a)$ , which is described in (91). This gives  $M_l(\bar{p}_c) = \alpha R_0 - \beta^{-1}$ , which must be negative in the over-investment case as shown in the proof of Corollary 1. Thus, both  $\epsilon$  and  $\phi$  are strictly positive and  $\Omega < 1$ , implying  $d\Pi_0^*/d\Delta < 0$ .

Finally, if  $M > 0$  and  $M_l < 0$ , then  $\partial I_0^*/\partial\Delta < 0$  and  $\partial I_0^*/\partial p_1 > 0$ . From (96), it follows that  $dp_1/d\Delta > 0$ .

□